N5 Mechanical Drawing & Design

Gateways to Engineering Studies - Chris Brink

HYBRID LEARNING SOLUTIONS

Gateways to Engineering Studies

Mechanical Drawing & Design

N5

Chris Brink

Published by
Hybrid Learning Solutions (Pty) Ltd

Email: urania@hybridlearning.co.za

ISBN: 978-1-928203-97-1

Editor: Urania Bellos
Proofreader: Urania Bellos
Book design: Sarah Buchanan
Cover design: Sarah Buchanan
Artwork: Wendi Wise / Sarah Buchanan

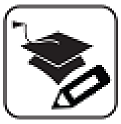
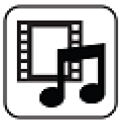
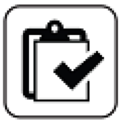
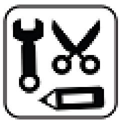
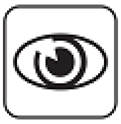
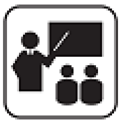
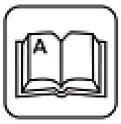
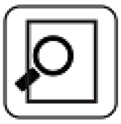
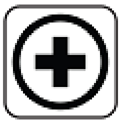
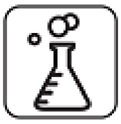
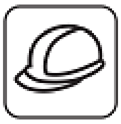
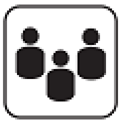
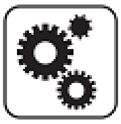
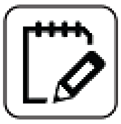
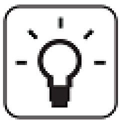
Printed and bound by: Formsxpress

Acknowledgements
Every effort is being made to trace the copyright holders. In the event of unintentional omissions or errors, any information that would enable the publisher to make the proper arrangements will be appreciated.

Table of Contents

Icons used in this book

We use different icons to help you work with this book; these are shown in the table below.

Icon	Description	Icon	Description
	Assessment / Activity		Multimedia
	Checklist		Practical
	Demonstration/ observation		Presentation/ Lecture
	Did you know?		Read
	Example		Safety
	Experiment		Site visit
	Group work/ discussions, role-play, etc.		Take note of
	In the workplace		Theoretical – questions, reports, case studies, etc.
	Keywords		Think about it

Introduction to Design Principles

Learning Outcomes
On the completion of this module the student must be able to:

- Describe the principles of practical design techniques
- Describe the general consideration and procedure affecting design
- Describe the general procedure in design

1.1 Introduction

In this subject it is not necessary to memorise any formula or method of calculation. You are merely expected to know where to find a certain formula which will be applicable to a certain problem, and how to solve this problem correctly.

Note:
During the examination you will be allowed to consult your personal notes and text books; therefore keep your notes neat and tidy and learn to use them quickly and correctly during your studies.

1.2 Principles of practical design techniques

To design is to take decisions. Thus, when the designer is faced with a certain practical problem, he has to rely on his own knowledge (theoretical as well as practical) to solve it.

In practice, we nowadays find, for example, more than one kind of water pump, and although they serve the same purpose, namely, to pump water, their designs may be completely different.

We can thus come to the conclusion that for every problem in design technique there is a different approach, because different decisions are taken by the designers.

The study of mechanical design is therefore the consideration of different factors which determine the mechanical arrangement of the parts in a machine, and such an arrangement again determines the sizes, form and material of the part concerned.

Think about it!
A bad decision will obviously result in a poor design and product.

Otherwise, the principle of decision-making is a principle of meeting a compromise, for example, any designer's aim is to use the strongest material for his design, but this material could be too expensive and uneconomical, consequently he must compromise and use a weaker material, with the result that the size, and perhaps the form of the part will have to be altered.

Each problem you will be faced with can roughly be approached as follows.

1.2.1 Preparation and analysis
Analyse the problem and make sure that you understand what is required. Gather the information in a logical sequence.

1.2.2 Synthesis
Gather all possible information and approach the problem from different angles. For this purpose you will make certain assumptions.

1.2.3 Test
Test your result in the light of various practical applications. For example decide whether the particular part will fit and whether it will perform the work economically.

1.2.4 Modification
Modify in case the test fails.

After the various parts for a machine have been manufactured, they have to be assembled; thus, during the designing process, the designer must make sure that:
- the various parts fit together correctly and precisely
- the parts can be assembled easily and quickly by the factory personnel
- parts which must be replaced during maintenance, are within easy reach
- adjustment points are fitted in convenient places

The design of a machine or structure implies full responsibility of the designer for complete drawings as part of the specifications, for the buildings of models if necessary, for suggestions, advice and inspection during erection, and, finally, the release of the finished machine to the owner.

After a machine has been tested under working conditions, as a critical proof test of what be expected in service, it is considered to be a marketable product.

The arrangements for building similar machines in large numbers is an engineering problem of "production" and the important factors involved are closely related to the following considerations, although not all of them necessarily apply to each design problem.

1.3 General considerations and procedure affecting design

- The determination of the motion of parts, or kinematics of the machine.
- The selection of materials from which the machine is to be constructed.
- The determination of the form and size of the machine parts.
- A study of the frictional resistance of moving parts and the means of lubrication.
- A study of convenient-and economical features in the operation and maintenance of the machine.
- A consideration of the employment of standard parts.
- A consideration of the-safety of the operator of the machine.
- A study of the facilities of the shop in which the machine or structure is to be fabricated.
- A consideration of the number of articles to be manufactured.
- A study of the cost of construction and the cost of operation.
- A study of the assembling of parts for the finished machine.
- A consideration of the transportation of the machine.

1.3.1 Kinematics

Geometry, trigonometry and the calculus are used to determine the location of centres of rotation and changes in the position of parts. The velocities and displacements should be thoroughly worked out, with no thought at this stage of the work as to materials, form, size and strength of parts.

1.3.2 Selection of materials

It is essential that a general knowledge of the properties of materials and their behaviour under working conditions should serve as a guide to their proper selection.

Some of the important characteristics of materials are:

- strength
- durability
- flexibility
- mass
- resistance to heat and corrosion
- ability to be cast, welded, or hardened
- machinability
- electrical conductivity
- insulating capacity
- cost.

Brass, for instance, may be used in place where cast iron would be used if it were more resistant to corrosion.

At high speeds non-metallic gears are used to reduce the noise caused by metal gears. Usually unlike metals are used to reduce friction and ensuing wear, but an exception to this is the use of cast iron on cast-iron.

A notable example of research into industrial applications in design is the discovery of high-speed tool steel which increased production by as much as 500 per cent. A later development is stellite cemented tungsten carbide and similar alloys for cutting edges.

Definition: Stellite alloy
A range of cobalt-chromium alloys designed for wear resistance. It may also contain tungsten or molybdenum and a small but important amount of carbon. It is a trade marked name of the Kennametal Stellite Company and was invented by Elwood Haynes in the early 1900s as a substitute for cutlery that stained (or that had to be constantly cleaned).

1.3.3 Form and size of parts
Some parts of a machine require little, if any, consideration of strength.

In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain, and it is important to anticipate any suddenly applied or impact load, which might cause failure.

Normal Loads are often exceeded by conditions which are outside the usual range, and while only momentarily applied, they dictate the working unit stress which should be used.

An example of this is the starting of a machine under full load, in which case the forces it is subjected to are considerably greater than those required for normal running.

Did you know?
The necessity of reducing vibrations by absorption may require that the machine frames be made heavier than would be required if only strength were considered.

Each part of a machine should be the simplest resistant member that will safely withstand the stresses imposed by the load, and the general shape should conform to what usage and tradition have prescribed for machine parts.

1.3.4 Frictional resistance and lubrication
There is always a loss of power owing to frictional resistance, and it should be noted that the friction of starting is higher than running friction. Careful attention should be given to the matter of lubricating all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

The selection of the proper provision for lubrication will often radically influence the design of a machine. The drawings should indicate the size and location of oil holes and oil grooves for all parts that require them.

1.3.5 Convenient and economical factors

Facility for adjustment for wear must be provided, employing various take up devices and arranging them so that alignment of parts will be preserved.

If parts have to be changed for different products or replaced on account of wear of breakage, easy access should be provided, and the necessity of removing other parts to accomplish this should be avoided if possible.

Think about it!
The economical operation of a machine should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

1.3.6 Use of standard parts

This is closely related to costs, and a study of this matter may avoid needless expense, since the cost of standard or stock parts is frequently only a fraction of the cost of similar parts made to order.

Standard or stock parts should be used whenever possible: parts for which patterns are already in existence, such as gears, pulleys, and bearings; and parts which may be selected from regular shop stock, such as screws, nuts and pins.

Note:
The use of special bolts, studs and pins should be avoided. Bolts and studs should be of as few sizes as possible so as to avoid the delay caused by changing drills, reamers and taps.

1.3.7 Safety of operation

The design engineer does not always have a free hand in this matter, because safety devices add to the cost of the machine, but it is his duty to provide for the safety of the operator to the greatest possible extent.

Any moving part of a machine that is within the zone of a worker is considered an accident hazard, and may be the cause of an injury. Investigation has shown that three-quarters of all accidents are preventable; half of them by the removal of the hazards and half by safety education.

Note:
Safety devices should in no way interfere with the operation of the machine, slow up production, or inconvenience the worker unnecessarily.

1.3.8 Shop facilities

The design engineer should be familiar with the limitations of his employer's shop, in order to avoid the necessity of having work done in some other shop.

Think about it!
It is sometimes necessary to plan and supervise shop operations and to draft methods for casting, handling and machining special parts.

1.3.9 Number of articles to be manufactured

This affects the design in a number of ways. The engineering and shop costs which are called fixed charges or "overhead expenses" are distributed over the number of articles which are manufactured, so that if only a few are to be made, extra expense is not-justified, unless the machine is large or of special design, and therefore the designer, must, as far as possible, restrict his specification to standard parts.

1.3.10 Cost of construction and operation

Many of the matters already discussed have a direct bearing on cost, and this is one of the most important considerations involved in design. In some cases it is quite possible that the high cost of an article would immediately bar it from further consideration.

Under all conditions the design engineer should use all his skill in an endeavour to reduce the cost of the following items:

- design
- material
- shop processes
- assembling
- testing
- transportation
- the up-keep of the machine in the hands of the purchaser.

1.3.11 Assembling and transportation

Every machine must be assembled as a unit before it can function. Large units often have to be assembled in the shop, tested, and then taken apart to be transported to their place of service. Large flywheels are made in sections, partly to meet the restriction of transportation.

1.4 General procedure in design

The first consideration in the design of any machine is to understand the requirements of the problem. Design in general, involves the application of known engineering principles to certain problems.

Did you know?

No engineering design ever failed where correct theory was used by the designer, because correct theory must agree throughout with correct practice.

The design engineer must be certain that his assumptions are correct and his data as complete as possible before proceeding with the design.

1.4.1 Rational design

When mathematics can be employed to determine the form and size of parts, the design is called a rational design; but unfortunately rational design cannot be applied to the solution of all problems.

When rational design is used, the size of the parts should be calculated from the forces to which the part will be subjected, and on the basis of safe limits of stress intensity. Machines in which rigidity is of prime importance often do not lend themselves to mathematical treatment, except as a check.

In general, they are designed to give satisfactory operating results, and, considering strength only, some parts may have various degrees of oversize.

1.4.2 Empirical design

For cases in which rational design cannot be applied, the design engineer should make use of the proportions of a similar machine which may have been developed by a process of evolution.

Did you know?
Empirical design is the result of using data derived from machines and designs in actual use, and such information is usually tabulated in various handbooks for ready reference.

In empirical design, a survey of machines similar to the one contemplated, leads to the incorporation of the good features and the avoidance of the bad features of existing machines.

If an actual machine cannot be studied, recourse should be had to the technical press, catalogues, photographs or verbal descriptions.

It is always legitimate to take advantage of the experience which others have gained in the same field, and in this way costly errors may often be avoided.

1.4.3 Designing by experience

The early years of employment of design engineers are often spent in a number of plants, and this results in first-hand experience in a broad field of design.

The designer's mind is stocked with information, factual data and principles of procedure, and when he wishes to use a certain machine element, he recalls the material, form and dimensions from former association and then adapts them to the problem.

Before the development of the principles of mechanics as applied to strength of materials, all design was necessarily based on experience.

Activity 1.1

1. Name five general considerations and methods of procedure affecting design. Discuss each one briefly.
2. Write brief notes on:
   2.1 The form and size of parts.
   2.2 Frictional resistance and lubrication.
   2.3 The use of standard parts.

Self-Check

I am able to:	**Yes**	**No**
• Describe the principles of practical design techniques		
• Describe the general consideration and procedure affecting design		
• Describe the general procedure in design		

If you have answered ‘no’ to any of the outcomes listed above, then speak to your facilitator for guidance and further development.

Module 2

Ultimate and Working Stress

Learning Outcomes
On the completion of this module the student must be able to:

- Describe the types of stress
- Describe bending and twisting
- Demonstrate stress calculations
- Describe shear stress
  - Single
  - Double
- Describe crushing (bearing) on a curved surface
- Describe the behaviour of materials under a tensile test
- Describe working stress
  - Describe factors of safety

2.1 Introduction

Ultimate strength is the point at which a structure will fail. Working stress is the point at which the designed structure will operate at full stress load. This is usually a factor of safety of 3 to 5 times less than the ultimate strength.

If a load on a beam will be expected to handle a working stress of 23 000 kg with potential shock loading of 68 000 kg then the ultimate strength of the beam to use should be 340 000 kg with a factor of safety designed in of 5 times the maximum shock load. This factor of safety is often enough to prevent failure of the structure resulting from fatigue and corrosion.

2.2 Stress

Consider a bar (say steel) subjected to a pull P, as shown in **Figure 2.1**. P is called the load, and is measured in Newtons.

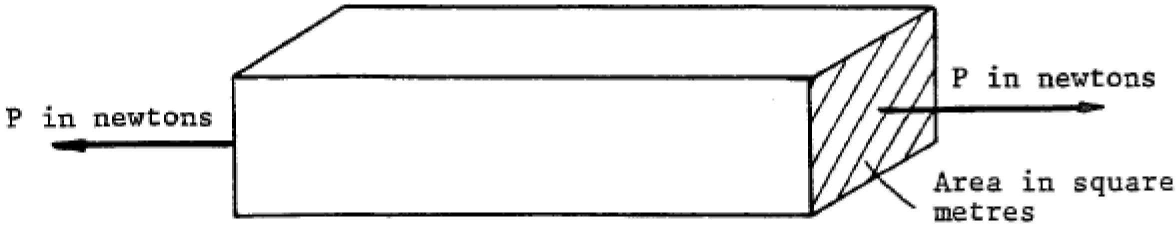


Figure 2.1

This load tends to stretch the bar, and is resisted by the adhesion between the particles of the material. This resistance is called stress.

If the bar has a cross-sectional area of A square metres, then the intensity of stress (σ) in the material is $\frac{P}{A}$ newtons per square metre (N/m²). The word "intensity" is usually omitted, and the word "stress" is taken to mean "intensity of stress".

⚠	**Important Note!** Load and stress are two different things. **Load** is the **force** applied, and is measured in **Newtons**. **Stress** is the **resistance** to the load, and is measured as $\frac{Load}{Cross-sectional\ area}$ newtons per square metre (N/m²).

2.3 Types of stress

There are four main types of stress caused by four different ways of loading, namely:

2.3.1 Tension

When the load tends to stretch the material.

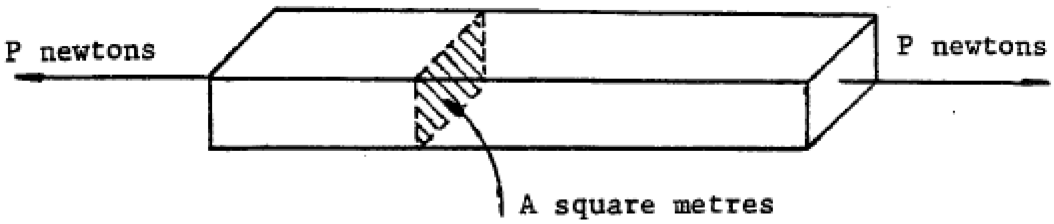


Figure 2.2

Tensile load = P newtons
Cross-sectional area = A square metres
Tensile stress = σ_t newtons per square metres

Tensile stress = $\frac{Tensile\ load}{Cross-sectional\ area}$

$\sigma_t = \frac{P}{A} N/m^2$

2.3.2 Compression

When the load tends to shorten the body.

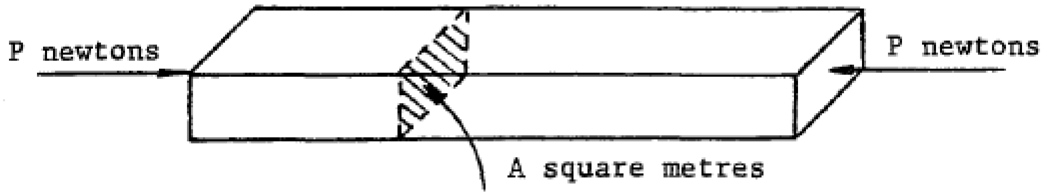


Figure 2.3

Compressive load = P newtons
Cross-sectional area = A square metres
Compressive stress = σ_c newtons per square metres

Compressive stress = $\frac{Compressive\ load}{Cross-sectional\ area}$

$\sigma_c = \frac{P}{A} N/m^2$

2.3.3 Shear

When the load tends to slide (shear), one portion of the body over the other.

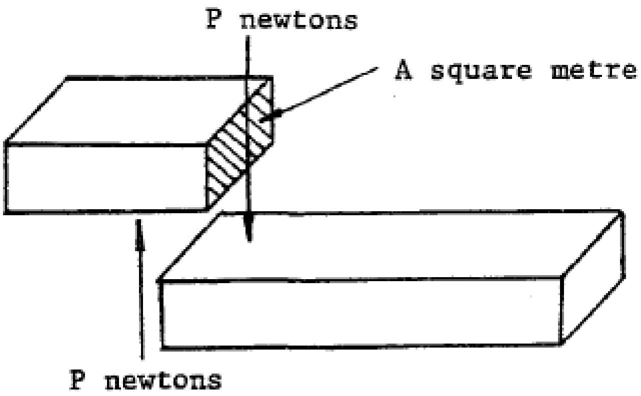


Figure 2.4

Shear load = P newtons
Cross-sectional area = A square metres
Shear stress = τ newtons per square metres

Shear stress = $\frac{Shear\ load}{Shear\ area}$

$\tau = \frac{P}{A}\ N/m^2$

2.3.4 Crushing

When two pieces of material are pressed together and there is a tendency to squeeze them out over the area of contact.

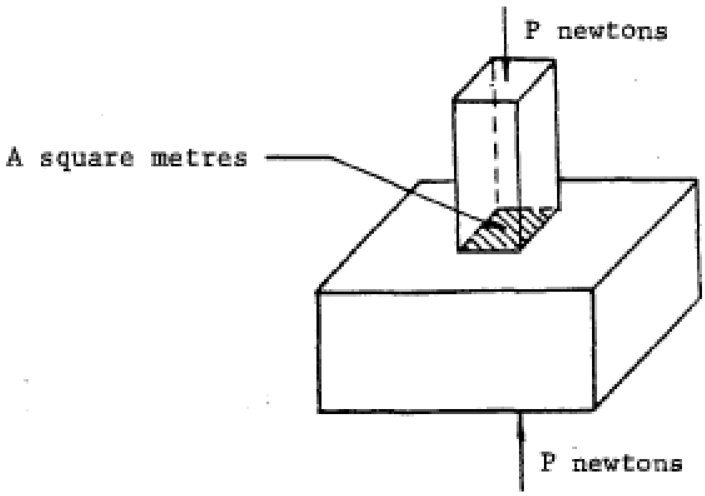


Figure 2.5

Crushing load = P newtons
Cross-sectional area = A square metres
Crushing stress = σ_c newtons per square metres

Shear stress = $\frac{Crushing\ load}{Area\ of\ contact\ between\ two\ pieces}$

$\sigma_c = \frac{P}{A}\ N/m^2$

2.4 Bending and twisting

In addition to the above four ways in which a piece of material may be loaded, it can also be bent or twisted. These two ways however, do not produce special stresses.

When a bar is bent, one side is put into tension and the other side into compression, as shown in **Figure 2.6**. Bending is not dealt with in this course.

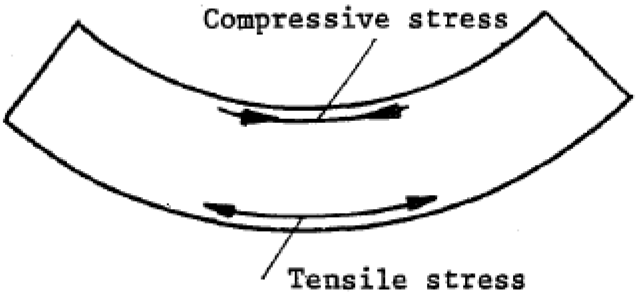


Figure 2.6

When a piece of material (say a shaft) is twisted, each cross-section tends to slide over the adjacent one and a shear stress is produced. This will be dealt with later.

2.5 Calculations

One of the main essentials of design is speed and accuracy in calculations. It is of the utmost importance that the student should understand what is meant by "accuracy". A problem which asks for the necessary diameter of shaft will end up, say, as follows:
Diameter of shaft= 93,726 mm (make it 95 mm).

The fact that the nearest practical size is taken in the final answer does not, however, mean that wild approximations can be taken in the intermediate working.

On the other hand, it is not necessary to work things out to a long string of figures.

A good general rule is to work all calculations to three significant figures. The following examples illustrate what is meant by "correct to three significant figures".

Full Number	Correct to three significant figures
2576.15	2580
17.423	17.4
3061.243	3060
0.014678	0.0147
0.00032134	0.000321

In the worked examples in this and the following lectures, note how all intermediate steps are worked out. This is much better than carrying on long string of fractions and working out the final answer only.

Note:
The units (newton, metre, etc.) are stated at each step. This must be done in all examples worked out.

Worked Example 2.1

A tie-rod of 60 mm diameter carries a pull of 180 kN. Find the tensile stress in the material.

Note: A tie-rod is a member designed to carry tension only. If used in compression, it would buckle, owing to its slenderness.

Solution:

Cross-sectional area of rod $= \frac{\pi d^2}{4}$

$= \frac{\pi \times (0,06)^2}{4}$

Tensile stress $= \frac{Tensile\ load}{Cross\text{−}sectional\ area}$

$= \frac{180 \times 10^3 N}{0{,}00283 m^2}$

$= 63\ 604\ 240\ N/m^2$

Say $= 63{,}6 \times 10^6 N/m^2$

$= 63{,}6\ MPa$

Note:
1 N/m² equals 1 pascal (Pa).

Worked Example 2.2

What diameter mild steel rod is required to carry a pull of 120 kN, if the tensile stress in the steel is not to exceed 62 MPa?

Solution:

Tensile stress $= \frac{Tensile\ load}{Cross\text{-}sectional\ area}$

∴ Cross-sectional area $= \frac{Tensile\ load}{Tensile\ stress}$

$= \frac{120 \times 10^3 N}{62 \times 10^6\ N/m^2}$

$= 0{,}001936\ m^2$

But cross-sectional area $= \frac{\pi d^2}{4}$

Now $\frac{\pi d^2}{4}$ = 0,001936 m²

$d^2 = \frac{0{,}001936 \times 4}{\pi} m^2$

$= 0{,}00246\ m^2$

$d = \sqrt{0{,}00246\ m^2}$

= 0,0496 say 50 mm diameter rod

Worked Example 2.3

What is the maximum Load that can be carried by a hollow cast iron column of outside diameter 254 mm and inside diameter 200 mm, if the compressive stress in the material is not to exceed 52 MPa?

Solution:

Cross-sectional area of column $= \frac{\pi}{4}(D^2 - d^2)$

where D = outside diameter

and d = inside diameter

Cross-sectional area of column $= \frac{\pi}{4}(0{,}254^2 - 0{,}2^2)$

= 0,0193 m²

Compressive stress $= \frac{Compressive\ load}{Cross-sectional\ area}$

∴ Compressive load = Compressive stress x Cross-sectional area

$= 52 \times 10^6\ N/m^2 \times 0{,}0193\ m^2$

= 1004 N

Worked Example 2.4

A steel rod is 30 mm diameter for part of its length and 45 mm diameter for the remainder. A pull is applied, which produces a tensile stress of 77 MPa in the thinner portion. Calculate this pull and the stress in the thicker portion.

Solution:

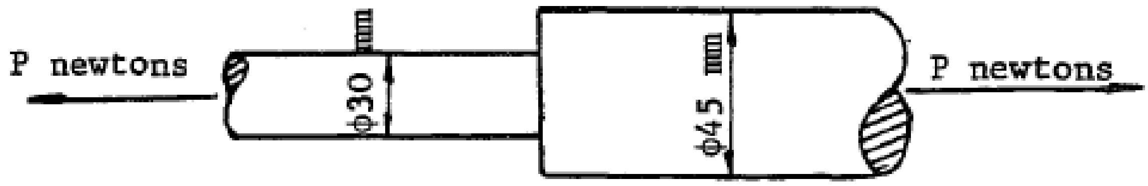


Figure 2.7

30 mm Diameter portion

Cross-sectional area $= \frac{\pi d^2}{4}$

$= \frac{\pi \times (0{,}03\text{m})^2}{4}$

$= 7{,}069 \times 10^{-4}\text{m}^2$

Tensile stress $= \frac{\text{Tensile load}}{\text{Cross-sectional area}}$

Tensile load = Tensile stress x Cross-sectional area

$= 77 \times 10^6\text{N/m}^2 \times 7{,}069 \times 10^{-4}\text{m}^2$

$= 54{,}4 \times 10^3\text{N}$

$= 54{,}4\ \text{kN}$

This is the pull P applied to the 30 mm diameter portion, and therefore it must also be applied to the 45 mm portion, since the pull at one end of the bar cannot be greater than that at the other end.

45 mm diameter portion

Cross-sectional area $= \frac{\pi d^2}{4}$

$= \frac{\pi \times (0{,}045\text{m})^2}{4}$

$= 1{,}59 \times 10^{-3}\text{m}^2$

Tensile stress $= \frac{\text{Tensile load}}{\text{Cross-sectional area}}$

$= \frac{54{,}4 \times 10^3 N}{1{,}59 \times 10^{-3} m^2}$

$= 34{,}22\ MPa$

Note:
The load on each portion of the rod is the same, but that the stresses are different.

Worked Example 2.5

In order to punch a 22 mm diameter hole in a steel plate, a force of 500 kN is required on the punch.

Determine:
(a) the crushing stress on the punch.
(b) The maximum thickness of plate that can be punched if the shear stress in the material is 300 MPa.

<u>Solution:</u>

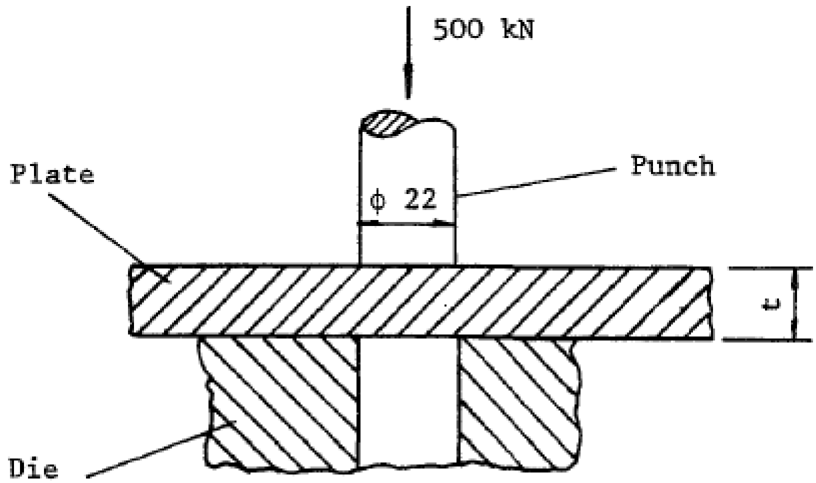


Figure 2.8

(a) Area of contact between punch and plate $= \frac{\pi d^2}{4}$

$= \frac{\pi \times (0{,}022\text{m})^2}{4}$

$= 3{,}8\ \times 10^{-4}\text{m}^2$

Crushing stress $= \frac{Crushing\ load}{Area\ of\ contact\ between\ punch\ and\ plate}$

$= \frac{500 \times 10^3 N}{3{,}8 \times 10^{-4} m^2}$

$= 1316\ MPa$

(b) Shear area = circumference of hole x thickness

$= \pi\text{d} \times \dagger$

$= 3{,}14\ \times 0{,}022\ \text{m} \times \dagger$

$= 0{,}0691\ m\ \times \dagger$

$$\text{Shear stress} = \frac{Sheer\ load}{Shear\ area}$$
$$\therefore \text{Shear area} = \frac{Sheer\ load}{Shear\ stress}$$
$$0{,}0691\ m \times \dagger = \frac{500 \times 10^3 N}{300 \times 10^6 N/m^2}$$
$$= 1{,}667 \times 10^{-3} m^2$$
$$\dagger = \frac{1{,}667 \times 10^{-3} m^2}{0{,}0691\ m}$$
$$= 0{,}0241\ m$$
$$= 24{,}1\ mm$$

2.6 Single and double shear

The following two sketches (**Figure 2.9** and **Figure 2.10**) illustrate the difference between single and double shear:

2.6.1 Single shear

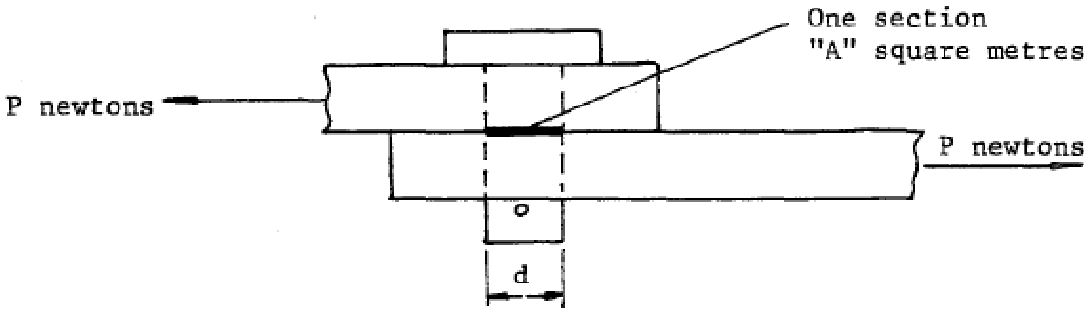


Figure 2.9

In the case of single shear, the load is borne by one section of the pin and the

$$\text{Shear stress} = \frac{Shear\ load}{Cross-sectional\ area}$$
$$\tau = \frac{P}{A}\ N/m^2$$
$$= \frac{P}{\frac{\pi d^2}{4}}\ N/m^2$$

2.6.2 Double shear

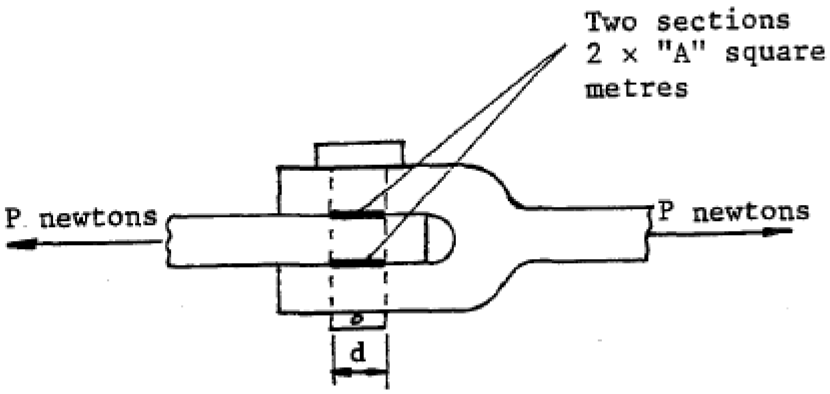


Figure 2.10

In the case of double shear, the load is borne by two sections of the pin and the

$$\text{Shear stress} = \frac{Shear\ load}{Cross\text{-}sectional\ area}$$

$$\tau = \frac{P}{2 \times A}\ N/m^2$$

$$= \frac{P}{2 \times \frac{\pi d^2}{4}}\ N/m^2$$

2.7 Crushing (or bearing) on a curved surface

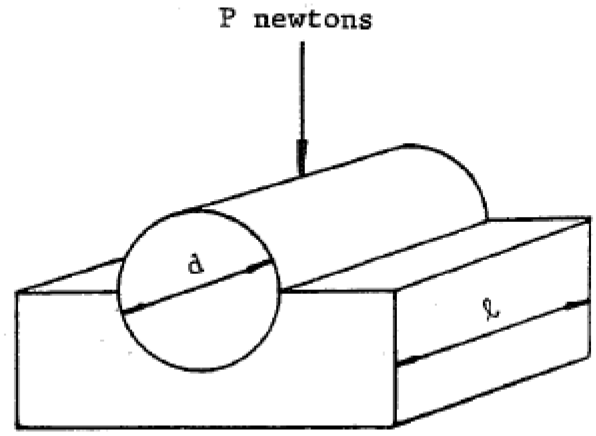


Figure 2.11

The actual area of contact between the two objects (**Figure 2.11**) is the curved surface $\frac{\pi d}{2} \times l$. For various reasons the pressure between the surfaces will not be uniform. It will be greater at the bottom of the groove than at the sides.

If therefore, the load is divided by the curved surface area, the answer obtained will be too small. In order to achieve a more reasonable result the area used in design is the "projected" area, ie $d \times l$, and we have

$$\text{Crushing or Bearing stress} = \frac{Bearing\ load}{Projected\ area}$$

$$\sigma_c = \frac{P}{d \times l}\ N/m^2$$

Did you know?
The word "crushing" is generally used when there is no movement between the surfaces, and "bearing" when the cylinder can turn in the groove.

2.8 Behaviour of materials under a tensile test

When a gradually increasing tensile load is applied to a bar of material (say mild steel), it stretches. At first, when the load, and hence the stress, is low, the stretch is very small, and a special apparatus, called an extensometer, must be used to measure it.

Definitions:

Up to a certain point the material is **elastic**, that is, if the load is removed, all the stretch will disappear and the bar will return to its original length. The stress in the material at this point is called the **elastic limit**.

Just past the elastic limit the stretch suddenly becomes comparatively large, and this is called the **yield point**. If the load is removed after the elastic limit or yield point have been passed, it will be found that not all the stretch disappears. **"Permanent set"** is said to have occurred, and the material is no longer perfectly elastic.

If further load is applied, the bar continues to stretch in comparatively large amounts until no more load can be applied.

The bar narrows down (or **"waists"**) at some point and very soon breaks. This maximum load that can be applied is called the **ultimate load**, and the corresponding stress is called the **ultimate stress** or **ultimate strength** of the material.

Figure 2.12

The foregoing is a description of the behaviour of mild steel under a tensile test. Other materials behave more or less in the same way. Some have no marked yield point, while others are elastic almost up to the ultimate stress and break suddenly with no waisting.

2.9 Working stress, factor of safety

In designing a part of a machine or structure it is obvious that the stress in the material must not be permitted to reach anywhere near the ultimate stress.

In fact, it must not be allowed to reach even the elastic limit, otherwise when the load was applied permanent deformation would occur and the part would be useless.

Actually it would not even be safe to design to a stress slightly below the elastic limit, since any miscalculation or unexpected load might cause an excessive stress. Also, if the part is to be subjected to repeated or fluctuating loads, the material would suffer from fatigue and failure would eventually occur.

Definition: Fatigue
A weakening of the material as a result of a change in its structure in the form of a type of crystallization.

For a number of reasons, therefore, the stress in a material must be kept well below the ultimate stress, and such a stress is called the safe or working stress. It is found by dividing the ultimate stress by a figure called the ***Factor of Safety***.

Thus we have:

$$\text{Safe or working stress} = \frac{Ultimate\ stress}{Factor\ of\ safety}$$

$$\text{Also, since stress} = \frac{Load}{Area}, \text{ we have}$$

$$\text{Safe or working stress} = \frac{Safe\ or\ working\ load}{Area}$$

$$\text{And Ultimate stress} = \frac{Ultimate\ load}{Area}$$

In the foregoing discussion we have referred particularly to tensile stresses. The same applies in most respects to compression, shear and crushing, and in each case a factor of safety must be used to fix the safe working stress to be adopted.

2.10 Choice of factor of safety

The factor of safety used in practice depends upon a large number of considerations, some of which are:-

- Consequences of failure - if someone is liable to be killed or injured a high factor must be used.
- Nature of load - steady, alternating, or shock. An alternating load causes fatigue in the material, and a shock load may be hard to estimate.
- Accuracy with which loads may be estimated. If much guess work is involved, a high factor must be used.
- Reliability of materials. Castings may have blow holes in them and temperature stresses as a result of irregular cooling; timber may have weak spots.
- Length of time member has to last. This applies particularly to wearing parts. If they are to last a long time, the crushing or bearing stresses must be kept low.
- Amount of care and attention the object will receive. Compare the bearings of a generating set in a power station operating under ideal conditions, with those of a windmill which is probably greased only when they squeak.

These, and many other items, must be considered when deciding what factor or safety (and hence what working stresses) should be used in a design. Such a decision can be made only with personal practical experience or by using the experience of others as published in various engineering handbooks.
At this level you are not expected to have such experience, and the factor of safety or the working stress to be used, will always be specified.

Worked Example 2.6

Two steel plates, each 20 mm thick, are joined together by a Lap joint with 3 24-mm-diameter rivets. Find the shear stress in the rivets and the crushing stress between the rivets and the plates when the joint is carrying a pull of 105 kN

<u>Solution:</u>

Load carried by each rivet $= \frac{105 \times 10^3}{3} N$

$= 35$ kN

Cross-sectional area of rivet $= \frac{\pi d^2}{4}$

$= \frac{\pi \times (0{,}024\ m)^2}{4}$

$= 4{,}524 \times 10^{-4} m^2$

Since it is a lap joint, the rivets are in single shear.

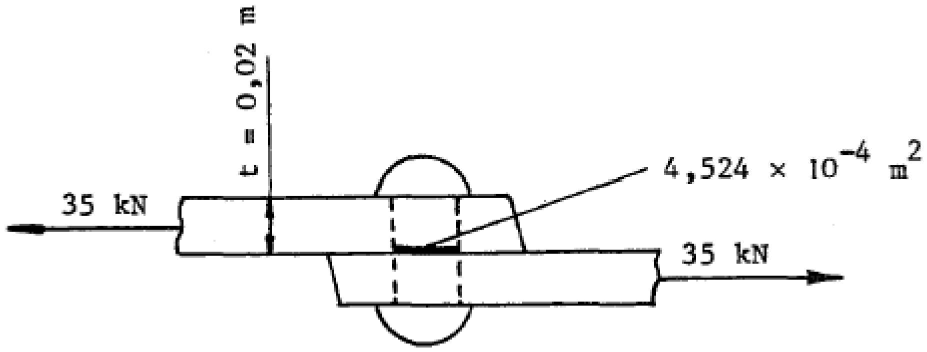


Figure 2.13

Shear stress on rivet $= \frac{Shear\ load}{Cross-sectional\ area}$

$= \frac{35 \times 10^3 N}{4{,}524 \times 10^{-4} m^2}$

$= 77{,}4$ MPa

Note that the contact area between the rivet and the plates is a curved surface, therefore the projected area must be used. Also the pull of 35 kN comes on to half the length of the rivet, and so the projected area of contact is diameter of rivet x thickness of plate.

Projected area of contact $= d \times$ †

= 0,024 m x 0,02 m

$= 4{,}8\ x\ 10^{-4} m^2$

Crushing stress $= \frac{Crushing\ load}{Projected\ area}$

$= \frac{35 \times 10^3 N}{4{,}8 \times 10^{-4} m^2}$

$= 72{,}9$ MPa

Worked Example 2.7

A knuckle joint has the following dimensions: Diameter of pin 20 mm, width of eye 22 mm, width of forks 12 mm.

When this joint is carrying a pull of 50 kN, determine:
1. the shear stress in the pin;
2. the bearing stress between the. pin and the eye;
3. the bearing stress between the pin and the fork.

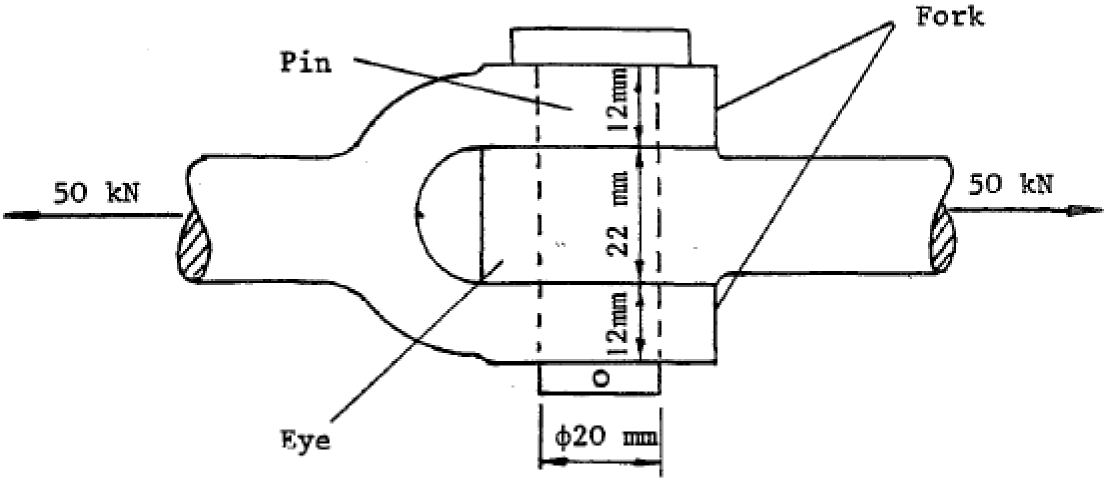


Figure 2.14

Solution:

1. The pin is in double shear

$$\text{Shear stress} = \frac{Shear\ load}{Cross\text{–}sectional\ area}$$
$$= \frac{P}{2 \times A}\ N/m^2$$
$$= \frac{50 \times 10^3 N}{2 \times \frac{\pi}{4} \times (0{,}02\ m)^2}$$
$$= 79{,}6 \text{ MPa}$$

2. The contact area between the pin and the eye is a curved surface, 20 mm in diameter and 22 mm in length.

$$\text{Projected area of contact} = d \times l$$
$$= 0{,}02 \text{ m x } 0{,}022 \text{ m}$$
$$= 4{,}4\ x\ 10^{-4} m^2$$

$$\text{Bearing stress} = \frac{Bearing\ load}{Projected\ area}$$
$$= \frac{50 \times 10^3 N}{4{,}4 \times 10^{-4} m^2}$$
$$= 113{,}6 \text{ MPa}$$

3. The contact area between the pin and the fork is two curved surfaces, each 20 mm in diameter and 12 mm long.

Projected area of contact $= 2 \times d \times l$
$= 2 \times 0{,}02\ m \times 0{,}012\ m$
$= 4{,}8 \times 10^{-4} m^2$

Bearing stress $= \frac{Bearing\ load}{Projected\ area}$
$= \frac{50 \times 10^3 N}{4{,}8 \times 10^{-4} m^2}$
$= 104{,}2$ MPa

Worked Example 2.8

What diameter mild steel tie-rod would you use to carry a load of 150 kN if the ultimate tensile stress of mild steel is 496 MPa and a factor of safety of 4 is to be used? What load would break the rod you are using?

<u>**Solution:**</u>

Working stress $= \frac{Ultimate\ stress}{Factor\ of\ safety}$
$= \frac{496 \times 10^6 N/m^2}{4}$
$= 124$ MPa

Working stress $= \frac{Working\ load}{Cross\text{-}sectional\ area}$

$\therefore$ Cross-sectional area $= \frac{Working\ load}{Working\ stress}$

$= \frac{150 \times 10^3 N}{124 \times 10^6 N/m^2}$
$= 1{,}21 \times 10^{-3} m^2$

But area $= \frac{\pi d^2}{4}$

$\therefore \frac{\pi d^2}{4} = 1{,}21 \times 10^{-3} m^2$

$d^2 = \sqrt{\frac{1{,}21 \times 10^{-3} m^2 \times 4}{\pi}}$

d $= \sqrt{1{,}541 \times 10^{-3} m^2}$

d $= 0{,}0393$ m

Say 40-mm-diameter rod

Using a 40 mm diameter rod:

Cross-sectional area $= \frac{\pi d^2}{4}$
$= \frac{\pi \times (0{,}04\ m)^2}{4}$
$= 1{,}257 \times 10^{-3} m^2$

Ultimate stress $= \frac{Ultimate\ load}{Cross-sectional\ area}$

∴ Ultimate load = Ultimate stress x Cross-sectional area

$= 496\ \times\ 10^6 N/m^2 \times 1{,}257 \times\ 10^{-3} m^2$

= 623,3 kN

Worked Example 2.9

A mild steel bar, 20 mm square, is tested in shear, and is found to break under a Load of 156 kN. One 24-mm-diameter rivet of the same material is used in a lap joint to carry a load of 35 kN. What factor of safety does this represent?

Solution:

Shear stress $= \frac{Shear\ load}{Cross-sectional\ area}$

$= \frac{156\ \times 10^3 N}{0{,}02\ m \times 0{,}02\ m}$

= 390 MPa

Since 156 kN is the load which *broke* the bar, it follows that 390 MPa is the *ultimate* stress of the material. The rivet is in single shear (see **Figure 2.9**).

Cross-sectional area of rivet $= \frac{\pi d^2}{4}$

$= \frac{\pi \times (0{,}04\ m)^2}{4}$

$= 4{,}524 \times 10^{-4} m^2$

Working stress in rivet material $= \frac{Shear\ load}{Cross-sectional\ area}$

$= \frac{35 \times 10^3 N}{4{,}524 \times 10^{-4} m^2}$

= 77, 4 MPa

The rivet material, therefore, has an ultimate stress of 390 MPa and a working stress of 77,4 MPa.

Working stress $= \frac{Ultimate\ stress}{Factor\ of\ safety}$

∴ Factor of safety $= \frac{Ultimate\ stress}{Working\ stress}$

$= \frac{390 \times 10^6 Pa}{77{,}4 \times 10^6 Pa}$

= 5

Activity 2.1

1. A length of steel tubing with an outside diameter of 30 mm and an inside diameter of 22 mm is subjected to an axial pull of 15 kN. Determine the stress in the steel.
2. What diameter mild steel tie rod is required to carry a pull of 150 kN if the tensile stress in the steel is not to exceed 60 MPa?
3. A Length of shafting has diameters of 50 mm, 57 mm and 70 mm at various places. It carries a compressive Load, which produces a stress in the 57 mm diameter portion of 80 MPa. Determine the stresses in the other two portions.
4. A short column consists of two sections welded together as shown. Both sections are square and hollow. The Lower section is 150 mm square outside and 140 mm square inside. If the compressive stress in the lower section is 24MPa, determine the load on the column. Also determine the inside measurement of the upper section so that the stress in it will be the same as in the lower section.

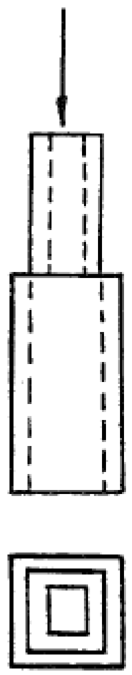


Figure 2.15

5. A knuckle joint has to carry a pull of 80 kN, and the shear stress in the pin is not to exceed 85 MPa. What diameter pin must be used?
   If the eye is 28 mm wide and the forks 16 mm wide, what will be the crushing stresses between the pin and the eye and between the pin and the fork when the joint is carrying the 80 kN pull?
6. A steel tie-bar, 25 mm in diameter, was designed to carry a pull of 50 kN with a factor of safety of 4. Determine the working stress in the bar and the ultimate tensile stress of the steel.
   If this bar is replaced by one of 20 mm in diameter of the same material, what load can it carry if the factor of safety is to remain the same?

7. The maximum pressure on a 380-mm-diameter piston of a steam engine is 820 kPa.
8. Calculate the required piston rod diameter when the ultimate stress of the material equals 275 MPa and a factor of safety of 5 is used.

Self-Check

I am able to:	**Yes**	**No**
• Describe the types of stress		
• Describe bending and twisting		
• Demonstrate stress calculations		
• Describe shear stress		
o Single		
o Double		
• Describe crushing (bearing) on a curved surface		
• Describe the behaviour of materials under a tensile test		
• Describe working stress		
• Describe factors of safety		
If you have answered 'no' to any of the outcomes listed above, then speak to your facilitator for guidance and further development.		

Module 3

Lap and Butt Joints

Learning Outcomes
On the completion of this module the student must be able to:

- Describe rivet heads
- Describe types of riveted joints
- Describe the design of riveted joints
    - Methods of failure of single-riveted lap joints
    - Working stress to be used if not given
    - Strength of solid plate
    - Efficiency of riveted joints
- Describe the methods of failure of double-riveted lap joints
- Describe butt joints
- Describe lozenge joints
- Describe riveted joints on high pressure cylinders
    - Thin cylinders
    - Joints for cylindrical pressure vessels, ie boilers and tanks
- Describe fasteners
    - Threaded
    - Proportions of ISO thread
    - Designation of ISO screws
    - Specifications
    - Bolts and studs in tension
    - Bolts in shear
    - Length of threaded part on studs and tap bolts
    - Covers for steam engine cylinders
    - Covers for inspection holes
    - Design of bolts for fixing covers to steam engine cylinders and manholes
- Describe studs or bolts used for steam cylinder covers and manhole doors
    - Number
    - Size
    - Pitch

3.1 Introduction

Rivets, known as permanent fasteners, are an effective means of joining plates and steel sections. Plate work such as boilers, air receivers, tanks, ship's plates, etc. Steel sections are used in bridgework, headgears, steel-framing of all types, and reinforcing for large construction works.

Usually only the thickness (t) of the plate or in steelwork the load (F) to be carried, is given.

From this known data we can design an effective joint as follows:
- Size of rivet (d).
- Pitch of rivets (p).
- Distance between rows of rivets (Pr).
- Distance from edge of plate to centre of rivets (y).
- Type of joint (if not stated).

3.2 Rivet heads

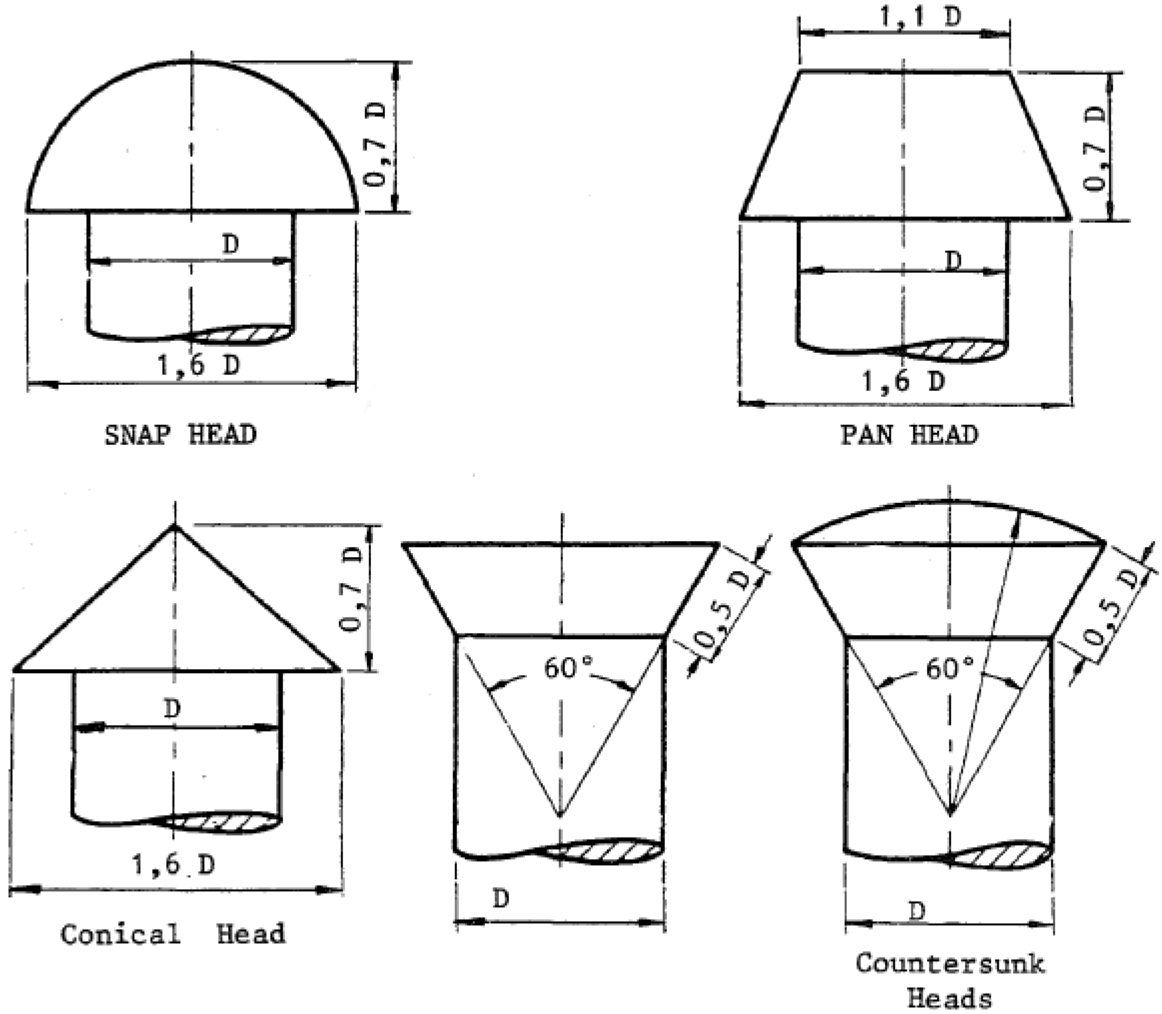


Figure 3.1

No absolutely definite sizes can be given for rivet heads, since they depend to some extent on the manufacturer.

The following proportions shown in **Figure 3.1** are suitable for sketching.

3.3 Types of riveted lap joints

Joints are classified as either lap joints or butt joints.

3.3.1 Lap Joints

In lap joints, one plate overlaps the other and the rivets pass through both plates.

3.3.1.1 Single-riveted lap joint

The two plates overlap and are joined by means of one row of rivets.

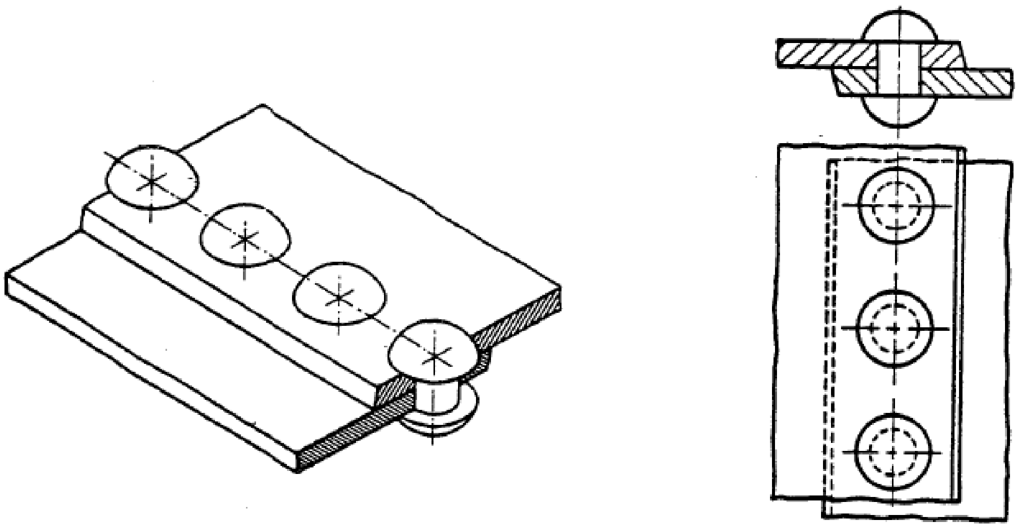


Figure 3.2

3.3.1.2 Double-riveted lap joint (chain riveting)

The two plates overlap and are joined by means of two rows of rivets. The rivets are placed next to each other.

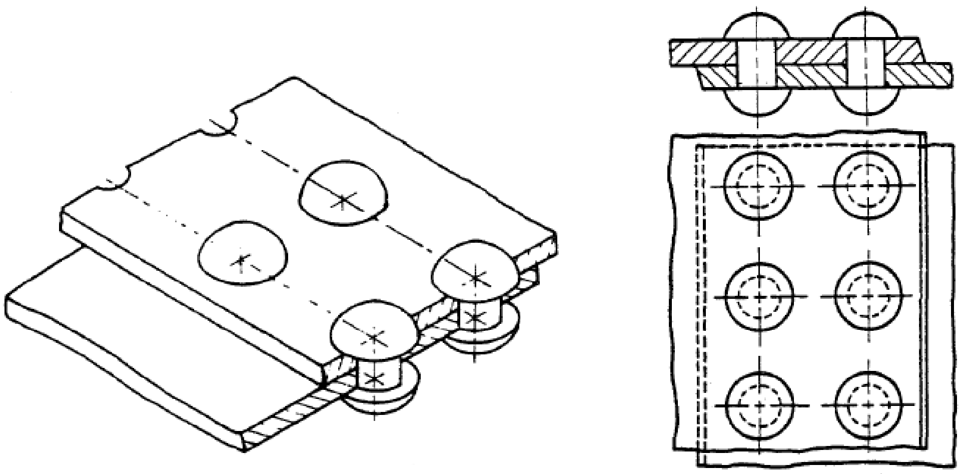


Figure 3.3

3.3.1.3 Double-riveted lap joint (zig-zag riveting)

The two plates overlap and are joined by means of two rows of rivets. The rivets are staggered in the plates.

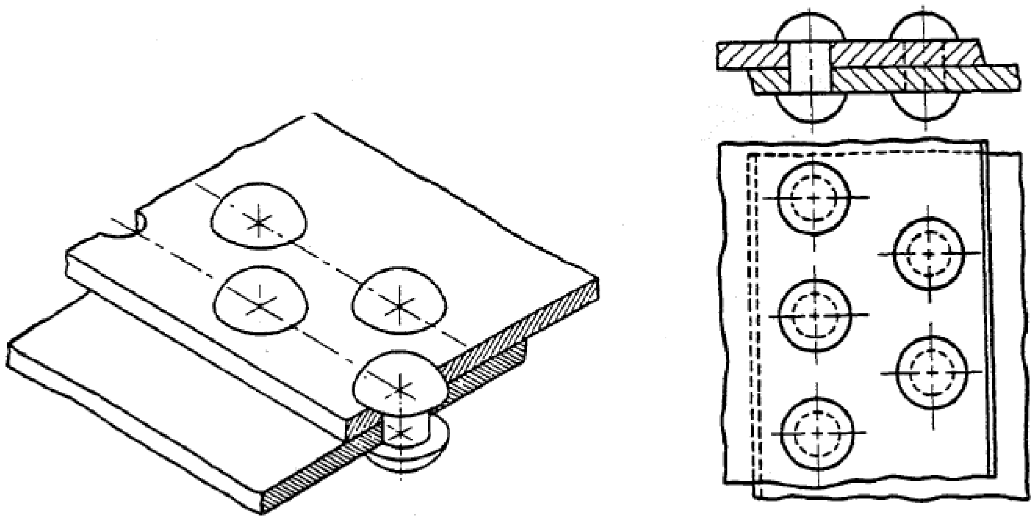


Figure 3.4

In all lap joints the straining forces in the plates are not in the same plane and they produce a couple, tending to bend the joint.

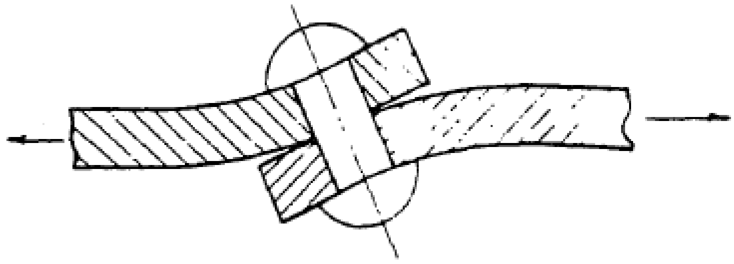


Figure 3.5

3.3.2 Butt joints
In butt riveting, the plates are kept in alignment and a butt strap or cover plate is placed over the joint and riveted to each plate; frequently two straps are used, one placed on each side of the plates.

When a single cover strap is used in a butt joint, the same tendency can be observed as in a lap joint, where the straining forces in the plates tend to bend the joints. The bending action may be avoided in butt joints by using two straps.

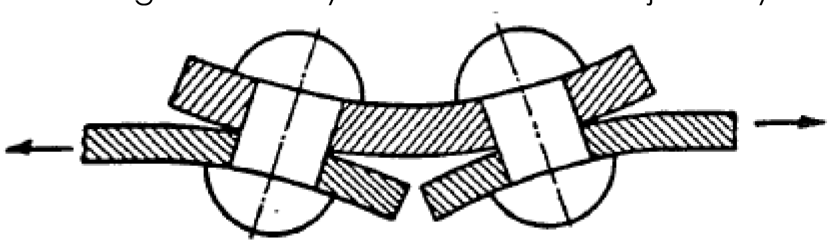


Figure 3.6

3.3.2.1 Single-riveted butt joint

Figure 3.7 shows a butt joint, and since only one row of rivets passes through each plate, it is a single-riveted joint.

In the isometric drawing two cover straps are shown. In the orthographic drawing only one strap was used.

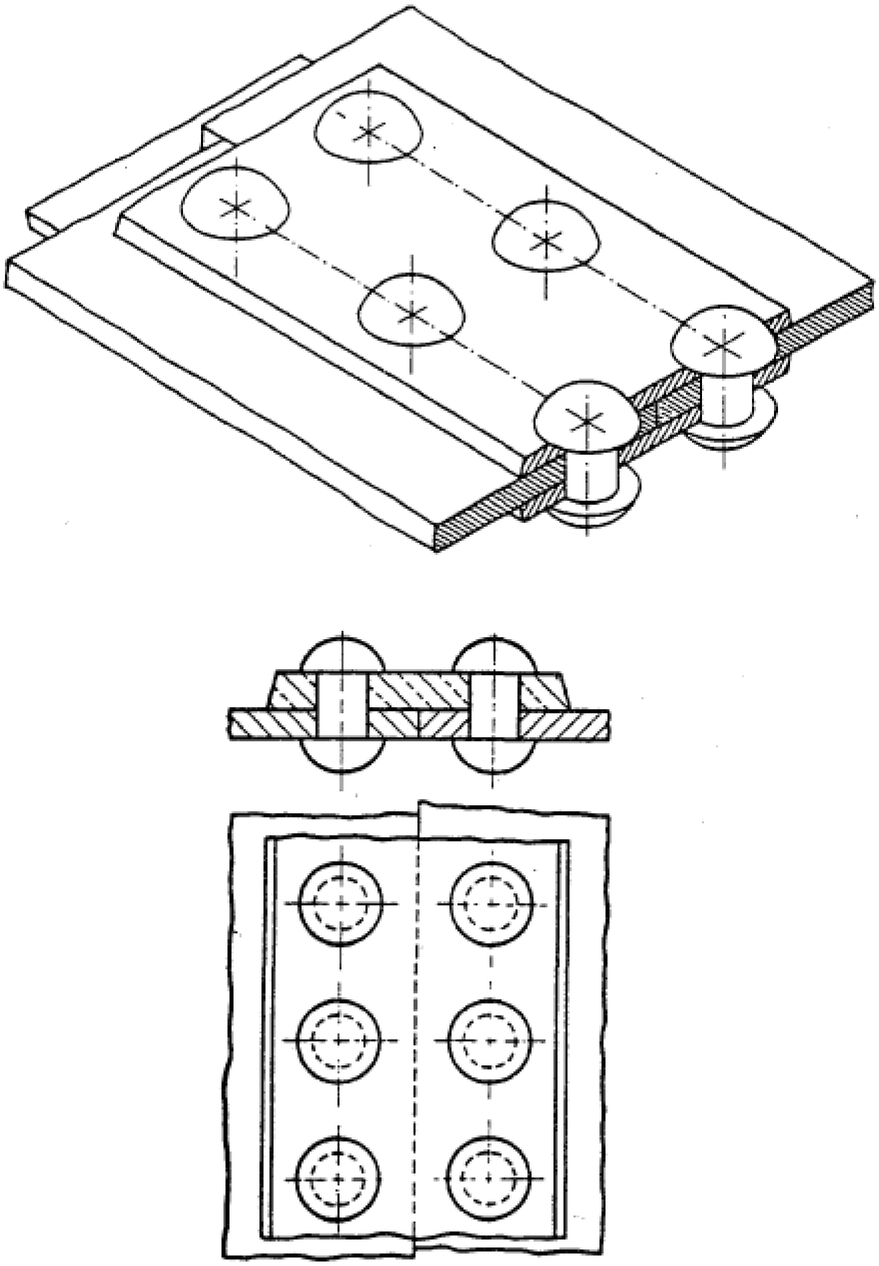


Figure 3.7

3.3.2.2 Double-riveted butt joint (chain riveting)

Two rows of rivets pass through each plate, and the rivets are arranged directly opposite each other.

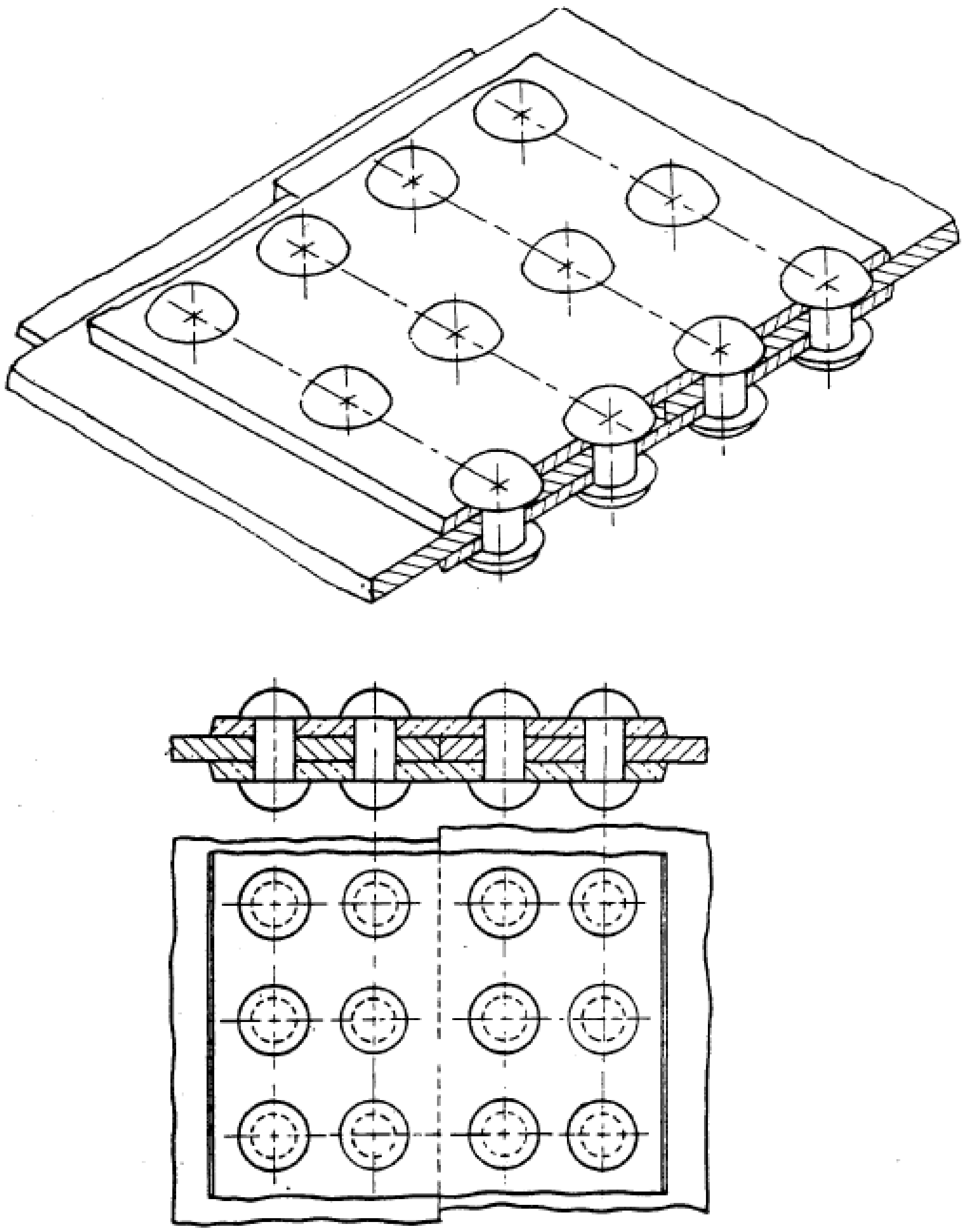


Figure 3.8

3.3.2.3 Double-riveted butt joint (zig-zag riveting)
Two rows of rivets pass through each plate. The rivets are staggered in the plates.

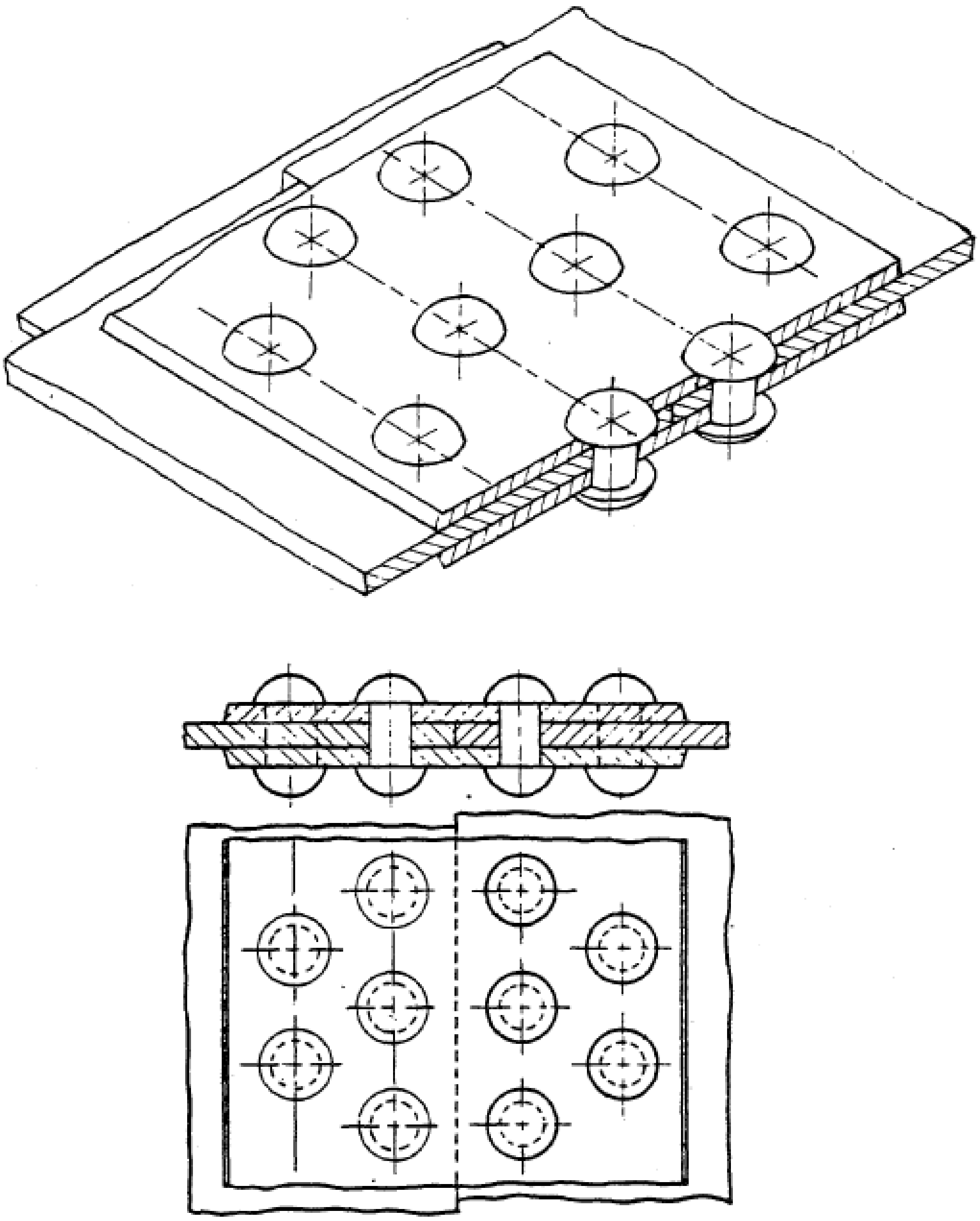


Figure 3.9

3.3.2.4 Treble-riveted butt joint

In modern high-pressure boiler work, treble-riveted butt joints having two cover straps are often used.

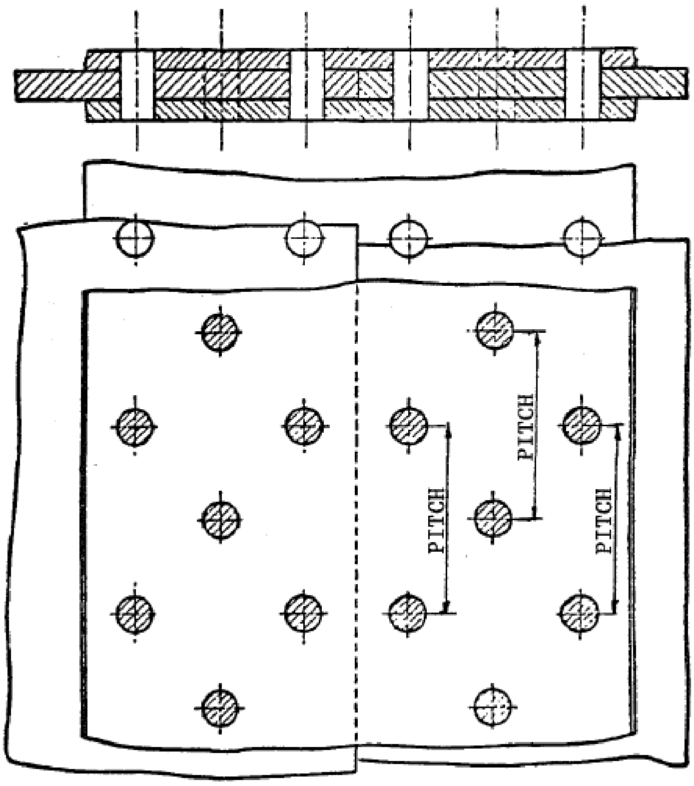


Figure 3.10

From **Figure 3.11** it can be seen that the pitch of the outer row of rivets is twice that of the inner rows. By omitting alternate rivets, less metal is removed by drilling, and a stronger joint is obtained.

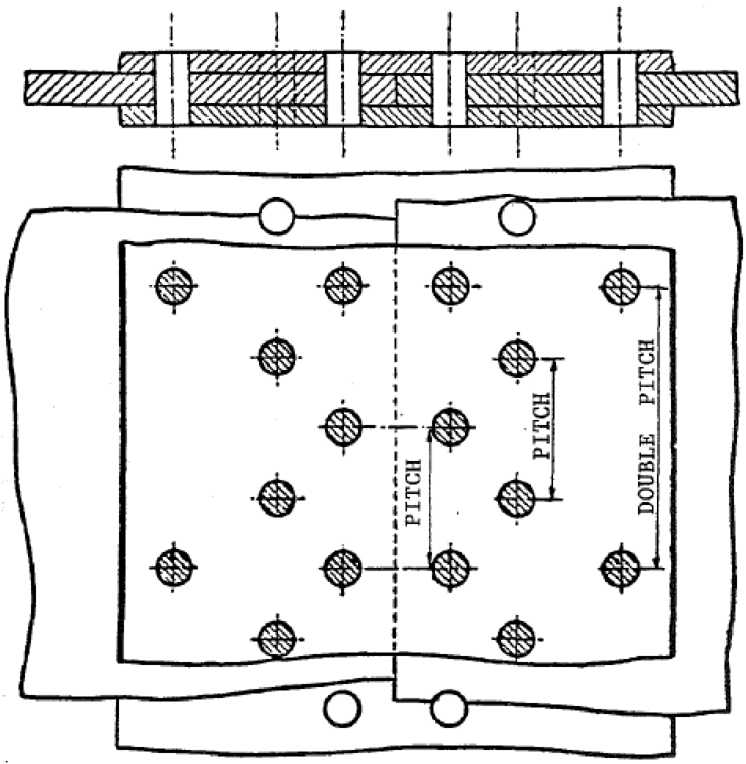


Figure 3.11

Sometimes the width of the outer strap is made narrower than the width of the inner one.

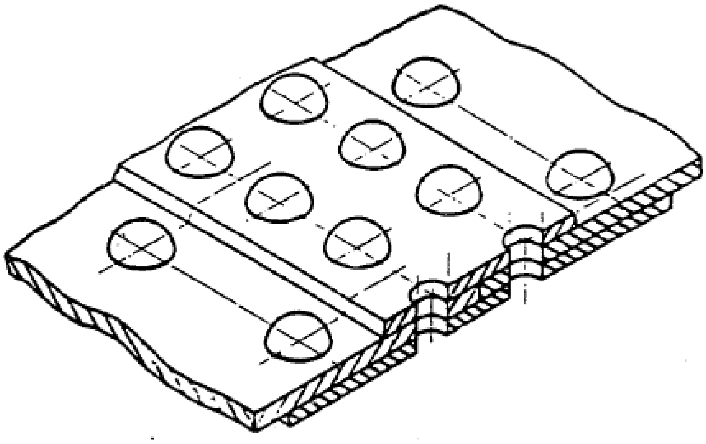


Figure 3.12

3.3.2.5 Lozenge joint

This joint is used to connect flat tie-bars in bridge and other structural work. This joint is of nearly uniform strength throughout.

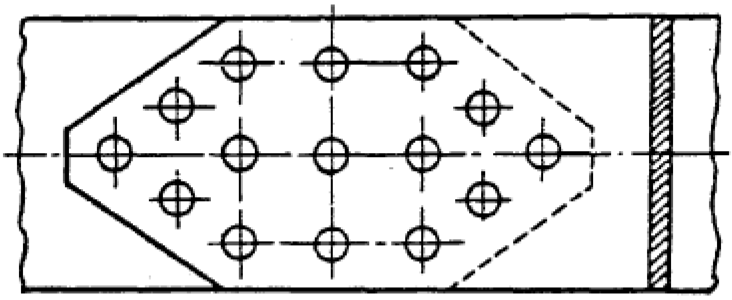


Figure 3.13

3.4 Definitions and basic concepts

3.4.1 Thickness of plates (t)

The thickness is used as the basis for designing riveted joints.

Standard plate thicknesses in millimetres are 6, 8, 10, 12, 15, 18, 20, 22, 25, 28, 30, 32, 35, 38, 40, 45, 50.

3.4.2 Thickness of cover plates (t_1)

The cover plates are usually thinner than the main plates. They are made 0,625 to 1 times the thickness of the main plates. The ends are chamfered at an angle of 80°. For treble-riveted joints where alternate rivets is omitted from the outer row, see **Figure 3.10**.

$$t_1 = \frac{5 \times t(p-d)}{8 \times (p-2d)}$$

3.4.3 The diameter of the rivets (d)

The diameter of the rivets depends on the thickness of the main plates used, and may be determined by using the empirical formula:

$$d = 6\sqrt{t}$$

Standard rivet diameters in millimetres are 6, 8, 10, 12, 16, 20, 24, 30, 36.

3.4.4 -The pitch of the rivets (p)
This is the distance between the centre of two successive rivets in the same row.

The pitch is calculated with regard to the strength of the rivets and the plates, the diameter of the rivets and the thickness of the plates. The pitch should never be less than 2 x d, so as to allow for the head to be formed.

3.4.5 Diagonal pitch (pd) (zig-zag riveting)
This is the distance diagonally from the centre of a rivet in one row to the centre of a rivet in the next row, in a zig-zag rivet arrangement. It need not be calculated, for it works out automatically by placing the second row of rivets equally spaced between the first row. When calculated it, use the formula:

$$= \sqrt{\left(\frac{p}{2}\right)^2 + (P_r)^2}$$

3.4.6 The margin of the plate (y)
This margin is the distance measured from the edge of the plate to the centre line of the rivet holes. To ensure that the rivets will not tear out at the side of the holes, the margin is made equal to 1,5 x d.

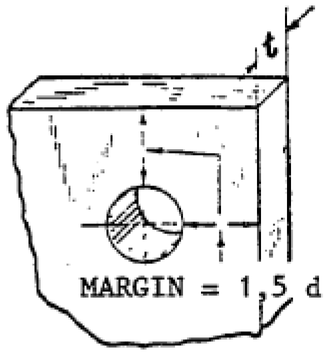


Figure 3.14

3.4.7 Distance between the rows of rivets (P_r)
This is the distance between the two centre lines of the rows of drilled rivet holes. The minimum value in chain riveting is usually taken as 2 x d, and in zig-zag riveting as 2 x d or 0,6 x p.

3.4.8 Overlap of plates
This is the distance the plates overlap in a rivet joint.
Single-riveted lap joint = $2 \times y$.
Double- riveted lap joint = $2 \times y + P_r$.

3.5 Design of riveted joints

Before designing a joint, it would be wise to investigate all the ways in which a joint may fail.

3.5.1 Methods of failure of single-riveted lap joints

We will consider a very simple joint consisting of two strips of plate with one rivet joining them.

3.5.1.1 Tearing of plate at rivet hole

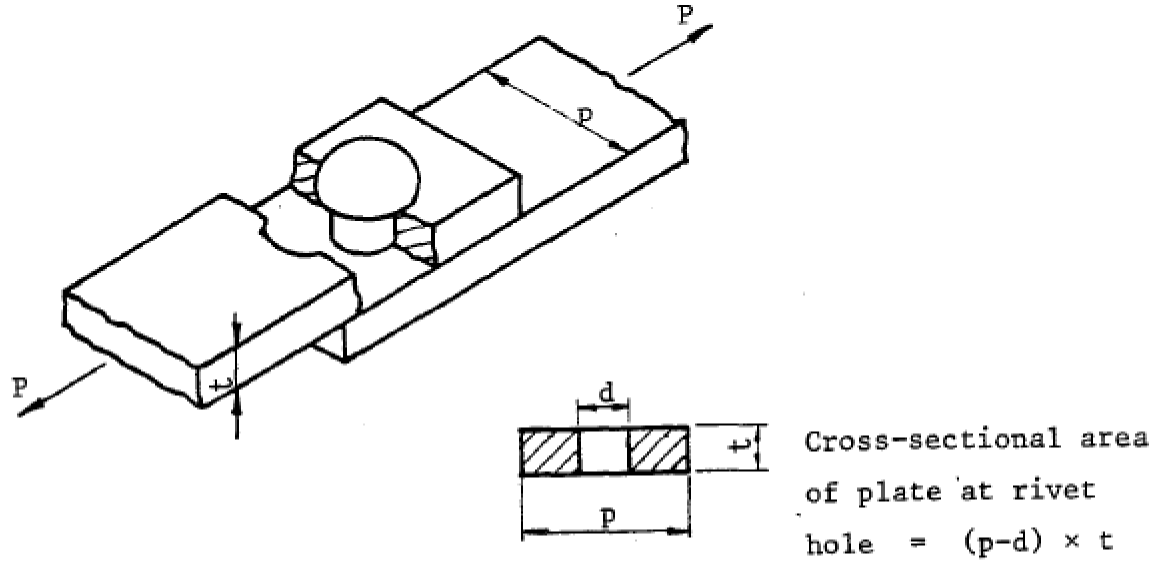


Figure 3.15

The plate can fail in tension at the line through the rivet hole. It will naturally fail here and not elsewhere, as it is weakened by the hole.

$$\text{Ultimate tensile stress} = \frac{\text{Ultimate tensile load}}{\text{Cross-sectional area of plate at rivet hole}}$$

∴ Ultimate tensile load = Ultimate tensile stress x Cross-sectional area of plate at rivet hole

= Ultimate tensile stress x (p - d) x t

or

Allowable tensile load = Allowable tensile stress x (p - d) x t

3.5.1.2 Shearing of rivet

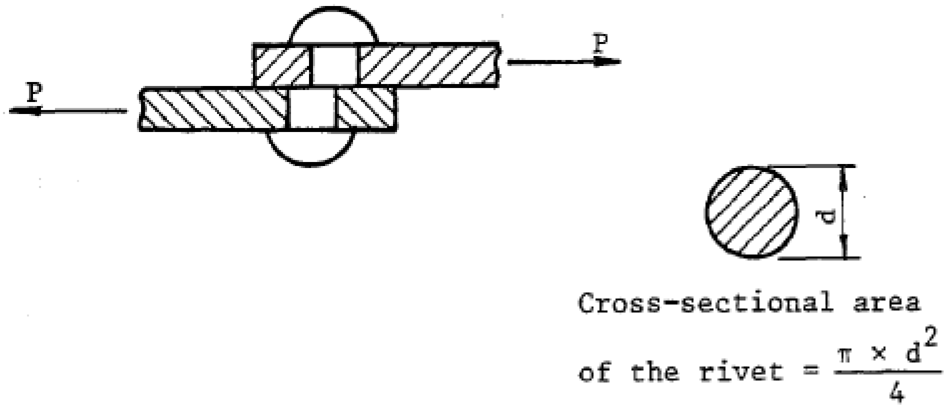


Figure 3.16

The rivet may shear at the joint of the two plates.

Ultimate shear stress $= \frac{Ultimate\ shear\ load}{Cross\text{-}sectional\ area\ of\ rivet}$

∴ Ultimate shear load = Ultimate shear stress x Cross-sectional area of rivet

= Ultimate shear stress x $\frac{\pi d^2}{4}$

or

Allowable shear load = Allowable shear stress x $\frac{\pi d^2}{4}$

3.5.1.3 Crushing of rivet or plate

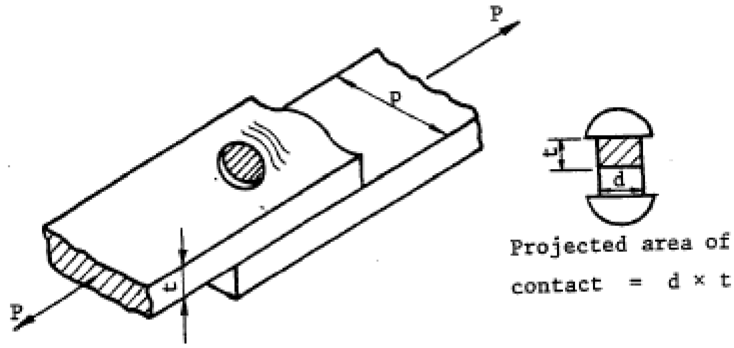


Figure 3.17

The plate or rivet may crush, with the result that the joint will become loose. In **Figure 3.17** the rivet head has not been drawn, so as to show what happens.

Ultimate crushing stress $= \frac{Ultimate\ \text{crushing}\ load}{Projected\ area\ of\ contact}$

∴ Ultimate crushing load = Ultimate crushing stress x Projected area of contact

= Ultimate crushing stress x d x t

or

Allowable crushing load = Allowable crushing stress x d x t

In each of the above cases, if σ_t, τ and σ_c are the allowable stresses, then P is the allowable load, and if σ_t, τ and σ_c are the ultimate stresses, then P is the ultimate load.

3.5.1.4 Cross-Tearing of the Plate

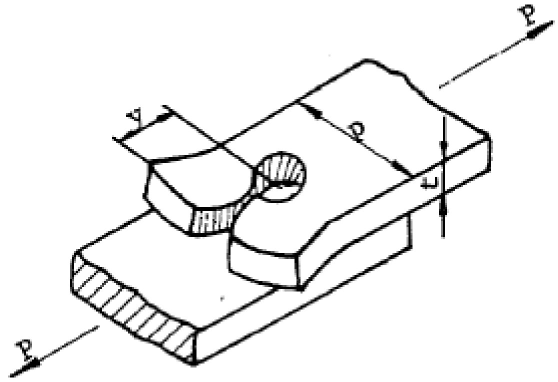


Figure 3.18

Failure may occur as a result of the rivet tearing the plate at the margin. This will never occur if the distance from the centre of the rivet to the edge of the plate is 1,5 x d.

3.5.2 Working stresses to be used if not given

Tensile-stress in the plate .... 77 MPa
Shear stress in the rivets .... 108 MPa
Crushing stress between plate and rivet......... 154 MPa

3.5.3 Strength of solid plate

Ultimate tensile stress in solid plate $= \frac{Ultimate\ tensile\ load}{Cross-sectional\ area\ of\ plate}$

∴ Ultimate tensile load = Ultimate tensile stress in solid plate x Cross-sectional area of plate

= Ultimate tensile stress in solid plate x p x t

or

Working load = Working tensile stress in solid plate x p x t

3.5.4 Efficiency of riveted joints

The efficiency of a riveted joint is given by the ratio of the least strength in a pitch width to the strength of a pitch width of solid plate.

The efficiency will be given by the smaller of the following values.

3.5.4.1 Tearing efficiency $= \frac{Strength\ of\ pierced\ plate}{Strength\ of\ solid\ plate}$

$$= \frac{Ultimate\ tensile\ stress\ of\ plate \times Cross-sectional\ area\ of\ plate\ at\ rivet\ hole}{Ultimate\ tensile\ stress\ of\ plate \times Cross-section\ area\ of\ plate}$$

$$= \frac{\sigma_t \times (p-d) \times t}{\sigma_t \times p \times t}$$

$$= \frac{p-d}{p}$$

3.5.4.2 Shearing efficiency $= \frac{Strength\ of\ rivet}{Strength\ of\ solid\ plate}$

$$= \frac{Ultimate\ shear\ stress\ of\ rivet \times Cross-sectional\ area\ of\ rivet}{Ultimate\ tensile\ stress\ of\ plate \times Cross-sectional\ area\ of\ plate}$$

$$= \tau \times \frac{\pi \times d^2}{4}$$

$$= \frac{4}{\sigma_t \times p \times t}$$

3.5.4.3 Crushing efficiency $= \frac{Crushing\ strength}{Strength\ of\ solid\ plate}$

$$= \frac{Ultimate\ crushing\ stress \times projected\ area\ of\ contact}{Ultimate\ tensile\ stress\ of\ plate \times Cross-sectional\ area\ of\ plate}$$

$$= \frac{\sigma_c \times d \times t}{\sigma_t \times p \times t}$$

$$= \frac{\sigma_c \times d}{\sigma_t \times p}$$

In designing riveted joints, the following efficiencies will serve as a guide to a well-designed joint:

Single-riveted lap joint 56%
Double-riveted lap joint 66%
Single-riveted butt joint .... 66% } 2 cover straps
Double-riveted butt joint .... 75% }

An examination of the following sketches shows that a riveted joint consists of a number of "pitch lengths", all identical.

Think about it!
If, therefore, we design one pitch length, we have designed the whole joint.

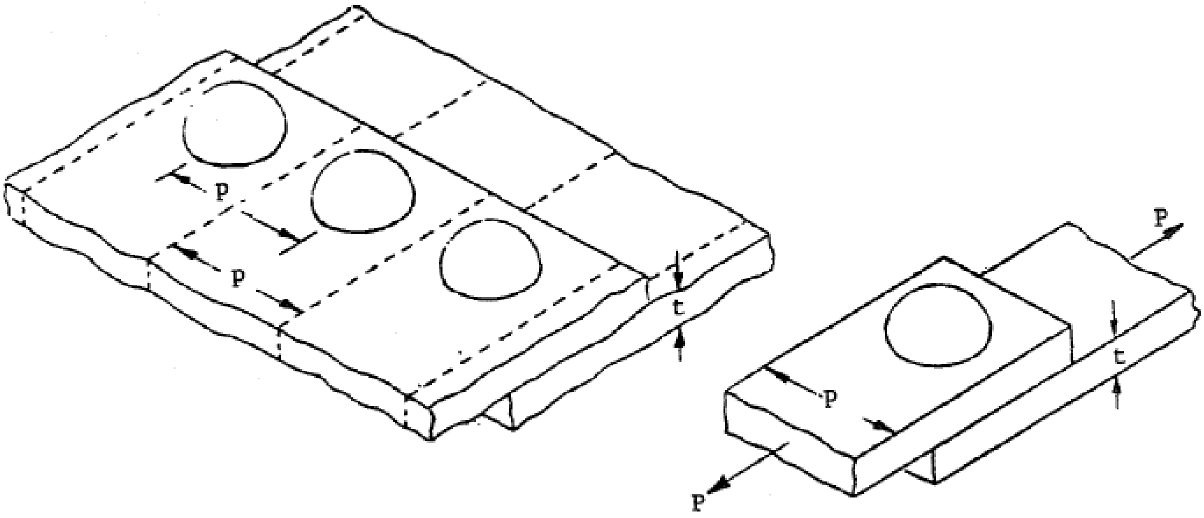


Figure 3.19 Single-riveted lap joint

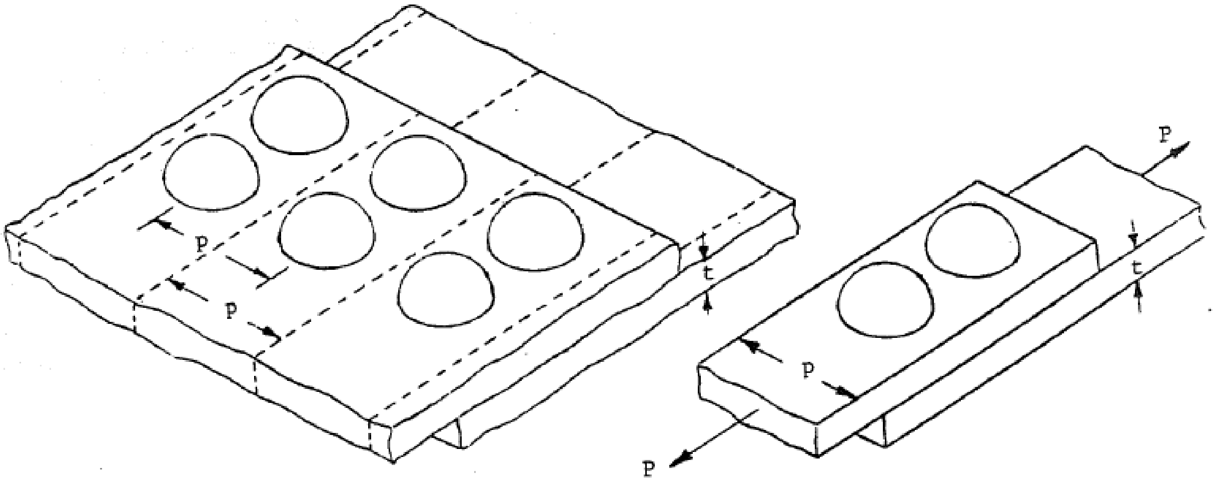


Figure 3.20 Double-riveted lap joint (chain riveting)

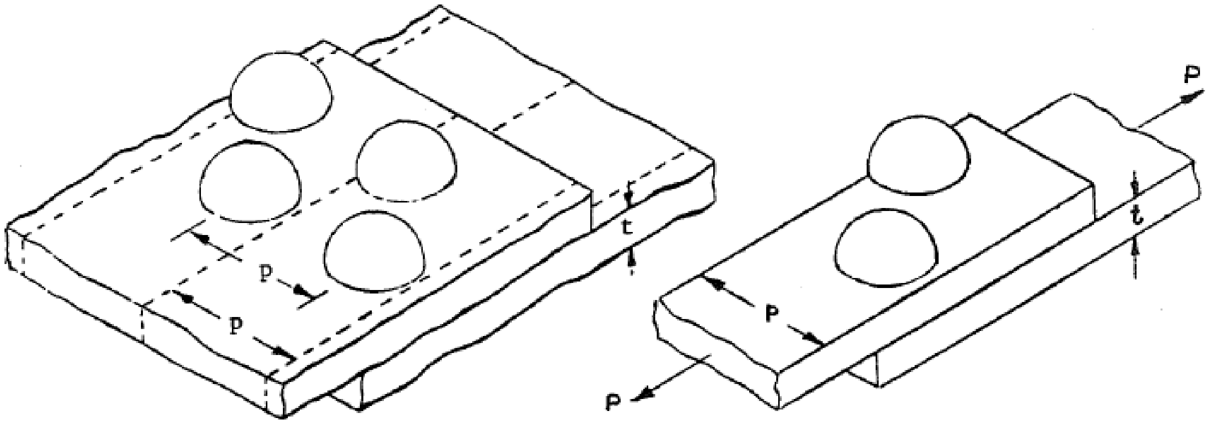


Figure 3.21 Double-riveted lap joint (zig-zag riveting)

	Worked Example 3.1

Two portions of a tie-plate 50 mm wide and 12 mm thick, are joined by a lap joint with one 20 mm diameter rivet. If the plates are carrying a pull of 30 kN, find:

i) the tensile stress in the plates;
ii) the shear stress in the rivet;
iii) the crushing stress between the plate and the rivet.

In practice, of course, one rivet would never be used, but we must deal with a simple case before proceeding to more difficult ones.

Solution:

i. Tensile stress

$$= \frac{Tensile\ load}{Cross\text{-}sectional\ area\ of\ plate}$$
$$= \frac{P}{(p-d)\times t}$$
$$= \frac{30 \times 10^3 N}{(0{,}05\ m - 0{,}02\ m) \times 0{,}012\ m}$$
$$= 83{,}3 \times 10^6\ N/m^2$$
$$= 83{,}3\ MPa$$

ii. Shear stress

$$= \frac{Shear\ load}{Cross\text{-}sectional\ area\ of\ rivet}$$
$$= \frac{P}{\frac{\pi d^2}{4}}$$
$$= \frac{30 \times 10^3 N}{\frac{\pi \times (0{,}02\ m)^2}{4}}$$
$$= 95{,}5 \times 10^6\ N/m^2$$
$$= 95{,}5\ MPa$$

iii. Crushing stress

$$= \frac{Crushing\ load}{Projected\ area\ of\ contact}$$
$$= \frac{P}{d \times t}$$
$$= \frac{30 \times 10^3 N}{0{,}02\ m \times 0{,}012\ m}$$
$$= 125 \times 10^6\ N/m^2$$
$$= 125\ MPa$$

Worked Example 3.2

If, in **Worked Example 3.1**, the materials have the ultimate stresses given below, at what load will the joint fail and in what manner? What will be the safe working load with a factor of safety of 4? What is the efficiency of the joint?

Ultimate tensile stress of plate steel	= 464 MPa
Ultimate shear stress of rivet steel	= 384 MPa
Ultimate crushing stress of plate and rivet	= 620 MPa

Solution:

Plate in tension

Ultimate tensile load = Ultimate tensile stress x cross-sectional area of plate at rivet hole

= $\sigma_t \times (p - d) \times t$

= 464 x 10^6 N/m² x (0,05 m – 0,02 m) x 0,012 m

=167,04 x 10^3 N

= 167,04 kN

Rivet in shear

Ultimate shear load = Ultimate shear stress x cross-sectional area of rivet

= $\tau \times \frac{\pi d^2}{4}$

= 384 x 10^6 N/m² x $\frac{\pi \times (0{,}02)^2}{4}$

=120,64 x 10^3 N

= 120,64 kN

Crushing rivet on plate

Ultimate crushing load = Ultimate crushing stress x Projected area of contact

= $\sigma_c \times d \times t$

= 620 x 10^6 N/m² x $0{,}02\ m \times 0{,}012\ m$

= 148,8 x 10^3 N

= 148,8 kN

So we see that to break the joint in the three different ways requires loads of 167,04 kN, 120,64 kN, and 148,8 kN, respectively. It is obvious, therefore, that it will fail by shearing the rivet at 120,64 kN.

Safe working load = $\frac{Ultimate\ load}{Factor\ of\ safety}$

= $\frac{120{,}24 \times 10^3\ N}{4}$

= 30,16 x 10^3 N

= 30,16 kN

The object of the joint was to join two plates 50 mm wide by 12 mm thick. Now the ultimate strength of these plates in tension is:

Ultimate tensile stress in plate x cross-sectional area of plate

= $\sigma_t \times p \times t$

= 464 x 10^6 N/m² x $0{,}05\ m \times 0{,}012\ m$

= 278,4 x 10^3 N

= 278,4 kN

If we had joined them with a joint with a strength of 278,4 kN, we would have had 100%-efficiency. But our joint has a strength of only 120,64 kN.

Therefore the efficiency of the joint is:

$$= \frac{120{,}64 \times 10^3\ N}{278{,}4 \times 10^3\ N} \times 100$$
$$= 43{,}33\%$$

Worked Example 3.3

A tie-plate is required to carry a pull of 110 kN. It is to be made of 10 mm thick plate and is to have a lap joint with a single row of 20 mm diameter rivets. The working stresses in the materials are:

Tensile stress (σ_t) in plate	= 124 MPa
Shear stress (τ) in rivets	= 100 MPa
Crushing stress (σ_c) between plates and rivets	= 154 MPa

Find the necessary width for the tie-plate and design the joint.

<u>Solution:</u>

<u>Number of rivets</u>

Working load for one rivet in shear = Working shear stress x cross-sectional area of rivet

$$= \tau \times \frac{\pi d^2}{4}$$
$$= 100 \times 10^6\ \text{N/m}^2 \times \frac{\pi \times (0{,}02)^2}{4}$$
$$= 31{,}42 \times 10^3\ \text{N}$$
$$= 31{,}42\ \text{kN}$$

No of rivets required to carry 110 kN $= \frac{110 \times 10^3\ N}{31{,}42 \times 10^3\ N}$
$= 3.5$

So 4 rivets must be used.

<u>Width of plate</u>

As can be seen from **Figure 3.22**, the cross-sectional area of the plate in tension is (w - 4d) x t.

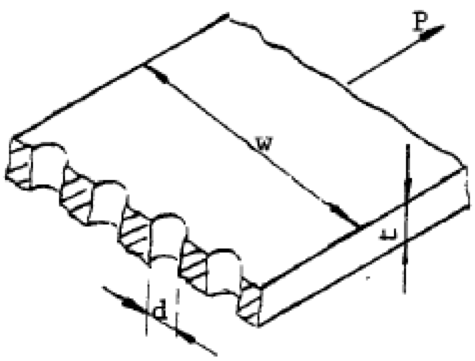


Figure 3.22

$$\text{Load} = \text{Cross-sectional area x Tensile stress}$$
$$P = (w - 4d) \times t \times \sigma_t$$
$$110 \times 10^3 N = (w - 4 \times 0{,}02\ m) \times 0{,}01\ m \times 124 \times 10^6 N/m^2$$
$$w - 4 \times 0{,}02\ m = \frac{110 \times 10^3 N}{0{,}01\ m \times 124 \times 10^6 N/m^2}$$
$$w - 0{,}08\ m = 0{,}0887\ \text{m}$$
$$w = 0{,}0887\ \text{m} + 0{,}08\ \text{m}$$
$$= 0{,}1687\ \text{m}$$

Make with of plate = 170 mm

Check for crushing of rivets between the plate
First method

Working load for one rivet in crushing = Working crushing stress x Projected area of contact

$$= \sigma_c \times d \times t$$
$$= 154 \times 10^6\ \text{N/m}^2 \times 0{,}02\ m \times 0{,}01\ m$$
$$= 30{,}8 \times 10^3\ \text{N}$$
$$= 30{,}8\ \text{kN}$$

No of rivets required for 110 kN $= \frac{110 \times 10^3 N}{30{,}8 \times 10^3 N}$

$= 3{,}57$

As we have 4 rivets, they are safe for crushing.

Alternative method
We have 4 rivets carrying 110 kN

∴ 1 rivet carries $\frac{110 \times 10^3 N}{4}$

$$= 27{,}5 \times 10^3 N$$
$$= 27{,}5\ kN$$

Which produces a crushing stress of $\frac{load}{projected\ area}$

$$= \frac{27{,}5 \times 10^3 N}{d \times t}$$
$$= \frac{27{,}5 \times 10^3 N}{0{,}02\ m \times 0{,}01\ m}$$
$$= 137{,}5 \times 10^6 N/m^2$$
$$= 137{,}5\ MPa$$

As the allowable crushing stress is 154 MPa, this is satisfactory.

Arrangement of rivets
Distance from centre of rivet to edge of plate must be 1,5 x d

y = 1,5 x 0,02 m
= 0,03 m
= 30 mm
So the lap = 2 x y
= 2 x 30 mm
= 60 mm

Now 4 rivets have to be fitted into a plate 170 mm wide.

A little consideration shows that the arrangement shown in **Figure 3.23** will be suitable. Note that the distance between the centres of the rivets is more than the minimum allowable, which is 2 x d.

p = 2 x 0,02 mm
= 40 mm

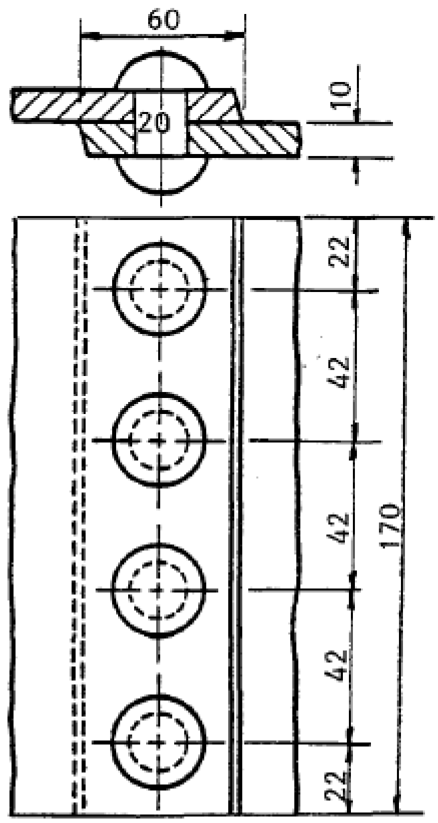


Figure 3.23

3.6 Methods of failure regarding double-riveted lap joints

Consider a simple joint consisting of two strips of plate joined by two rivets.

3.6.1 Tearing of Plate

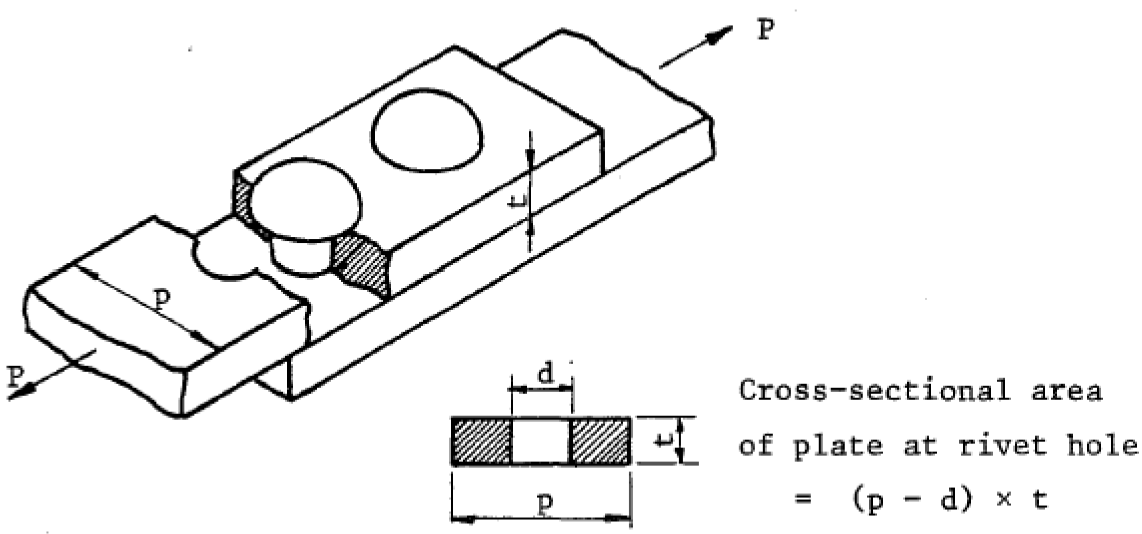


Figure 3.24

The plate may fail in tension at the rivet hole.

Note:
The fact that there are two rivets makes no difference.

Failure occurs at a section with one hole.

Ultimate tensile load = Ultimate tensile stress x Cross-sectional area of plate at rivet hole

P $= \sigma_t \times \ (p - d) \times t$

3.6.2 Shearing of rivets

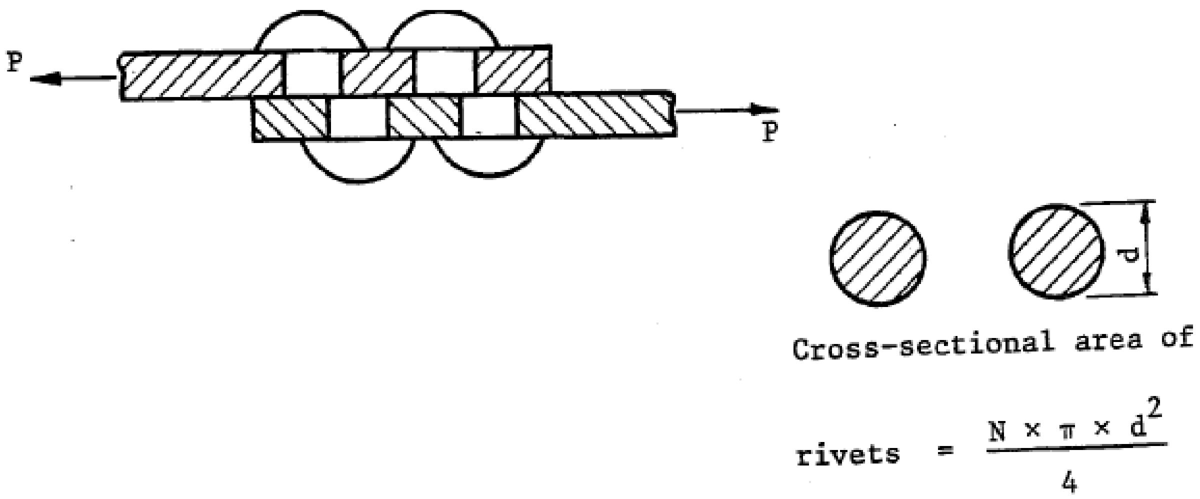


Figure 3.25

Both rivets have to shear.

Ultimate shear load = Ultimate shear stress x Cross-sectional area of rivets

$$P = \tau \times N \times \frac{\pi d^2}{4}$$

where N is the number of rivets in shear and in this case N will be 2

$$\therefore P = \tau \times 2 \times \frac{\pi d^2}{4}$$

3.6.3 Crushing rivets against the plate

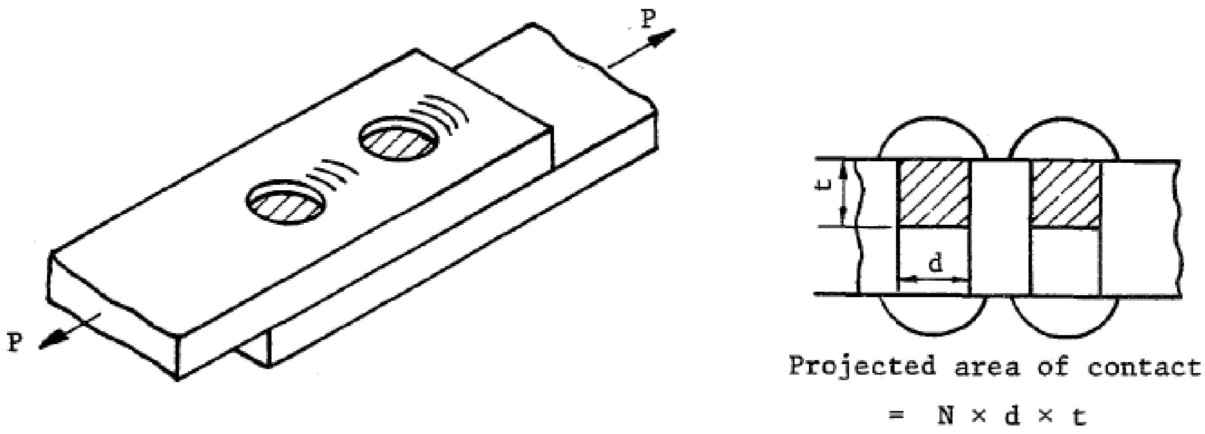


Figure 3.26

Crushing at both rivets has to occur. The heads have not been drawn in the isometric sketch, so as to show what happens.

Ultimate crushing load = Ultimate crushing stress x Projected area of contact

$$P = \sigma_c \times N \times d \times t$$

In this case N is again 2

$$P = \sigma_c \times 2 \times d \times t$$

Worked Example 3.4

A tie plate is to be designed to carry a pull of 220 kN. There is to be a double, chain-riveted lap joint in the plates, which are 20 mm thick. Design the plates and joint, using a factor of safety of 4 and ultimate stresses of 460 x 10^6, 388 x 10^6, 620 x 10^6 pascals for tension, shear and crushing.

Solution:

Rivet Diameter

Since no rivet diameter is specified, we must first decide on a suitable one.

$$d = 6\sqrt{t}$$
$$= 6 \times \sqrt{20}$$
$$= 6 \times 4{,}472\ mm$$
$$= 26{,}832\ mm$$

Use 24 mm standard diameter rivets.

Note that 30 mm standard diameter rivets could also be used.

Working Stresses

$$\sigma_t = \frac{Ultimate\ tensile\ stress}{Factor\ of\ safety}$$
$$= \frac{450 \times 10^6\ N/m^2}{4}$$
$$= 115 \times 10^6\ N/m^2$$

$$\tau = \frac{Ultimate\ shear\ stress}{Factor\ of\ safety}$$
$$= \frac{388 \times 10^6\ N/m^2}{4}$$
$$= 97 \times 10^6\ N/m^2$$

$$\sigma_c = \frac{Ultimate\ crushing\ stress}{Factor\ of\ safety}$$
$$= \frac{650 \times 10^6\ N/m^2}{4}$$
$$= 155 \times 10^6\ N/m^2$$

No of rivets

Working load for 1 rivet in shear $= \frac{\pi d^2}{4} \times \tau$

$= \frac{\pi \times (0{,}024\ m)^2}{4} \times 97 \times 10^6 N/m^2$

= 43,88 kN

Number of rivets to carry 220 kN $= \frac{220 \times 10^3}{43{,}88 \times 10^3\ N}$

= 5

Five rivets would do, but since the joint is chain-riveted, we must have an even number, so we use 6 rivets in two rows of 3.

Width of Plate

As can be seen from the sketch, the area of plate carrying the load is $(w - Nd)\ x\ t$.

Cross-sectional area of plate at rivet holes = (w - 3d) x t

Allowable tensile load = Allowable tensile stress x Cross-sectional area of the plate at rivet holes

$$P = \sigma_t \times (w - 3d) \times t$$

$$w - 3d = \frac{P}{\sigma_t \times t}$$

$$w = \frac{P}{\sigma_t \times t} + 3d$$

$$= \frac{220 \times 10^3 N}{115 \times 10^6\ N/m^2 \times 0{,}02\ m} + 3 \times 0{,}024\ m$$

= 0,096 m + 0,072 m

= 0,168 m

Say = 168 mm

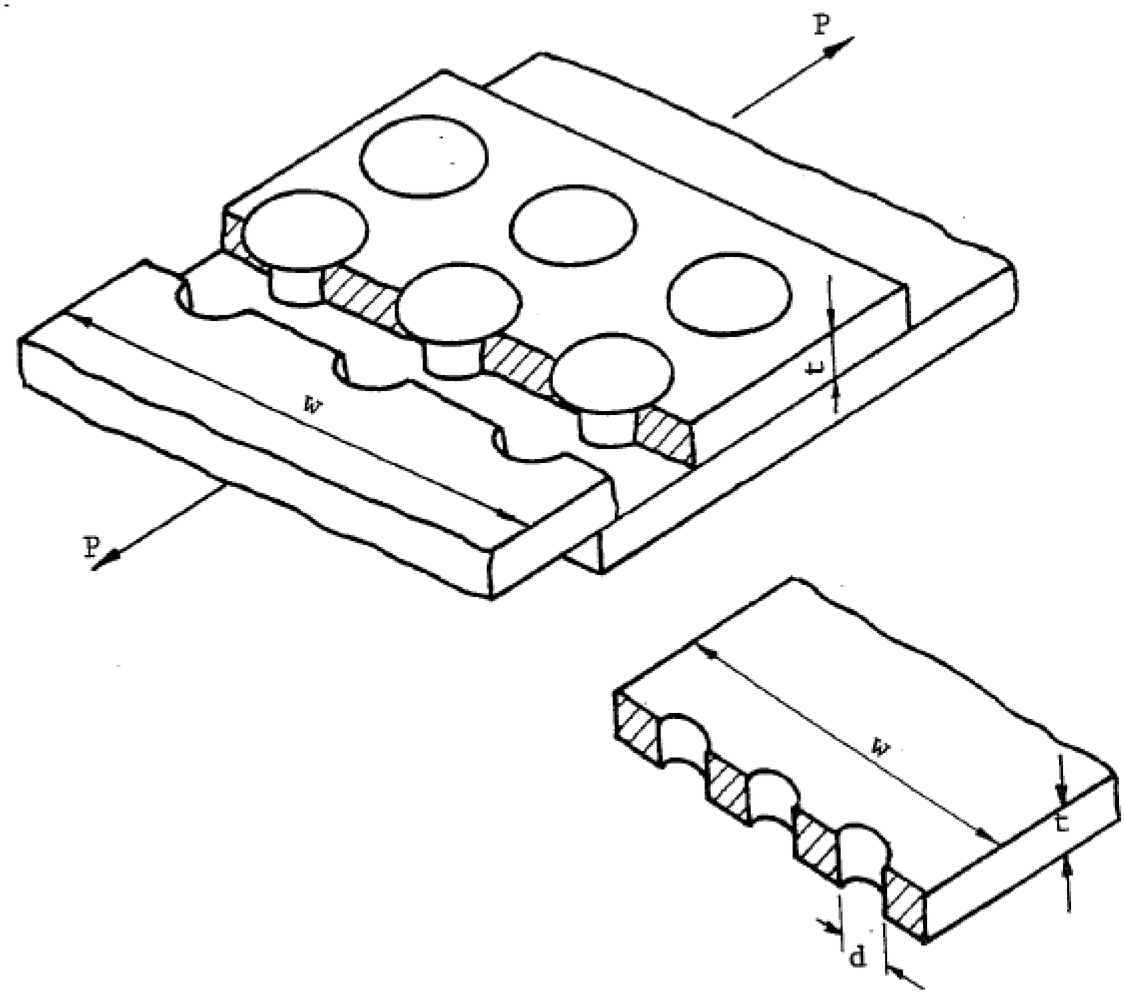


Figure 3.27

Check for crushing

Working load for 1 rivet in crushing $= \sigma_c \times d \times t$

$= 115 \times 10^6 N/m^2 \times 0{,}024m \times 0{,}02\ m$

= 74,4 kN

Number of rivets required $= \frac{220 \times 10^3 N}{74{,}4 \times 10^3 N}$
= 2,97

As we have 6, there will be no danger of crushing occurring.

Arrangement of Rivets

Distance from centre of rivet to edge of plate $= 1{,}5 \times d$

$y = 1{,}5 \times 0{,}024\ m$
= 0,036 m
= 36 mm

Minimum distance between rivet centres $= 2 \times d$

$P_r = 2 \times 0{,}024\ m$
= 0,048 m
= 48 mm

The following arrangement will therefore be satisfactory:

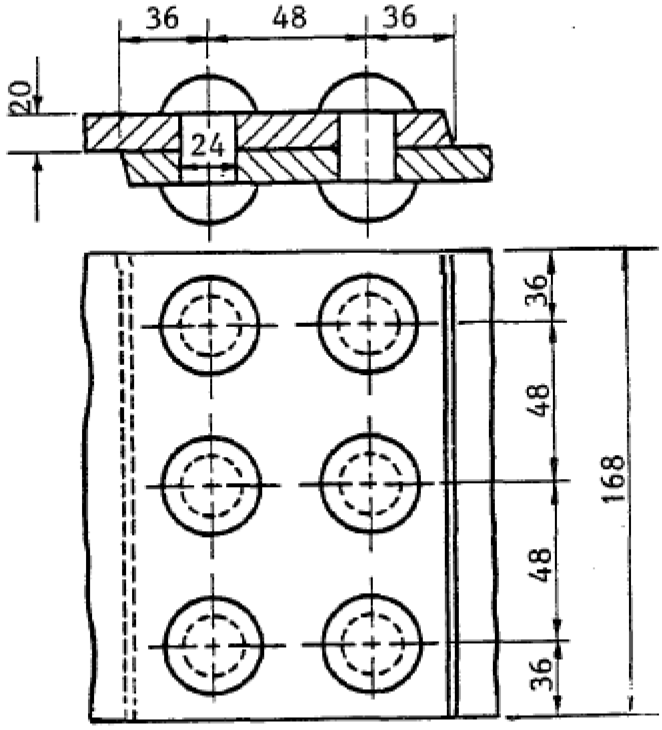


Figure 3.28

So far we have learnt how to design riveted joints for tie bars.

In these problems, a definite load was given, and the number of rivets and width of plate to carry this load were calculated.

We now have to deal with joints for boilers, compressed-air-receivers, etc. In this case, only the thickness of the plates is given, and we have to join them as efficiently as possible.

Thus we have to decide on the size of the rivets and how far apart they are to be spaced (ie their pitch). The size of the rivet is obtained from the empirical formula $d = 6\sqrt{t}$, so it remains only to find the pitch.

Now, if we make the pitch very large, we will have a joint which is strong in tension in the plates as there are very few rivet holes in it. It will, however, be weak in shear of the rivets, for there will not be enough of them.

If, on the other hand, we make the pitch small in order to get more rivets, then we are weakening the plates, owing to the large number of holes in them.

The ideal condition would be when a compromise is reached and we have equal strengths in the plates in tension and the rivets in shear. To obtain the best pitch, therefore, we write:

Strength of plate in tension = Strength of rivets in shear

Therefore:

For single-riveted lap joint

Strength of plate in tension = Strength of rivets in shear

$$\sigma_t \times (p - d) \times t = \tau \times \frac{\pi d^2}{4}$$

For double-riveted lap joint

Strength of plate in tension = Strength of rivets in shear

$$\sigma_t \times (p - d) \times t = \tau \times 2 \times \frac{\pi d^2}{4}$$

Note:
In theory, there is no difference in strength between chain and zig-zag riveting. In practice, zig-zag riveting gives a tighter joint and is used mainly in boiler work.

Worked Example 3.5

Calculate a suitable pitch for a single-riveted lap joint for 12 mm thick steel plates.

Tensile stress of steel = 480 MPa
Shear stress of steel = 384 MPa
Crushing stress of steel = 618 MPa

Factor of safety = 6

Calculate the failing loads and the working loads per pitch length for tension, shear and crushing. How would the joint in all probability fail and at what load? What is the safe load per pitch length?
Determine the efficiencies of the joint for tension, shear and crushing.
State the joint efficiency.

Solution:
Diameter of Rivets

$$\begin{aligned} d &= 6\sqrt{t} \\ &= 6 \times \sqrt{12} \\ &= 6 \times 3{,}464\ mm \\ &= 20{,}784\ mm \end{aligned}$$

Use 20 mm diameter standard-size rivets.

Stresses
Ultimate stresses:

$$\begin{aligned} \sigma_t &= 480\ Mpa \\ \tau &= 384\ Mpa \\ \sigma_c &= 618\ Mpa \end{aligned}$$

Working stresses:

$$\text{Allowable stress} = \frac{Ultimate\ stress}{Factor\ of\ safety}$$

$$\sigma_t = \frac{480 \times 10^6\ N/m^2}{6} = 80\ MPa$$

$$\tau = \frac{384 \times 10^6\ N/m^2}{6} = 64\ MPa$$

$$\sigma_c = \frac{618 \times 10^6\ N/m^2}{6} = 103\ MPa$$

Pitch
Consider one pitch length of the joint.

For the best pitch:

$$\text{Strength of plate in tension} = \text{Strength of rivets in shear}$$

$$\sigma_t \times (p - d) \times t = \tau \times \frac{\pi d^2}{4}$$

We can use either ultimate or working stresses because we are equating the tensile strength to the shear strength.

$$480 \times 10^6 N/m^2 \times (p - 0{,}02\ m) \times 0{,}012\ m = 384 \times 10^6 N/m^2 \times \frac{\pi \times (0{,}02\ m)^2}{4}$$

$$p - 0{,}02\ m = \frac{120637{,}2N}{480 \times 10^6 N/m^2 \times 0{,}012}$$

p = $0{,}021\,m + 0{,}02\,m$
= 0,041 m
Pitch = 41 mm

Check that this is above the allowable minimum.
Minimum pitch = $2\ x\ d$
= 2 x 20 mm
= 40 mm

Failing loads per pitch length
Ultimate stresses must be used.

Plate in tension
Ultimate tensile load = Ultimate tensile stress x Cross-sectional area of plate at rivet hole
$P = \sigma_t \times (p - d) \times t$
$= 480 \times 10^6 N/m^2 \times (0{,}041\,m - 0{,}02\,m) \times 0{,}012\,m$
= 120,96 kN

Rivet in shear
Ultimate shear load = Ultimate shear stress x Cross-sectional area of rivet
$P = \tau \times \frac{\pi d^2}{4}$
$= 384 \times 10^6 N/m^2 \times \frac{(0{,}02\,m)^2}{4}$
= 120,64 kN

Crushing of Rivets against the plate
Ultimate crushing load = Ultimate crushing stress x Projected area of contact
$P = \sigma_c \times d \times t$
$=618 \times 10^6 N/m^2 \times 0{,}02\,m \times 0{,}012\,m$
= 148,32 kN

The failing load is therefore 120,6 kN (the smallest of the three) and the joint would fail by shearing the rivets.

Working loads per pitch length
These may be found by either using working stresses or dividing the failing loads (ultimate loads) by the factor of safety.

Plate in tension
Allowable tensile load = Allowable tensile stress x Cross-sectional area of plate at rivet hole
$P = \sigma_t \times (p - d) \times t$
$= 80 \times 10^6 N/m^2 \times (0{,}041\,m - 0{,}02\,m) \times 0{,}012\,m$
= 20,16 kN

or

Allowable tensile load $= \frac{Ultimate\ tensile\ load}{Factor\ of\ safety}$

$P = \frac{120{,}96 \times 10^3\ N}{6}$

= 20,16 kN

Rivet in Shear

Allowable shear load = Allowable shear stress x Cross-sectional area of rivet

$P = \tau \times \frac{\pi d^2}{4}$

$= 64 \times 10^6 N/m^2 \times \frac{(0{,}02\ m)^2}{4}$

= 20,11 kN

Crushing of Rivet against the plate

Allowable crushing load = Allowable crushing stress x Projected area of contact

$P = \sigma_c \times d \times t$

$= 103 \times 10^6 N/m^2 \times 0{,}02\ m \times 0{,}012\ m$

= 24,72 kN

or

Allowable crushing load $= \frac{Ultimate\ crushing\ load}{Factor\ of\ safety}$

$P = \frac{148{,}32 \times 10^3\ N}{6}$

= 24,72 kN

The safe load is therefore 20,11 kN (the smallest of the three).

Efficiencies

Again we can work with either ultimate or working stresses.
Using ultimate stresses:

Strength of solid plate per pitch length

Ultimate tensile load = Ultimate tensile stress x Cross-sectional area of plate

$P = \sigma_t \times p \times t$

$= 480 \times 10^6 N/m^2 \times 0{,}041\ m \times 0{,}012\ m$

= 236,16 kN

If this strength were obtained for the joint it would be 100% efficient.

Strength of pierced plate in tension = 120,96 kN

Efficiency in tension $= \frac{Strength\ of\ pierced\ plate}{Strength\ of\ solid\ plate}$

$= \frac{120{,}96 \times 10^3\ N}{236{,}16 \times 10^3\ N}$

= 0,512

= 51,2%

Strength of rivet in shear = 120,64 kN

$$\text{Efficiency in shear} = \frac{Strength\ of\ rivet}{Strength\ of\ solid\ plate}$$
$$= \frac{120{,}64 \times 10^3}{236{,}16 \times 10^3}$$
$$= 0{,}511$$
$$= 51{,}1\%$$

Strength in crushing = 148,32 kN

$$\text{Efficiency in crushing} = \frac{Crushing\ strength}{Strength\ of\ solid\ plate}$$
$$= \frac{148{,}32 \times 10^3 N}{236{,}16 \times 10^3 N}$$
$$= 0{,}628$$
$$= 62{,}8\%$$

The joint efficiency is the lowest of these, namely 51,1%.

Worked Example 3.6

Design a double-riveted lap joint with staggered riveting for steel plates, 22 mm thick, using the following data:

Rivet diameter = $6\sqrt{t}$

Ultimate stresses –	tension	495 MPa
	shear	385 MPa
	crushing	650 MPa

Factor of safety = 5

Use the method of equating tensile and shear loads to find the pitch.
Check for crushing and find the joint efficiency. Find the working load per metre length of the joint. Sketch the joint freehand and insert all dimensions.

Solution:
Diameter of Rivets

$$d = 6\sqrt{t}$$
$$= 6 \times \sqrt{22}$$
$$= 28{,}143\ mm$$

Use 30 mm diameter standard-size rivets.

Pitch
Using ultimate stresses, consider one pitch length of the joint.

Strength of plate in tension = Strength of rivets in shear

$$\sigma_t \times (p-d) \times t = \tau \times N \times \frac{\pi d^2}{4}$$

$$495 \times 10^6 N/m^2 \times (p - 0{,}03\ m) \times 0{,}022\ m = 385 \times 10^6 N/m^2 \times \frac{\pi \times (0{,}03\ m)^2}{4}$$

$$p - 0{,}03\ m = \frac{544\ 281 N}{495 \times 10^6 N/m^2 \times 0{,}022}$$

$$p = 0{,}05\ m + 0{,}03\ m$$

= 0,08 m
= 80 mm

Check that this is above the allowable minimum.
Minimum pitch = $2\ x\ d$
= 2 x 30 mm
= 60 mm

Failing load per pitch length

Plate in tension
Ultimate tensile load = Ultimate tensile stress x Cross-sectional area of plate at rivet holes

$$P = \sigma_t \times (p-d) \times t$$
$$= 495 \times 10^6 N/m^2 \times (0{,}08\ m - 0{,}03\ m) \times 0{,}022\ m$$

= 544,5 kN

Rivet in shear
Ultimate shear load = Ultimate shear stress x Cross-sectional area of rivets

$$P = \tau \times N \times \frac{\pi d^2}{4}$$
$$= 385 \times 10^6 N/m^2 \times 2 \times \frac{\pi \times (0{,}03\ m)^2}{4}$$

= 544,28 kN

Crushing of Rivets against the plate
Ultimate crushing load = Ultimate crushing stress x Projected area of contact

$$P = \sigma_c \times N \times d \times t$$
$$= 650 \times 10^6 N/m^2 \times 0{,}03\ m \times 0{,}022\ m$$

= 858 kN

Joint Efficiency
Strength of solid plate per pitch length

Ultimate tensile load = Ultimate tensile stress x Cross-sectional area of plate

$P = \sigma_t \times p \times t$

$= 495 \times 10^6 N/m^2 \times 0{,}08\ m \times 0{,}022\ m$

= 871,2 kN

Efficiency $= \frac{Lowest\ of\ three\ ultimate\ loads}{Strength\ of\ solid\ plate}$

$= \frac{544{,}28 \times 10^3 N}{871{,}2 \times 10^3 N}$

= 0,625

= 62,5%

Check for crushing

This has been done, as the failing load in crushing has been shown to be higher than those for tension or shear.

Working load per metre length

Failing load per pitch length = 544,28 kN

Working load per pitch length $= \frac{Failing\ loads}{Factor\ of\ safety}$

$= \frac{544{,}28 \times 10^3 N}{5}$

= 108,86 kN

Since the pitch is 0,08 m, the working load for 0,08 m length = 108,86 kN

Working load per 1 m length $= 108{,}86 \times 10^3\ N \times \frac{1\ m}{0{,}08\ mm}$

= 1 360,75 kN

Sketch

Distance from centre of rivets to edge of plate $= 1{,}5\ x\ d$

= 1,5 x 30 mm

= 45 mm

Distance between rows $= 2\ x\ d$

= 2 x 30 mm

= 60 mm

As no particular heads are specified, assume they are snap heads.

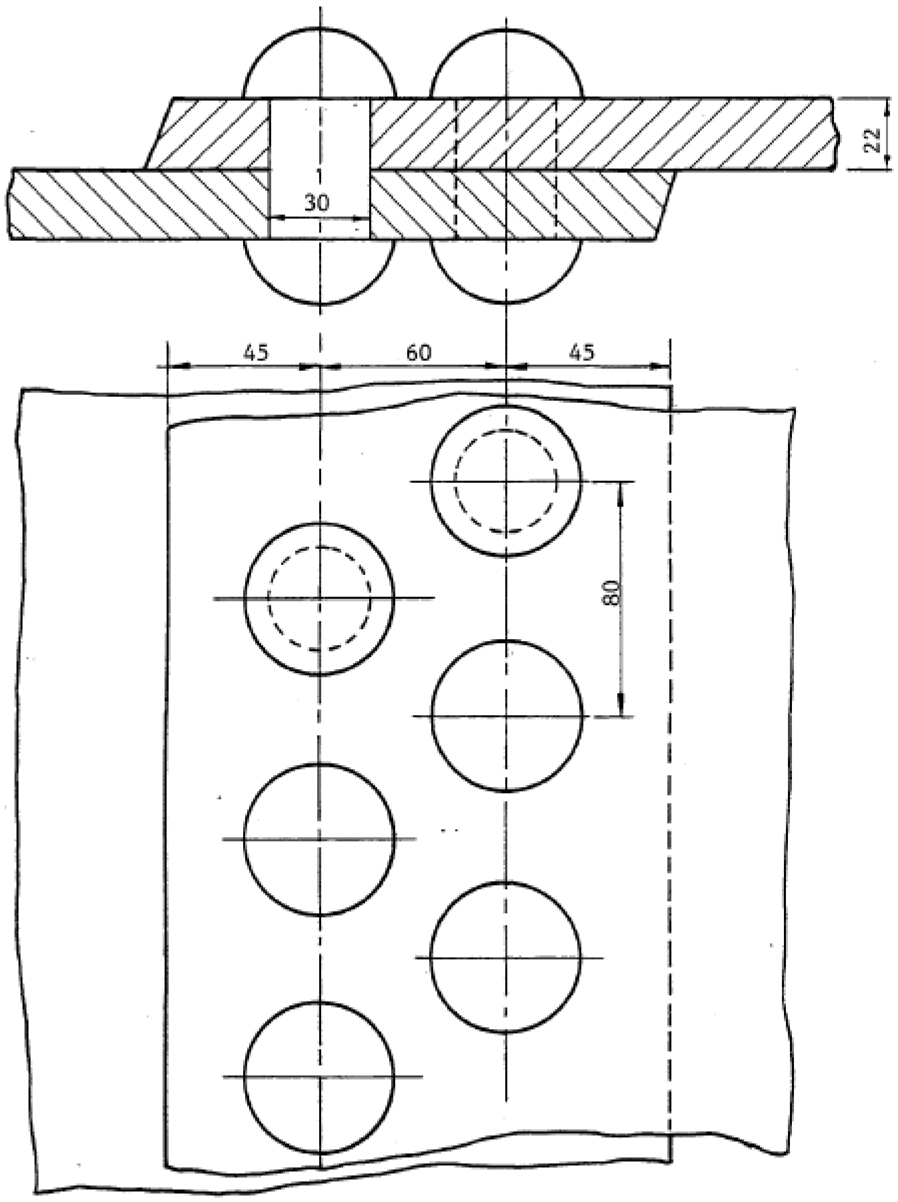


Figure 3.29

	Worked Example 3.7

Two steel plates, each 10 mm thick, are joined by a double-riveted lap joint. The rivets having snapheads to be arranged in zig-zag form having a pitch of 60 mm.

Use the following working stresses: σ_t = 108 MPa, τ = 85 MPa, σ_c = 148 MPa and

Calculate:
(a) a suitable diameter for the rivets by equating the tensile and shearing loads;
(b) the working strength of the solid plate;

(c) the working strength of the pierced plate;
(d) the working shear strength of the rivets;
(e) the working crushing strength of the rivets against the plate;
(f) the allowable safe load for the joint;
(g) the efficiency of the joint.

Solution:

(a) Diameter of rivets

Strength of plate in tension = Strength of rivets in shear

$$\sigma_t \times (p - d) \times t = \tau \times N \times \frac{\pi d^2}{4}$$

$$108 \times 10^6 N/m^2 \times (0{,}06\, m - d) \times 0{,}01\, m = 85 \times 10^6 N/m^2 \times \frac{2 \times \pi \times d^2}{4}$$

$$64\,800 - 1\,080\,000\, d = 133\,517\,688\, d^2$$

$$133\,517\,688\, d^2 + 1\,080\,000\, d - 64\,800 = 0$$

$$1d^2 + 0{,}0081\, d - 0{,}00049 = 0$$

$$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = +1$

$b = +0{,}0081$

$c = -0{,}000491$

$$d = \frac{-0{,}0081 \pm \sqrt{+0{,}0081^2 - [4 \times 1 \times (-0{,}00049)]}}{2 \times 1}$$

$$= \frac{-0{,}0081 \pm \sqrt{+0{,}00203}}{2}$$

$$= \frac{-0{,}03691}{2}$$

$= 0{,}0185$ Use the 20 mm diameter standard-size rivets

(b) Working strength of solid plate

$$P = \sigma_t \times p \times t$$

$$= 108 \times 10^6 N/m^2 \times 0{,}06\, m \times 0{,}01\, m$$

= 64,8 kN

(c) Working strength of pierced plate

$$P = \sigma_t \times (p - d) \times t$$

$$= 108 \times 10^6 N/m^2 \times (0{,}06\, m - 0{,}02\, m) \times 0{,}01\, m$$

= 43,2 kN

(d) Working shear strength of rivets

$$P = \tau \times N \times \frac{\pi d^2}{4}$$

$$= 85 \times 10^6 N/m^2 \times 2 \times \frac{\pi \times (0{,}02\, m)^2}{4}$$

= 53,41 kN

(e) Working crushing strength of rivets against the plate

$$P = \sigma_c \times N \times d \times t$$
$$= 148 \times 10^6 \mathrm{N/m^2} \times 0{,}02\ \mathrm{m} \times 0{,}01\ \mathrm{m}$$
$$= 59{,}2\ \mathrm{kN}$$

(f) The allowable safe load for the joint

It is the-smallest of the three working strengths of the joint, namely 43,2 kN.

(g) Efficiency of joint

$$\text{Efficiency} = \frac{\text{Working strength of rivets in shear}}{\text{Strength of solid plate}}$$
$$= \frac{43{,}2 \times 10^3 \mathrm{N}}{64{,}8 \times 10^3 \mathrm{N}} \times 100$$
$$= 66{,}7\%$$

Worked Example 3.8

Design a triple-riveted lap joint in which the pitch of the rivets in the outer rows is twice the pitch of the rivets in the inner row for plates 22 mm in thickness.

Determine the pitch and diameter of the rivets, taking the tensile stress in the plates as 93 MPa, the shearing stress in the rivets as 70 MPa and the crushing stress of the rivets against the plate as 118 MPa. Also find the efficiency of the joint.

Solution:

$$\text{Rivet diameter} = 6\sqrt{t}$$
$$= 6 \times \sqrt{22}$$
$$= 28{,}14\ mm$$

Use 30 mm standard-diameter rivets.

Pitch

$$\text{Strength of plate in tension} = \text{Strength of rivets in shear}$$
$$\sigma_t \times (p - d) \times t = \tau \times N \times \frac{\pi d^2}{4}$$

$$93 \times 10^6 N/m^2 \times (p - 0{,}03\ m) \times 0{,}022\ m = 70 \times 10^6 N/m^2 \times \frac{\pi \times (0{,}03\ m)^2}{4}$$
$$p = \frac{197\ 920\ N}{93 \times 10^6 N/m^2 \times 0{,}022\ m} + 0{,}03\ m$$
$$= 0{,}0967\ m + 0{,}03\ m$$
$$= 0{,}127\ \mathrm{m}$$
$$= 127\ \mathrm{mm}$$

Check that this is above the allowable minimum.

$$\text{Minimum pitch} = 2\ x\ d$$

= 2 x 30 mm
= 60 mm

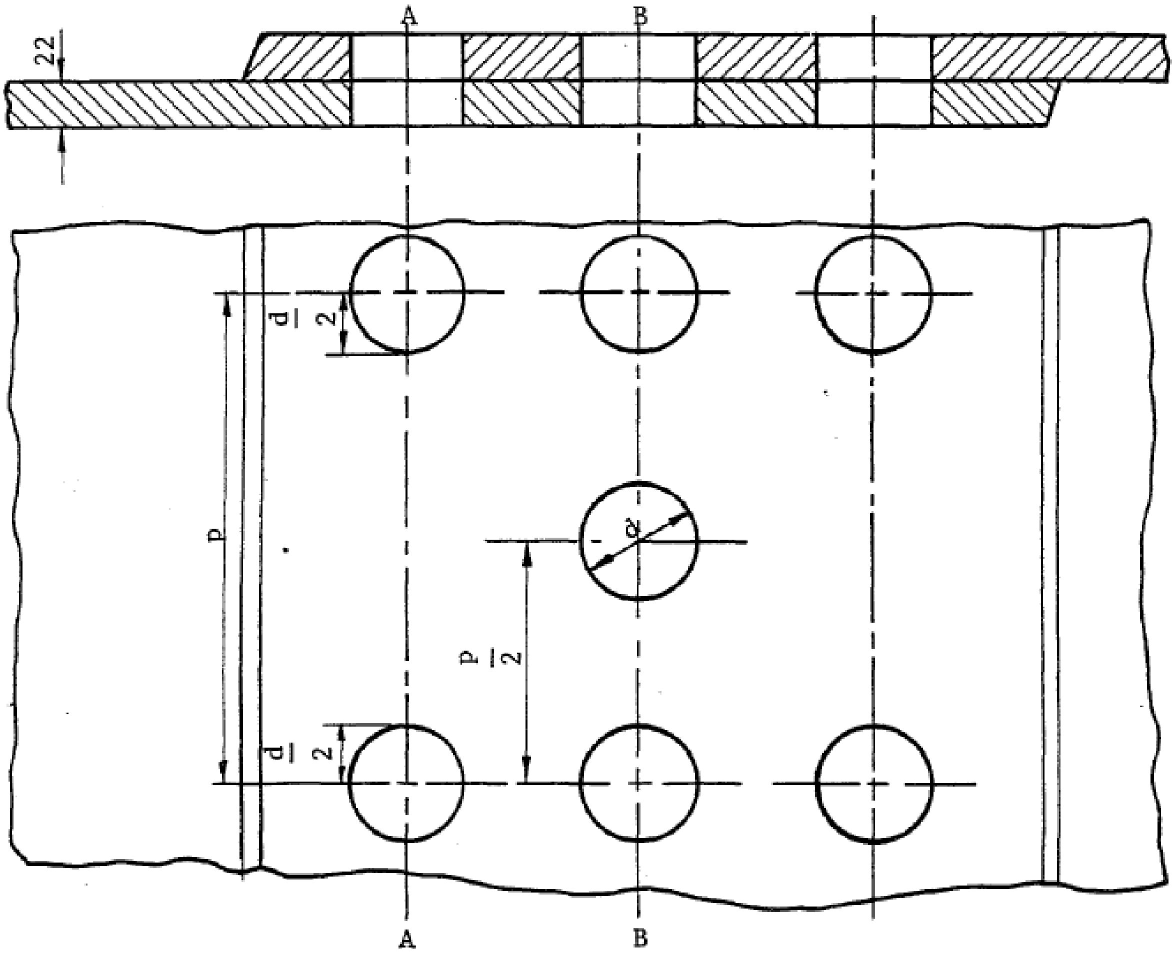


Figure 3.30

Strength of plate against tearing at AA $= \sigma_t \times (p - d) \times t$
$= 93 \times 10^6 N/m^2 \times (0{,}127\ m - 0{,}03\ m) \times 0{,}022\ m$
= 198,5 kN

Strength of plate against tearing at BB $= \sigma_t \times (p - d2) \times t$ + Strength of One rivet at AA in shear

$= \sigma_t \times (p - d2) \times t + \tau \times N \times \frac{\pi d^2}{4}$

$= 93 \times \frac{10^6 N}{m^2} \times (0{,}127\ m - 0{,}03\ m) \times 0{,}022\ m + 70 \times 10^6 N/m^2 \times 1 \times \frac{\pi \times (0{,}03 m)^2}{4}$

$= 137\ 082\ N + 49\ 480\ N$

$= 186{,}562\ kN$

Strength of rivets against shearing $= \tau \times N \times \frac{\pi d^2}{4}$
$= 70 \times 10^6 \mathrm{N/m^2} \times 4 \times \frac{\pi \times (0{,}03 \mathrm{m})^2}{4}$
$= 197{,}92\ \mathrm{kN}$

Strength of rivets against crushing $= \sigma_c \times N \times d \times t$
$= 118 \times 10^6 \mathrm{N/m^2} \times 0{,}03\ \mathrm{m} \times 0{,}022\ \mathrm{m}$

$= 311{,}52$ kN

Strength of solid plate $= \sigma_t \times p \times t$

$= 93 \times 10^6 \text{N/m}^2 \times 0{,}127 \text{ m} \times 0{,}022 \text{ m}$

$= 259{,}842$ kN

Therefore failing will occur in tearing the plate at BB 186,562 kN.

Efficiency of joint $= \dfrac{\textit{Failing load}}{\textit{Strength of solid plate}}$

$= \dfrac{186{,}562 \times 10^3}{259{,}842 \times 10^3 N}$

$= 0{,}718$

$= 71{,}8\%$

3.7 Butt joints

So far only lap joints have been discussed. The only difference in the calculation of butt joints with two cover straps is in shear. It should be remembered that each rivet shears twice.

The resistance offered by the rivet in butt joints is twice the resistance offered by the rivet in lap joints.

To be on the safe side, the rivet is not taken as being twice as strong as in single shear, but 1,75 times as strong, unless another value is mentioned in a question.

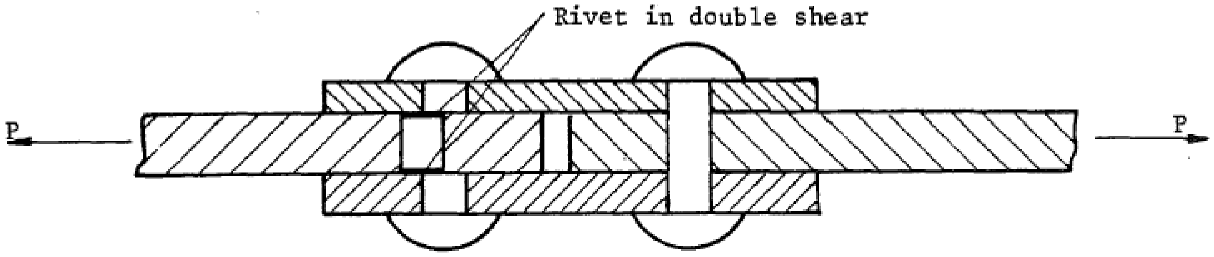


Figure 3.31

Cross-sectional area of rivet in double shear = 1,75 x Cross-sectional area of rivet in single shear

$= 1{,}75 \times \dfrac{\pi d^2}{4}$

Worked Example 3.9

Two steel plates, each 10 mm thick, are joined by a single-riveted butt joint using two cover plates. Determine the diameter and pitch of rivets by equating shear to tensile strength. Also determine the strengths and efficiencies of the joint.

Ultimate stresses: σ_t = 432 MPa

τ = 355 MPa
σ_C = 710 MPa

Solution:

Diameter of rivet $= 6\sqrt{t}$
$= 6 \times \sqrt{10}$
$= 18{,}97\ mm$

Use 20 mm diameter standard rivets.

Strength of plate in tension = Strength of rivets in shear

$\sigma_t \times (p-d) \times t = \tau \times N \times 1{,}75 \times \frac{\pi d^2}{4}$

(Single rivet, double shear)

$$432 \times 10^6 N/m^2 \times (p - 0{,}02\ m) \times 0{,}01\ m = 356 \times 10^6 N/m^2 \times 1 \times 1{,}75 \times \frac{\pi \times (0{,}02\ m)^2}{4}$$

$$p = \frac{195{,}17 \times 10^3\ N}{432 \times 10^6 N/m^2 \times 0{,}01\ m} + 0{,}02\ m$$

$$= 0{,}065\ m$$

Use 65 mm pitch.

Tensile strength of joint

$$P = \sigma_t \times (p-d) \times t$$
$$= 432 \times 10^6 N/m^2 \times (0{,}065\ m - 0{,}02\ m) \times 0{,}01\ m$$
= 194,4 kN

Shear strength of rivets

$$P = \tau \times N \times 1{,}75 \frac{\pi \times d^2}{4}$$
$$= 355 \times 10^6 \times 1 \times 1{,}75 \times \frac{\pi \times (0{,}02\text{m})^2}{4}$$
= 195,2 kN

Bearing strength of rivets against the plate

$$P = \sigma_c \times d \times t$$
$$= 710 \times 10^6 \text{N/m}^2 \times 0{,}02\ \text{m} \times 0{,}01\ \text{m}$$
= 142 kN

Strength of solid plate

$$P = \sigma_t \times p \times t$$
$$= 432 \times 10^6 \text{N/m}^2 \times 0{,}065\ \text{m} \times 0{,}01\ \text{m}$$
= 208,8 kN

Efficiencies

$$\text{Tensile} = \frac{194{,}4 \times 10^3 N}{280{,}8 \times 10^3 N} \times 100$$
= 69,23%

Shear $= \frac{195{,}2 \times 10^3 N}{280{,}8 \times 10^3 N} \times 100$
$= 69{,}52\%$

Crushing $= \frac{142 \times 10^3 N}{280{,}8 \times 10^3 N} \times 100$
$= 50{,}57\%$

Efficiency of joint is lowest calculated as above.
Efficiency of joint = 50,57%.

Worked Example 3.10

Design a double-riveted butt joint to connect two 12 mm steel plates, using two cover straps and the following ultimate stresses:

Tensile stress = 462 MPa
Shear stress = 372 MPa
Crushing stress = 648 MPa
Factor of Safety = 6

Solution:

Rivet diameter $= 6\sqrt{t}$
$= 6 \times \sqrt{12}$
$= 20{,}78\ mm$

Use 20 mm diameter standard-size rivets.

Pitch of rivets

Strength of plate in tension = Strength of rivets in shear

$$\sigma_t \times (p - d) \times t = \tau \times N \times 1{,}75 \times \frac{\pi d^2}{4}$$

$$462 \times 10^6 N/m^2 \times (p - 0{,}02\ m) \times 0{,}012\ m = 372 \times 10^6 N/m^2 \times 2 \times 1{,}75 \times \frac{\pi \times (0{,}02\ m)^2}{4}$$

(Double rivets, double shear)

$$p = \frac{409\ 035\ N}{462 \times 10^6 N/m^2 \times 0{,}012\ m} + 0{,}02\ m$$
$$= 0{,}093\ 7\ m$$

Use a pitch of 94 mm.

Working stresses

$$\sigma_t = \frac{462 \times 10^6 N/m^2}{6} = 77\ MPa$$
$$\tau = \frac{372 \times 10^6 N/m^2}{6} = 62\ MPa$$
$$\sigma_C = \frac{648 \times 10^6 N/m^2}{6} = 108\ MPa$$

Working loads

Tensile working load in solid plate $= \sigma_t \times p \times t$

$P = 77 \times 10^6 N/m^2 \times 0{,}094\ m \times 0{,}012\ m$

= 86,86 kN

Tensile working load in pierced plate $= \sigma_t \times (p - d) \times t$

$P = 77 \times 10^6 N/m^2 \times (0{,}094\ m \times 0{,}02\ m) \times 0{,}012m$

= 68,38 kN

Shear working load on rivets $= \tau \times N \times 1{,}75 \times \frac{\pi \times d^2}{4}$

$P = 62 \times 10^6 N/m^2 \times 2 \times 1{,}75 \times \frac{\pi \times (0{,}02\ m)^2}{4}$

= 68,71 kN

Crushing working load $= \sigma_C \times \mathrm{N} \times \mathrm{d} \times t$

$P = 108 \times \frac{10^6 N}{m^2} \times 2 \times 0{,}02\ \mathrm{m} \times 0{,}012\ \mathrm{m}$

= 51,84 kN

Efficiencies

Tensile $= \frac{68{,}38 \times 10^3 N}{86{,}86 \times 10^3 N} \times 100$

= 78,7%

Shear $= \frac{68{,}17 \times 10^3 N}{86{,}86 \times 10^3 N} \times 100$

= 78,5%

Crushing $= \frac{51{,}84 \times 10^3 N}{86{,}86 \times 10^3 N} \times 100$

= 59,7%

3.8 Lozenge joint

When two tie-bars (ie steel plates to carry tension such as occurs in a bridge) have to be connected by a riveted joint, the usual arrangement is a “lozenge” joint.

That this is the most efficient arrangement may be seen by considering the sketch shown in **Figure 3.32**. Sufficient rivets can be put in to make the joint as strong as, or stronger than, the solid plate in shearing and bearing.

As holes have to be punched in the plate, however, it cannot be made 100% strong in tension.

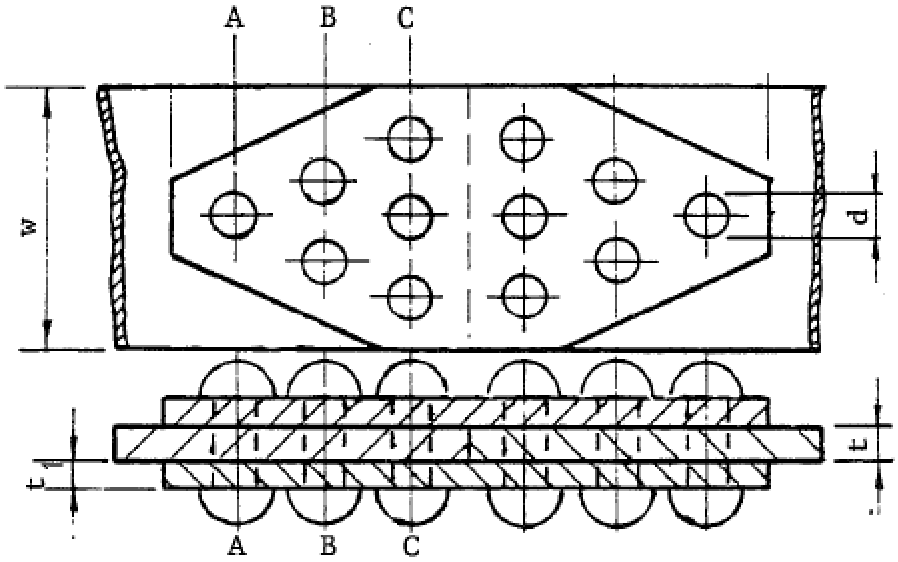


Figure 3.32

By arranging as shown, there is only one hole at section A, while at B, where there are two holes, failure cannot occur without shearing the rivet at A as well as tearing the plate at B.

At C there are three holes weakening the plate, but the 2 rivets at B and 1 rivet at A, a total of 3 rivets, have to be shorn before failure can occur at C.

Theoretically, the most efficient joint is given by a very large number of very small rivets. As this is impracticable, a reasonable size rivet has to be selected. This is best done by using the Formula$d = 6 \times \sqrt{t}$.

Always remember that the pitch must not be less than $2 \times d$ and the margin not less than $1{,}5 \times d$. Thickness of cover plates for a lozenge joint may be taken as $\frac{5 \times t(w-d)}{8(w-Nd)}$, where N = number of rivets at section CC.

Calculation of loads

At section A, load = stress x area

$$P = \sigma_t \times (w - d) \times t$$

At section B, load = stress x area + load of one rivet in double shear

$$P = \sigma_t \times (w - 2d) \times t + \tau \times N \times 1{,}75 \times \frac{\pi d^2}{4}$$

At section C, load = stress x area + load of three rivets in double shear

$$P = \sigma_t \times (w - 3d) \times t + \tau \times 3 \times 1{,}75 \times \frac{\pi d^2}{4}$$

Calculation of rivet diameter

To find rivet diameter, equate shearing load on one rivet to the crushing load on one rivet.

$$\tau \times 1{,}75 \times \frac{\pi d^2}{4} = \sigma_C \times d \times t$$

or

$$d = 6\sqrt{t}$$

Worked Example 3.11

Design a butt joint to connect two 178 mm x 10 mm tie-plates. Working stresses are: Tension 108 x 10^6 N/m², Shear 62 x 10^6 N/m² and Crushing 170 x 10^6 N/m².

Solution:

Rivet diameter

Shearing load on one rivet = Crushing load on one rivet

$$\tau \times 1{,}75 \times \frac{\pi d^2}{4} = \sigma_C \times d \times t$$

$$62 \times 10^6 N/m^2 \times 1{,}75 \times \frac{\pi \times d^2}{4} = 170 \times 10^6 N/m^2 \times d \times 0{,}010\ m$$

$$\frac{d^2}{d} = \frac{170 \times 10^6 N/m^2 \times 0{,}010\ m \times 4}{62 \times 10^6 N/m^2 \times 1{,}75 \times \pi}$$

$$d = 0{,}0199$$

Use 20 mm diameter standard-size rivets.

or $d = 6\sqrt{t}$

$= 6\sqrt{10}$

$= 18{,}97\ mm$ Say 20 mm diameter rivets

Safe tensile load of the plate at the section with one rivet hole

$$= \sigma_t \times (w - d) \times t$$

$$= 108 \times 10^6 N/m^2 \times (0{,}178\ m - 0{,}02) \times 0{,}010\ m$$

= 170,64 kN

Safe shearing load of one rivet in double shear $= \tau \times 1{,}75 \times \frac{\pi d^2}{4}$

$$= 62 \times 10^6 N/m^2 \times 1{,}75 \times \frac{\pi \times (0{,}02\ m)^2}{4}$$

= 34,09 kN

Safe crushing load of one rivet $= \sigma_C \times d \times t$

$$= 170 \times \frac{10^6 N}{m^2} \times 0{,}02\ m \times 0{,}01\ m$$

= 34 kN

Hence the crushing load determines the number of rivets required.
(**Note** that the crushing load is the minimum safe load).

Number of rivets $= \frac{Maximum\ safe\ load}{Minimum\ safe\ load}$

$= \frac{170{,}64 \times 10^3}{34 \times 10^3}$

$= 5{,}02$

Use 6 rivets arranged as in **Figure 3.33**.

Thickness of cover plates = $\frac{5 \times t(w-d)}{8(w-Nd)} = \frac{5 \times 0{,}01(0{,}178-0{,}02)}{8(0{,}178-3 \times 0{,}02)}$

$t_1 = 0{,}0084$

Say 10 mm

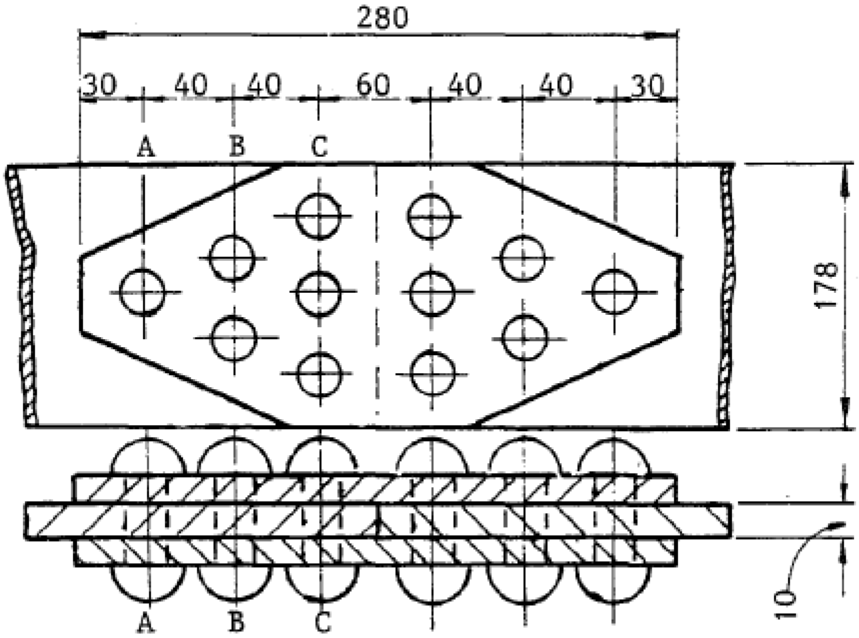


Figure 3.33

Safe tensile load at section AA $= \sigma_t \times (w-d) \times t$

$= 108 \times 10^6 N/m^2 \times (0{,}178\ m - 0{,}02\ m) \times 0{,}01\ m$

= 170,64 kN

Safe tensile load at section BB $= \sigma_t \times (p - 2d) \times t$ + Shear load of one rivet in double shear

$= \sigma_t \times (w - 2d) \times t + \tau \times 1{,}75 \times \frac{\pi d^2}{4}$

$= 108 \times 10^6 N/m^2 \times (0{,}178\ m - 2\ m \times 0{,}02\ m) + 0{,}01\ m$

$+62 \times 10^6 N/m^2 \times 1{,}75 \times \frac{\pi \times (0{,}02m)^2}{4}$

$= 149\ 040\ N + 34\ 086\ N$

$= 183{,}126\ kN$

Safe tensile load at section CC $= \sigma_t \times (w - 3d) \times t$ + Shear load of three rivets in double shear

$= \sigma_t \times (w - 3d) \times t + \tau \times 1{,}75 \times N \times \frac{d^2}{4}$

$= 108 \times 10^6 N/m^2 \times (0{,}178\ m - 3\ \times 0{,}02\ m) + 0{,}01\ m$

$+62 \times 10^6 N/m^2 \times 1{,}75 \times 3 \times \frac{\pi \times (0{,}02m)^2}{4}$

$= 127\ 440\ N + 102\ 259\ N$

$= 229{,}699\ kN$

Safe shear load for all rivets $= \tau \times 1{,}75 \times N \times \frac{d^2}{4}$

$= 62 \times 10^6 N/m^2 \times 1{,}75 \times 6 \times \frac{\pi\times(0{,}02m)^2}{4}$

$= 204{,}52\ kN$

Safe crushing load for rivets against main plate $= \sigma_C \times N \times d \times t$

$= 170\ \times 10^6 N/m^2 \times 6 \times 0{,}02\ m \times 0{,}01\ m$

= 204 kN

Safe crushing load for rivets against cover plates $= \sigma_C \times N \times d \times 2 \times t_1$

$= 170\ \times 10^6 N/m^2 \times 6 \times 0{,}02\ m \times 2 \times 0{,}010\ m$

= 408 kN

Safe tensile load for cover plates at section CC $= \sigma_t \times (w - 3d) \times d \times t$

$= 108\ \times 10^6 N/m^2 \times (0{,}178\ m - 3 \times 0{,}02\ m) \times 2 \times 0{,}010\ m$

= 254,9 kN

Hence weakest part is at AA.

Joint Efficiency $= \frac{170{,}64 \times 10^3 N}{w \times t \times \sigma_t}$

$= \frac{170{,}64 \times 10^3 N}{0{,}178\ m \times 0{,}01\ m \times 108 \times 10^6 N/m^2}$

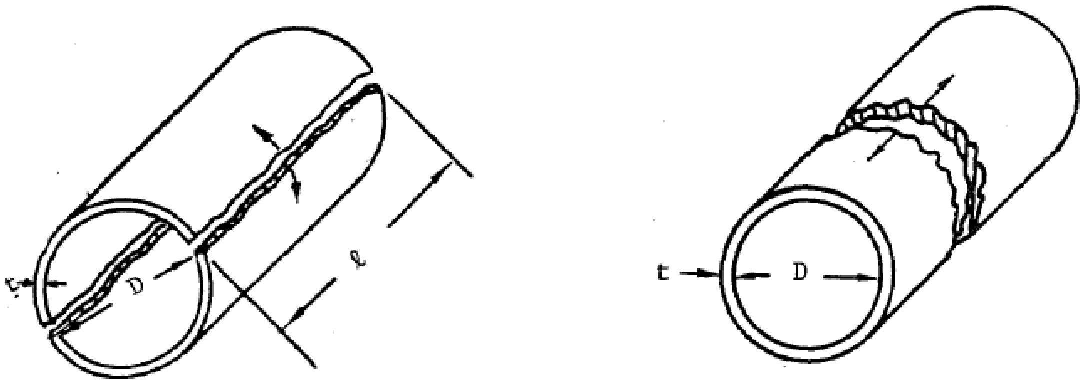
= 0,888

= 88,8%

3.9 Thin cylinders

When the thickness of the metal is small compared with the diameter of the cylinder (shell), the relationship between the internal pressure and the thickness of the metal may be calculated as follows.

There are two ways in which a thin cylinder or pipe subjected to internal pressure can burst. **Figure 3.34a** represents failure along a longitudinal section and **Figure 3.34b** on a circumferential section.



a. b.

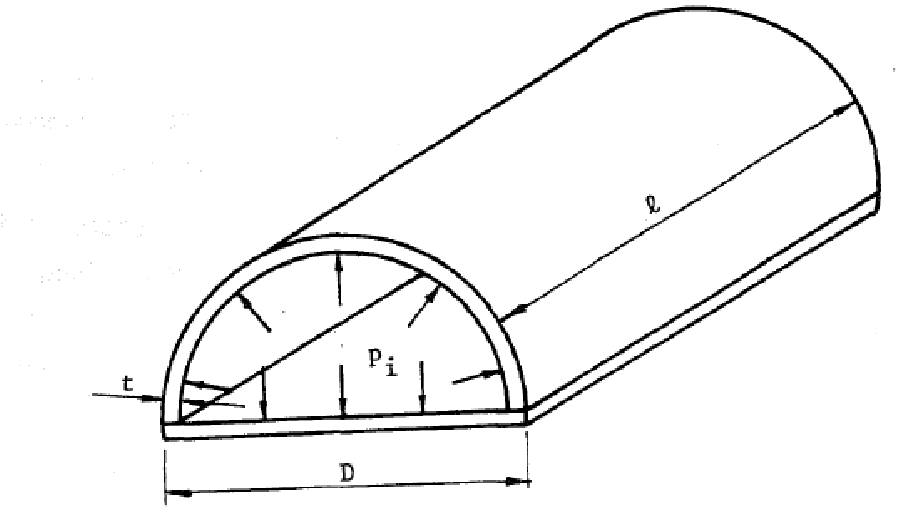
Figure 3.34

3.9.1 Longitudinal Section

The force caused by the internal pressure (p_i) tending to separate the two halves is resisted by the tensile stress in the metal across the longitudinal section.

A portion sufficiently far from the end plates to receive no assistance from them is considered.

Consider the equilibrium of one half of the cylinder with an imaginary steel plate attached as shown.



p_i = internal pressure (gauge pressure)
D = internal diameter
ℓ = length
t = thickness of metal

Figure 3.35

Note:
Sometimes the absolute pressure is given instead of gauge pressure therefore the following formula must be used to obtain the gauge pressure:

$$\textit{Gauge pressure} = \textit{Absolute pressure - Atmospheric pressure}$$

$$\textit{where 1 atmosphere} = 101{,}4\ kPA.$$

The resultant upward force on the curved surface must equal the downward force on the plate, namely, $P_i\ x\ D\ x\ l$ newtons.

Hence the force tending to force the two halves of the cylinder apart is $P_i\ x\ D\ x\ l$ newtons.

This force is resisted by the stress in the metal $\sigma_t\ N/m^2$ acting on two sections each $l \times t\ m^2$.

Hence

$$2 \times \sigma_t \times l \times t = P_i\ x\ D\ x\ l$$

$$\sigma_t = \frac{P_i\ x\ D\ x\ l}{2 \times l \times t}$$

$$\sigma_t = \frac{P_i\ x\ D}{2 \times t}$$

If the cylinder is constructed from riveted plates and the efficiency of the longitudinal joint is N_l then the average stress in the longitudinal joint is as follows:

$$\sigma_{t \times \pi \times D \times t} = \frac{P_i\ x\ D}{2 \times t \times n_l}$$

This is called the circumferential or hoop stress.

Note:
Circumferential or hoop stress acts on a longitudinal section.

3.9.2 Circumferential Section

The force tending to separate the two halves is the force on the end plates (see **Figure 3.34**), namely $\frac{\pi \times D^2}{4} \times P_i$

This is resisted by the stress (at) in the metal acting across a circumferential section of area $\pi \times\ D\ x\ t$.

$$\sigma_t \times \pi \times D \times t = \frac{\pi \times D^2}{4} \times P_i$$

$$\sigma_t = \frac{\pi \times D^2}{4} \times \frac{P_i}{\pi \times D \times t}$$

$$\sigma_t = \frac{P_i \times D}{4 \times t}$$

If the cylinder is constructed from riveted plates and the efficiency of the circumferential joint is n_C then the average stress in the joint is given by

$$\sigma_t = \frac{P_i \times D}{4 \times t \times n_C}$$

This is the longitudinal stress.

Note:
Longitudinal stress acts on a circumferential section.

3.10 Joints for cylindrical pressure vessels; boilers, tanks, etc

3.10.1 Longitudinal Joints

The type of joint (namely, lap or butt, single-riveted or multiple riveted) has to be settled. Usually a treble-riveted butt joint with double cover straps and zig-zag riveting is chosen, since this type of joint gives a high efficiency, which is required for the longitudinal seam.

It depends largely on the diameter of the vessel and the working pressure, and definite rules cannot be given. A joint efficiency must then be assumed, based on the following values:

Lap joints:	Single-riveted	55%
	Double-riveted	70%
	Treble-riveted	77%
Butt joints:	Single-riveted	65%
	Double-riveted	80%
	Treble-riveted	85%
	Quadruple-riveted	90%

The plate thickness t may be calculated from

$$\sigma_t = \frac{P_i \times D}{2 \times t \times n_l}$$

Taking $\sigma_t = 83\ MPa$ (for steel), and the rivet diameter, pitch, etc. may then be determined.

This may be made weaker than the longitudinal joint – compare

$\sigma_t = \frac{P_i \times D}{4 \times t}$ with $\sigma_t = \frac{P_i \times D}{2 \times t}$. It is usually of the lap type with rivets equal in diameter to those for the longitudinal joint.

The pitch may be settled for staunchness and the rivet shearing area checked. Thus, if p is the "ring pitch", and if there are n rows, the total number of rivets is

$$N_T \quad = \frac{n \times \pi \times D}{p}$$

The total shear area = $N_T \times \frac{nd^2}{4}$ and the total resistance to

shear = $\tau \times N_T \times \frac{nd^2}{4}$

The value of τ given by equating this resistance to the axial

bursting force $\frac{nD^2}{4} \times (p_i)$ should not exceed 65,5 MPa.

Worked Example 3.12

A cylindrical vessel having a diameter of 2m is subjected to an internal pressure of 1,25 MPa. The vessel is constructed from steel plates 16 mm thick which have an ultimate tensile stress of 450 MPa.

If the efficiencies of the longitudinal and circumferential joints are 80 and 60 per cent respectively, what is the factor of safety?

<u>Solution:</u>

<u>Longitudinal Joint</u>

$$\sigma_t = \frac{P_i \times D}{2 \times t \times n_l}$$

$$= \frac{1{,}25 \times 10^6 N/m^2 \times 2m}{2 \times 0{,}016\, m \times 0{,}8}$$

$$= 97{,}7\ MPa$$

$$Factor\ of\ safety = \frac{ultimate\ stress}{Working\ stress}$$

$$= \frac{450 \times 10^6 N/m^2}{97{,}7 \times 10^6 N/m^2}$$

<u>Circumferential Joint</u>

$$\sigma_t = \frac{P_i \times D}{4 \times t \times n_c}$$

$$= \frac{1{,}25 \times 10^6 N/m^2 \times 2m}{4 \times 0{,}016\, m \times 0{,}6}$$

$$= 65{,}1\ MPa$$

$$Factor\ of\ safety = \frac{ultimate\ stress}{Working\ stress}$$

$$= \frac{450 \times 10^6 N/m^2}{65{,}1 \times 10^6 N/m^2}$$
$$= 6{,}9\ MPa$$

Therefore the factor of safety is the lowest of the two, namely, 4,61.

Worked Example 3.13

A cylindrical vessel (1,8 m internal diameter), is required to withstand an internal pressure of 552 kPa. Determine the thickness of metal required and design the longitudinal and circumferential joints which are to be riveted. Take σ_t, τ and σ_c as 69 MPa, 52 MPa, 144 MPa, respectively.

Solution:

As the hoop stress is double the longitudinal stress, the required thickness of metal is determined from the hoop stress.

Longitudinal joint

Assume a double-riveted butt joint chain riveting with two cover straps, efficiency 80%

$$\sigma_t = \frac{P_i \times D}{2 \times t \times n_l}$$
$$t = \frac{552 \times 10^3 N/m^2 \times 1{,}8m}{2 \times 69 \times 10^6 N/m^2 \times 0{,}8}$$
$$= 0{,}009\ m$$

$$\text{Say} = 10\ mm\ plate\ thickness$$

$$Factor\ of\ safety = \frac{ultimate\ stress}{Working\ stress}$$

$$= \frac{450 \times 10^6 N/m^2}{97{,}7 \times 10^6 N/m^2}$$

$$Diameter\ of\ rivets = 6\sqrt{t}$$
$$= 6\sqrt{10}$$
$$= 18{,}97\ mm$$

Use 20 mm diameter rivets, standard size.

<u>*Pitch*</u>

Equating tensile strength to shear strength

$$\sigma_t \times (p - d) \times t = \tau \times N \times \frac{\pi d^2}{4} \times 1{,}75$$

$69 \times 10^6 N/m^2 \times (p - 0{,}02\ m) \times 0{,}01\ m = 52 \times 10^6 N/m^2 \times 2 \times \frac{\pi \times (0{,}02\ m)^2}{4} \times 1{,}75$

(Double rivets, double shear)

$$p = \frac{57\ 177\ N}{69 \times 10^6 N/m^2 \times 0{,}01\ m} + 0{,}02\ m$$
$$= 0{,}083\ m + 0{,}02\ m$$
$$= 0{,}103\ m$$
$$= 103\ m$$

Thus pitch will be strong enough but we have to check it for steam tightness.

$$n = \frac{p-d}{p}$$
$$0{,}8 = \frac{p - 0{,}02\ m}{p}$$
$$0{,}8\ p = p - 0{,}02\ m$$
$$p - 0{,}8 = 0{,}02\ m$$
$$0{,}2\ p = 0{,}02\ m$$
$$p = \frac{0{,}02\ m}{0{,}2}$$
$$p = 0{,}1$$

Make the pitch equal to 100 mm.

Actual Efficiency of Longitudinal joint

$$\text{Tension} = \frac{\sigma_t \times (p-d) \times t}{\sigma_t \times p \times t} = \frac{p-d}{p}$$
$$= \frac{0{,}1\ m - 0{,}02\ m}{0{,}1\ m}$$
$$= 0{,}8$$
$$= 80\%$$

$$\text{Shear} = \frac{\tau \times N \times \pi d^2 \times 1{,}75}{\sigma_t \times p \times t \times 4}$$
$$= \frac{52 \times 10^6 N/m^2 \times 2 \times \pi \times (0{,}02\ m)^2}{69 \times 10^6 N/m^2 \times 0{,}1\ m \times 0{,}01\ m \times 4} \times 1{,}75$$
$$= 0{,}829$$
$$= 82{,}9\%$$

$$\text{Crushing} = \frac{\sigma_c \times N \times d \times t}{\sigma_t \times p \times t}$$
$$= \frac{144 \times 10^6 N/m^2 \times 2 \times 0{,}02\ m}{69 \times 10^6 N/m^2 \times 0{,}1\ m}$$
$$= 0{,}835$$
$$= 83{,}5\%$$

The efficiency of the joint = 80%.

Thickness of cover plates (t_1)

$$t_1 = 0{,}625 \times t$$
$$= 0{,}625 \times 0{,}1\ m$$
$$= 0{,}0625\ m$$

Use 8 mm standard thickness plate.

Margin of plate

$$y = 1{,}5 \times d$$
$$= 1{,}5 \times 0{,}02\ m$$
$$= 0{,}03\ m = 30\ mm$$

Distance between rows

$$P_r = 2 \times d$$
$$= 2 \times 0{,}2\ m$$
$$= 0{,}04\ m$$
$$= 40\ mm$$

Circumferential joint

A single-riveted lap joint will be assumed using the same size of rivets as for the longitudinal joint.

As 10 mm plate is being used, a joint efficiency of $n_c = \frac{P_i \times D}{\sigma_t \times 4 \times t}$ is required.

$$= \frac{552 \times 10^3 N/m^2 \times 1{,}8\ m}{69 \times 10^6 N/m^2 \times 4 \times 0{,}01\ m} \times 1{,}75$$
$$= 0{,}36$$
$$= 36\%$$

Equating tensile strength to shear strength

$$\sigma_t \times (p-d) \times t = \tau \times N \times \frac{\pi d^2}{4}$$
$$69 \times 10^6 N/m^2 \times (p - 0{,}02\ m) \times 0{,}01\ m = 52 \times 10^6 N/m^2 \times 1 \times \frac{\pi \times (0{,}02\ m)^2}{4}$$
$$= \frac{16\ 336{,}3\ N}{69 \times 10^6 N/m^2 \times 0{,}01} + 0{,}02\ m$$
$$= 0{,}0437$$
$$= 43{,}7\ mm$$

Number of rivets in circumference $= \frac{\pi \times D}{p}$

$$N_T = \frac{\pi \times 1{,}8\ m}{0{,}0437}$$
$$= 129\ rivets$$

Use 129 rivets with an exact pitch of

$$= \frac{\pi \times 1{,}8\ m}{129} = 0{,}0438$$
$$= 43{,}83\ mm\ pitch$$

Check if 129 rivets is sufficient

$$\frac{\pi D^2}{4} \times pressure = \tau \times N_T \times \frac{\pi d^2}{4}$$

$$N_T = \frac{\pi D^2 \times p_i \times 4}{4 \times \tau \times \pi d^2}$$

$$= \frac{D^2 \times p_i}{\tau \times d^2}$$

$$= \frac{(1{,}8\ m)^2 \times 552 \times 10^3 N/m^2}{52 \times 10^6 N/m^2 \times (0{,}02\ m)^2}$$

$$= 86\ rivets$$

129 rivets is more than sufficient.

Actual Efficiency of Circumferential joint

$$\text{Tension} = \frac{\sigma_t \times (p-d) \times t}{p \times t \times \sigma_t} = \frac{p-d}{p}$$

$$= \frac{43{,}83\ mm - 20\ mm}{43{,}83\ mm}$$

$$= 0{,}544$$

$$= 54{,}4\%$$

$$\text{Shear} = \frac{\tau \times \frac{\pi d^2}{4}}{p \times t \times \sigma_t}$$

$$= \frac{52 \times 10^6 N/m^2 \times \pi \times (0{,}02\ m)^2}{0{,}04383\ m \times 0{,}01\ m \times 69 \times 10^6 N/m^2 \times 4}$$

$$= 0{,}54$$

$$= 54\%$$

$$\text{Crushing} = \frac{\sigma_c \times d \times t}{p \times t \times \sigma_t}$$

$$= \frac{144 \times 10^6 N/m^2 \times 0{,}02\ m \times 0{,}01\ m}{0{,}04383\ m \times 0{,}01\ m \times 69 \times 10^6 N/m^2}$$

$$= 0{,}952$$

$$= 95{,}2\%$$

The joint is satisfactory.

Worked Example 3.14

A boiler of 2 200 mm internal diameter is required to generate steam at a pressure of 1,7 MPa. Calculate the thickness of the shell plates and design the longitudinal and circumferential riveted joints.

The material used will be steel of ultimate tensile stress of 460 MPa and a factor of safety of 5 is to be used. The shearing stress is to be taken as $\frac{4}{5}$ of the tensile stress.

Solution:

Working stresses

$$\sigma_t = \frac{Ultimate\ tensile\ stress}{Factor\ of\ safety}$$
$$= \frac{460\times10^6 N/m^2}{5}$$
$$= 92\ MPa$$
$$\tau = \frac{4}{5}\times 92 \times 10^6 N/m^2$$
$$= 73{,}6\ MPa$$

Longitudinal joint

Assume a treble-riveted butt joint with double cover straps and diagonal riveting, having an efficiency of 85%. The joint riveting arrangement will be with alternate rivets omitted from the outer row.

Plate thickness

$$\sigma_t = \frac{P_i \times D}{2 \times t \times n_l}$$
$$t = \frac{P_i \times D}{2 \times \sigma_t \times n_l}$$
$$= \frac{1{,}7\times 10^6 N/m^2 \times 2{,}2\ m}{2 \times 92\ m \times 10^6 N/m^2 \times 0{,}85}$$
$$= 0{,}0239\ m$$

Use 24 mm standard thickness plate.

Diameter of rivets

$$d = 6\sqrt{t}$$
$$= 6\sqrt{24}$$
$$= 29{,}4\ mm$$

Use 30 mm diameter rivets

Pitch for outer-row rivets

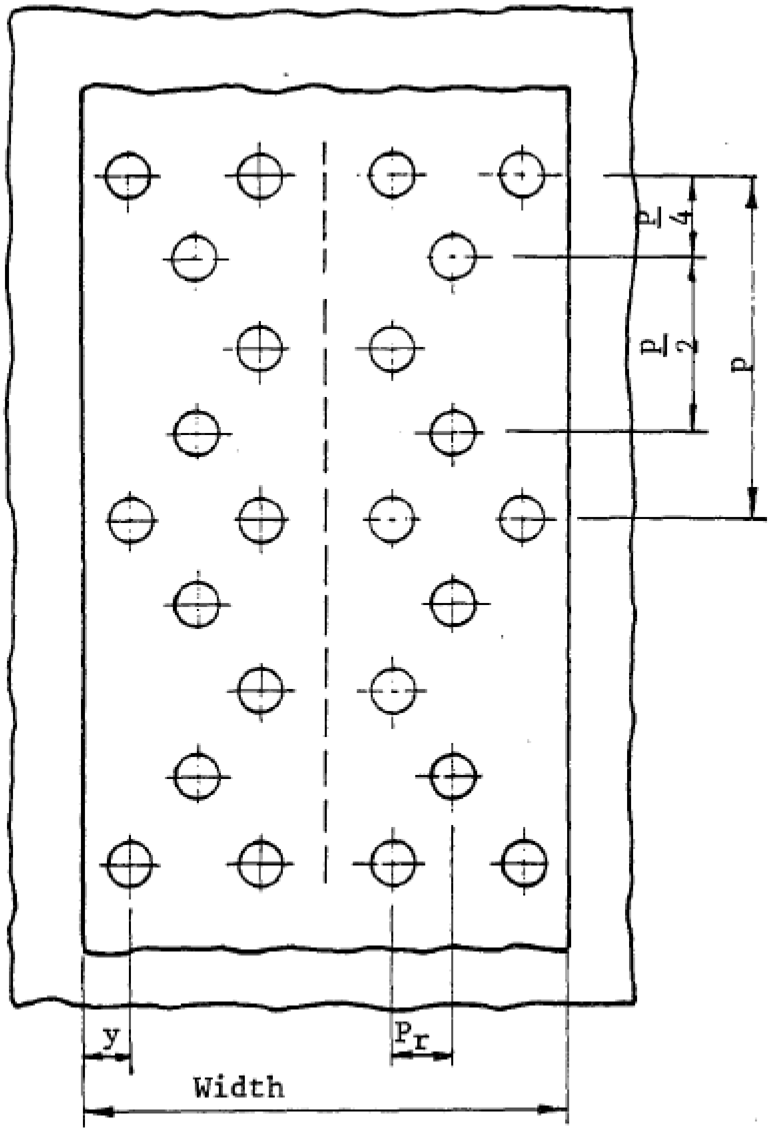


Figure 3.36

Equating tensile strength to shear strength

$$\sigma_t \times (p - d) \times t = \tau \times N \times \frac{\pi d^2}{4} \times 1{,}75$$

$$92 \times 10^6 N/m^2 \times (p - 0{,}030\ m) \times 0{,}024\ m = 73{,}6 \times 10^6 N/m^2 \times 5 \times \frac{\pi \times (0{,}03\ m)^2}{4} \times 1{,}75$$

p = 5 rivets contained in one pitch

$$p = \frac{455\ 217\ N}{92 \times 10^6 N/m^2 \times 0{,}024\ m} + 0{,}03\ m$$

$$= 0{,}236\ m$$

$$= 236\ mm$$

This pitch will be strong enough but we have to check it for steam tightness.

$$n = \frac{p-d}{p}$$

$$0{,}85 = \frac{p-d}{p}$$

$$0{,}85\,p = p - 0{,}03\,m$$
$$p - 0{,}85\,p = 0{,}03\,m$$
$$0{,}15\,p = 0{,}03\,m$$
$$p = \frac{0{,}03\,m}{0{,}15}$$
$$= 0{,}2\,m$$

Make the outer pitch equal to 200 mm.

Actual Efficiency of Longitudinal joint

Tensile strength of joint $= \sigma_t \times (p - d) \times t$

$= 92 \times \frac{10^6 N}{m^2} \times (0{,}2\,m - 0{,}03\,m) \times 0{,}024\,m$

$= 375{,}36\,kN$

Shear strength of joint $= \tau \times N \times \frac{\pi d^2}{4} \times 1{,}75$

$= 73{,}6 \times 10^6 N/m^2 \times 5 \times \frac{\pi \times (10{,}03)^2}{4} \times 1{,}75$

$= 455{,}22\,kN$

Thus the safe load which the joint can carry = 375,36 kN

Strength of solid plate $= \sigma_t \times p \times t$

$= 92 \times 10^6 N/m^2 \times 0{,}2\,m \times 0{,}024\,m$

$= 441{,}6\,kN$

Efficiency for tearing $= \frac{375{,}36 \times 10^3 \text{N}}{441{,}6 \times 10^3 \text{N}}$

$= 85\%$

Efficiency for shearing $= \frac{455{,}22 \times 10^3 \text{N}}{441{,}6 \times 10^3 \text{N}}$

$= 103{,}1\%$

Check crushing stress

$$\sigma_C = \frac{Safe\ load}{\text{Projected area rivets against plate}}$$
$$= \frac{375{,}36 \times 10^3 \text{N}}{5 \times \text{d} \times \text{t}}$$
$$= \frac{375{,}36 \times 10^3 \text{N}}{5 \times 0{,}03\text{ m} \times 0{,}024\text{ m}}$$

$= 104{,}3\,MPa$ well on the safe side

Thickness of cover plates

For joints of this kind

$$t_1 = \frac{5 \times t(p-d)}{8(p-2d)}$$
$$= \frac{5 \times 0{,}024\,m(0{,}2\,m - 0{,}03\,m)}{8 \times (0{,}2\,m - 2 \times 0{,}03\,m)}$$
$$= 0{,}0182\,m$$

Use 20 mm standard plate thickness.

Margin of plate

$$y = 1{,}5 \times d$$
$$= 1{,}5 \times 0{,}03\ m$$
$$= 45\ mm$$

Distance between rows of rivets

$$P_r = 2 \times d$$
$$= 2 \times 30\ mm$$
$$= 60\ mm$$

Diagonal pitch

$$pd = \sqrt{\left(\frac{p}{4}\right)^2 + (P_r)^2}$$
$$= \sqrt{\left(\frac{200}{4}\right)^2 + 60^2}$$
$$= 78{,}1\ mm$$

Width of butt straps

$$(4 \times y) + (4 \times P_r)$$
$$= (4 \times 45\ mm) + (4 \times 60\ mm)$$
$$= 420\ mm$$

Circumferential joint

A double-riveted lap joint with diagonal riveting is assumed. The same size of rivets as for the longitudinal joint is used.

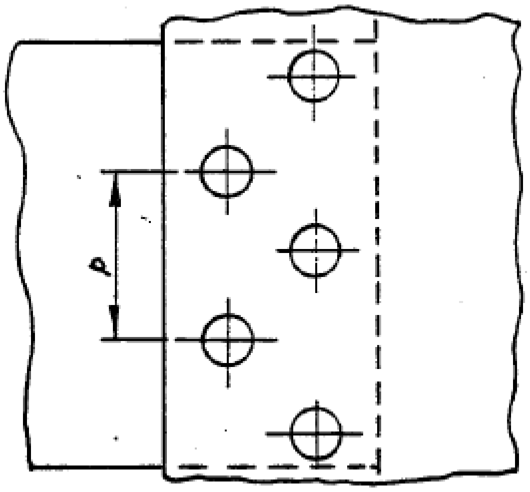


Figure 3.37

As 24 mm plate is being used, a joint efficiency of

$$n_c = \frac{P_i \times D}{\sigma_t \times 4 \times t}$$
$$= \frac{1{,}7 \times 10^6 N/m^2 \times 2{,}2\ m}{92 \times 10^6 N/m^2 \times 0{,}024\ m}$$
$$= 42{,}4\%$$

Equating tensile strength to shear strength.

$$\sigma_t \times (p-d) \times t = \tau \times N \times \frac{\pi d^2}{4}$$
$$92 \times 10^6 N/m^2 \times (p - 0{,}03\ m) \times 0{,}024\ m = 73{,}6 \times 10^6 N/m^2 \times 2 \times \frac{\pi \times (0{,}03\ m)^2}{4}$$
$$p = \frac{104\ 050\ N}{92 \times 10^6 N/m^2 \times 0{,}024\ m} + 0{,}03\ m$$
$$= 0{,}077\ m$$
$$= 77\ mm$$

Number of rivets in circumference $= \frac{\pi \times \pi \times D}{p}$

$$N = \frac{2 \times \tau \times 2{,}2\ m}{0{,}077}$$
$= 179{,}5\ rivets$ (say 180)

Use 90 rivets in each row

$$= \frac{\pi \times 2{,}2\ m}{90}$$
$$= 76{,}79\ mm$$

Check if 180 rivets is sufficient

$$\frac{\pi D^2}{4} \times P_i = \tau \times N \times \frac{\pi d^2}{4}$$
$$N = \frac{\pi D^2}{4} \times P_i \times \frac{4}{\tau \times \pi d^2}$$
$$= \frac{D^2 \times p_i}{\tau \times d^2}$$
$$= \frac{(2{,}2\ m)^2 \times 1{,}7 \times 10^6 N/m^2}{73{,}6 \times 10^6 N/m^2 \times (0{,}03\ m)^2}$$
$$= 124{,}2\ rivets$$

180 rivets is more than sufficient.

Actual Efficiency of Longitudinal joint

Tensile strength of joint $= \sigma_t \times (p-d) \times t$

$$= 92 \times 10^6 N/m^2 \times (0{,}07679\ m - 0{,}03\ m) \times 0{,}024\ m$$
$$= 103{,}31\ kN$$

Shear strength of joint $= \tau \times N \times \frac{\pi d^2}{4}$

$= 73{,}6 \times 10^6 N/m^2 \times 2 \times \frac{\pi \times (0{,}03)^2}{4}$
$= 104{,}05\ kN$

Strength of solid plate $= \sigma_t \times p \times t$
$= 92 \times 10^6 N/m^2 \times 0{,}07679\ m \times 0{,}024\ m$
$= 169{,}55\ kN$

Efficiency of circumferential joint

$$n_c = \frac{103{,}31 \times 10^3 N}{169{,}55 \times 10^3 N}$$
$$= 60{,}9\%$$

This is more than the 42,4% which is required.

$$pd = \sqrt{(P_r)^2 + \left(\frac{p}{2}\right)^2}$$
$$= \sqrt{60^2 + \left(\frac{76{,}79}{2}\right)^2}$$
$$= 71{,}23\ mm$$

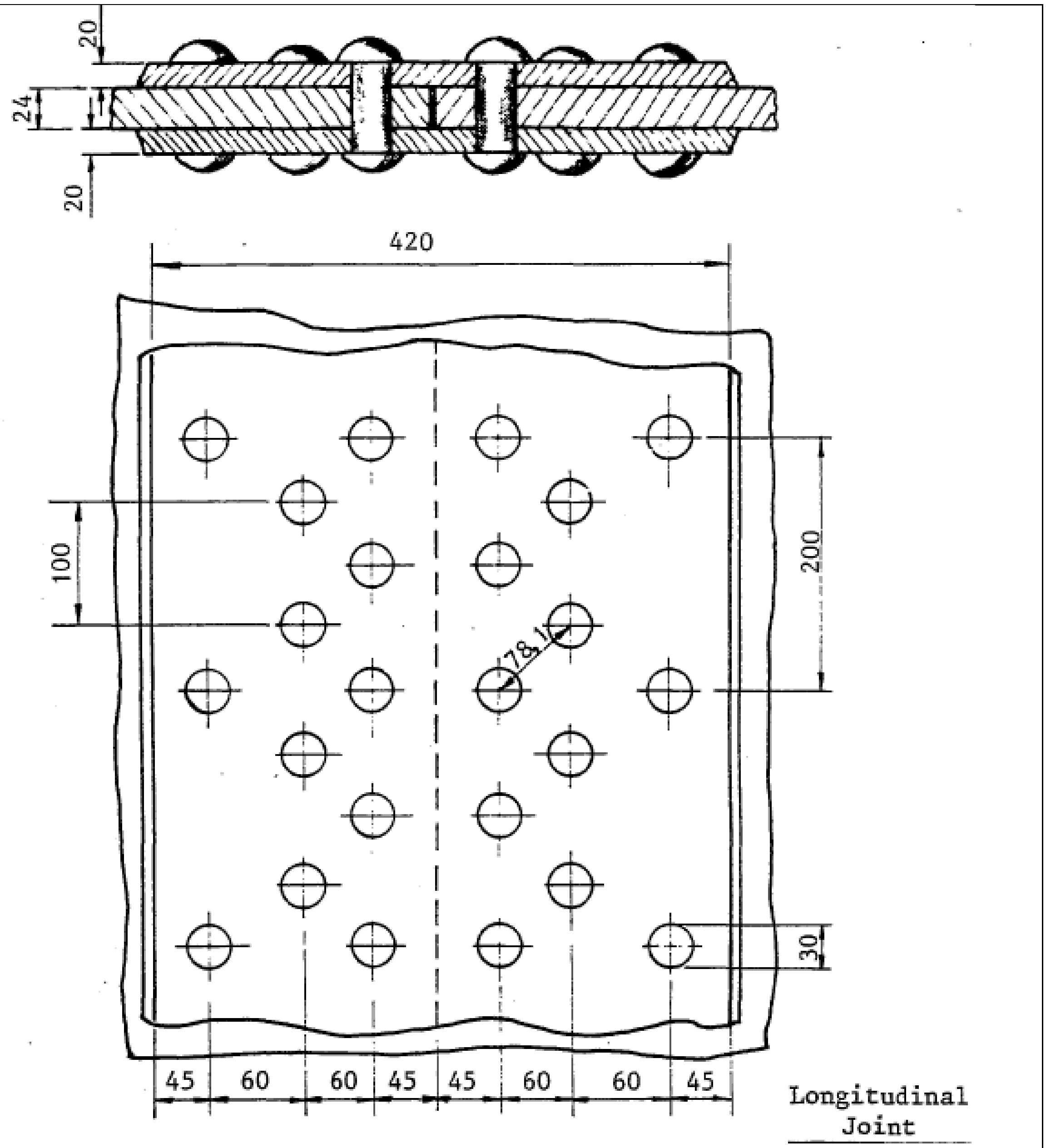

20
24
20
420
100
200
78,1
30
45
60
60
45
45
60
60
45
Longitudinal
Joint

Figure 3.38

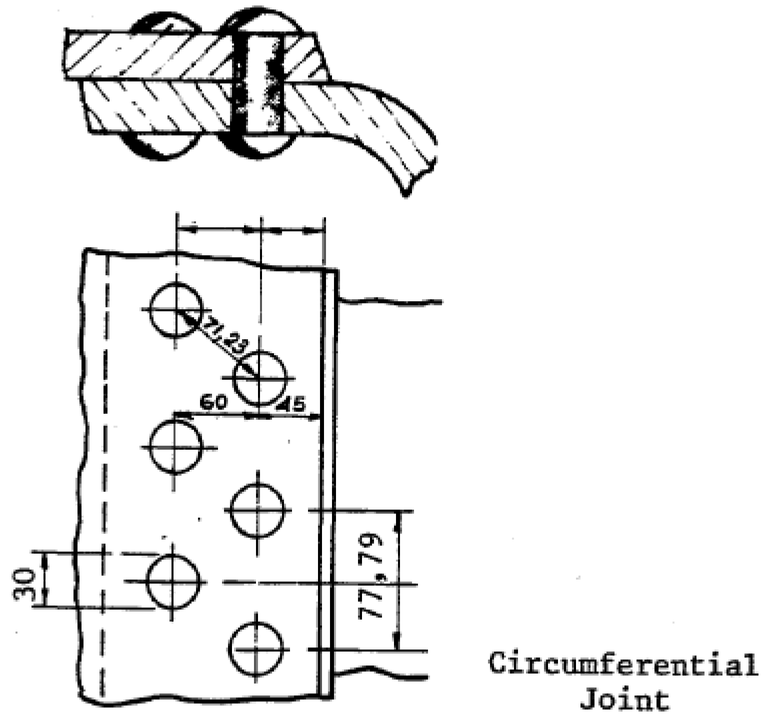


Figure 3.39

3.11 Fasteners

Fasteners are used to hold or fasten objects in a definite position with respect to other objects. There is a great variety of fasteners used today in the assembling of machines, building frames, bridges and other objects.

Fasteners are also used to hold objects in a definite place. Some of the fasteners are so widely used, that the manufacturers and other interested persons have agreed that they shall be made according to definite specifications as to size, shape and other details.

Think about it!
Such fasteners are regarded as being "standardised", because they are manufactured according to "standard" specifications.

3.11.1 Threaded fasteners

The threaded fasteners, such as screws, bolts and nuts, are most frequently used, and should be shown in position in an assembly drawing. All forms of the threaded fasteners depend upon the screw threads for their holding power.

Threads may be classified as either square-section or vee-section. The square-section threads are normally used for power transmission, while the vee-threads are used for nuts and bolts, pipe fixings and finer work.

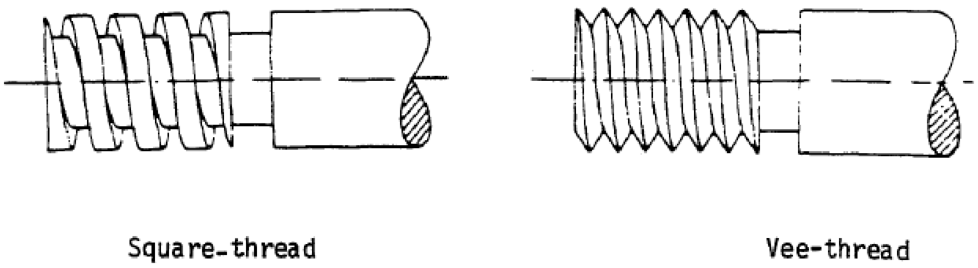


Figure 3.40

3.11.2 Proportions of ISO thread

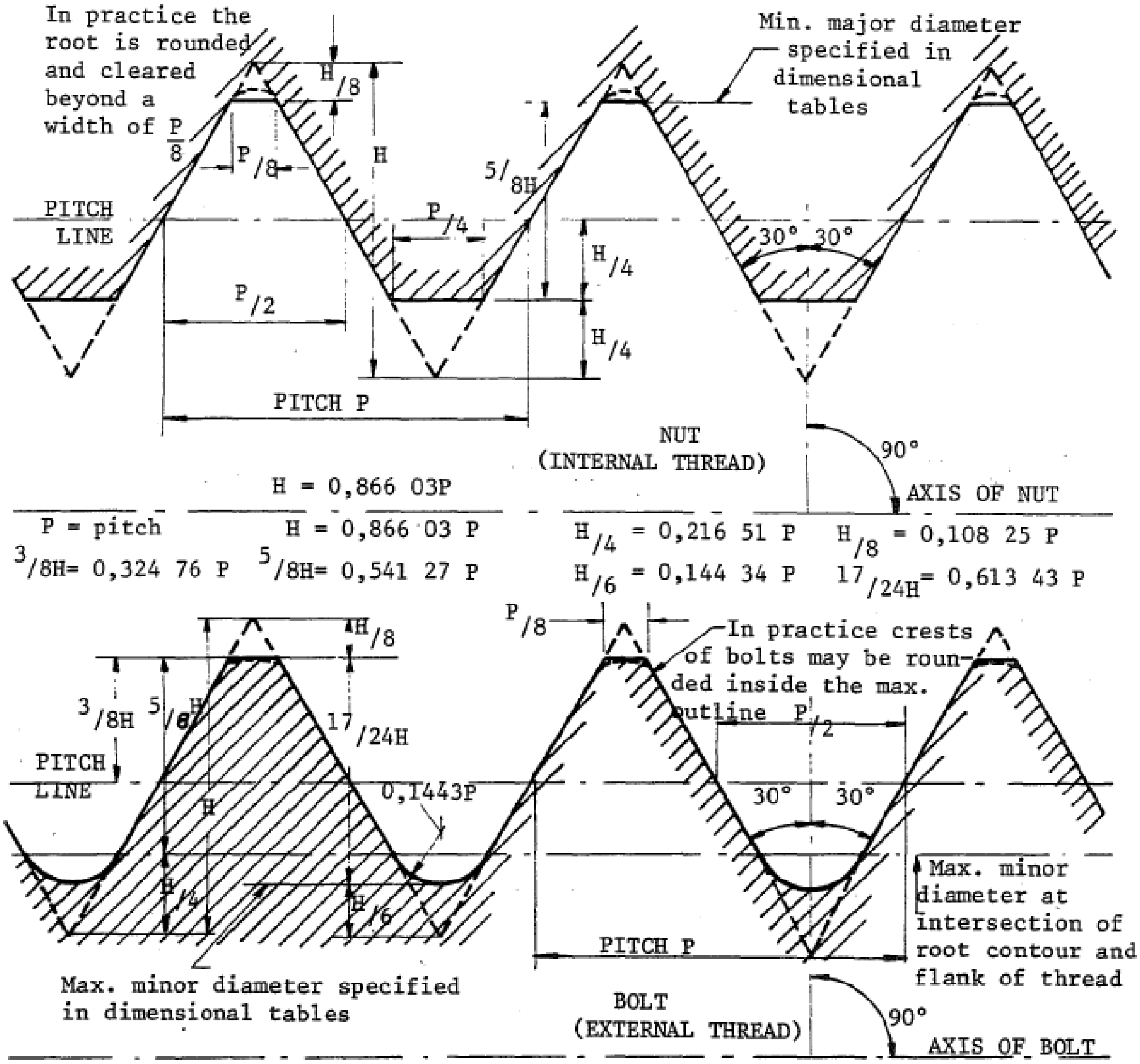


Figure 3.41 Design form of ISO metric thread

3.11.3 Designation of ISO screw threads

The International Organisation for Standardisation (ISO) has formulated a series of screw threads which should be used in countries using the metric system.

Metric screw threads may be one of three series, namely:

1. Coarse-pitch series, ISO thread.
2. Fine pitch series, ISO thread.
3. Constant-pitch series, ISO thread.

The coarse thread series, like the fine thread series, has a pitch which varies with the diameter of the bolt. In the constant thread series, the pitch remains constant irrespective of the diameter of the thread.

All the series, except the coarse thread series, are used in special circumstances. The vast majority of threads used, come from the coarse-thread series.

The method used on drawings for stating an ISO thread, is quite simple. Instead of stating the thread form and series, you need use only the letter "M". The diameter of the thread is stated immediately after "M'.

Thus M12 is ISO coarse-pitch thread form, 12 mm diameter thread, and M20 is ISO coarse-pitch thread form, 20 mm diameter thread. The pitch of the coarse thread series can be read off from tables.

If a thread is used from a constant-pitch series or fine-pitch series, the pitch is added after the diameter, so that M14 x 1,5 is a 14 mm diameter ISO thread, with a constant pitch of 1,5 mm, and M24 x 2 is a 24 mm diameter ISO thread with a fine pitch of 2 mm.

3.11.4 Specification: ISO metric screw threads

							Head of Bolt		Nut	Washer	
Nominal size diameter	Coarse			Fine			Width across flats	Height	Height	Thickness	Outside diameter
	Thread pitch	Core diameter (Bolt)	Core Area mm²	Thread pitch	Core diameter (Bolt)	Core Area mm²					
M1,6	0,35	1,171	1,07							0,3	4,0
M2	0,4	1,509	1,79							0,3	5,0
M2,5	0,45	1,948	2,98							0,5	6,5
M3	0,5	2,387	4,47				5,5	2,0	2,4	0,5	7,0
M4	0,7	3,141	7,75				7,0	2,8	3,2	0,8	9,0
M5	0,8	4,019	12,70				8,0	3,5	4,0	1,0	10,0
M6	1,0	4,773	17,90				10,0	4,0	5,0	1,5	12,5
M8	1,25	6,466	32,80	1,0	6,773	36,0	13,0	5,5	6,5	1,5	17,0
M10	1,5	8,160	52,30	1,25	8,466	56,3	17,0	7,0	8,0	2,0	21,0
M12	1,75	9,853	76,20	1,25	10,466	86,0	19,0	8,0	10,0	2,5	24,0
M14	2,0	11,546	105	1,5	12,160	116	22,0	9,0	11,0	2,5	28,0
M16	2,0	13,546	144	1,5	14,160	157	24,0	10,0	13,0	3,0	30,0
M18	2,5	14,933	175	1,5	16,160	205	27,0	12,0	15,0	3,0	34,0
M20	2,5	16,933	225	1,5	18,160	259	30,0	13,0	16,0	3,0	37,0
M22	2,5	18,933	282	1,5	20,160	319	32,0	14,0	18,0	3,0	39,0
M24	3,0	20,319	324	2,0	21,546	365	36,0	15,0	19,0	4,0	44,0
M27	3,0	23,319	427	2,0	24,546	473	41,0	17,0	22,0	4,0	50,0
M30	3,5	25,706	519	2,0	27,546	586	46,0	19,0	24,0	4,0	56,0
M33	3,5	28,706	647	2,0	30,546	733	50,0	21,0	26,0	5,0	60,0
M36	4,0	31,092	759	3,0	32,319	820	55,0	23,0	29,0	5,0	66,0
M39	4,0	34,092	913	3,0	35,319	980	60,0	25,0	31,0	6,0	72,0
M42	4,5	36,479	1050	3,0	38,319	1210	65,0	26,0	34,0		
M48	5,0	41,866	1380	3,0	44,319	1540	75,0	30,0	38,0		
M56	5,5	49,252	1910	4,0	51,093	2050	85,0	35,0	34,0		
M64	6,0	56,639	2520	4,0	59,093	2740	95,0	40,0	51,0		

Table 3.1

3.11.5 Bolts and studs in tension

When a bolt or stud is used in tension, the stress will be greater in the threaded than in the unthreaded portion, since the cross-sectional area is smaller at the root of the thread.

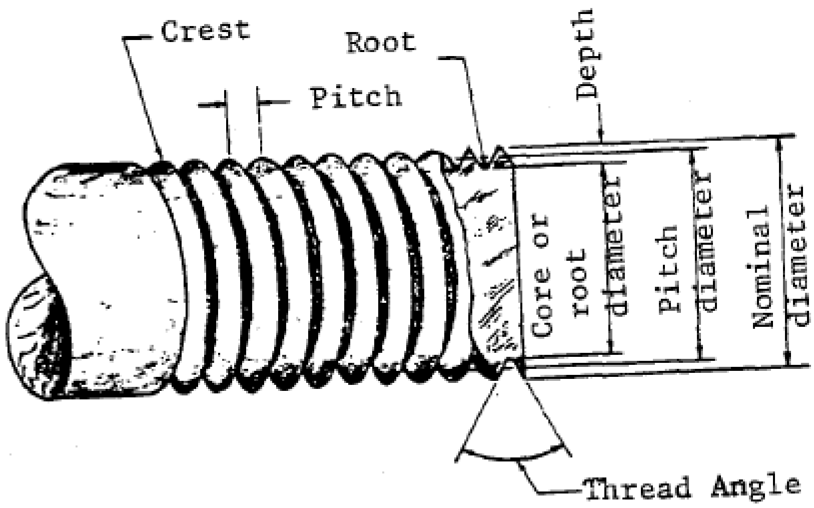


Figure 3.42

When dealing with bolts or studs in tension, therefore, we must always use core or root diameters and areas and not nominal diameters and areas.

The correct core diameters and core areas for ISO metric bolts and studs may be found in **Table 3.1**.

3.11.6 Bolts in shear

⚠	**Note:** It is only when bolts are in tension that we must take the core area into account.

When a bolt is used in shear, as in **Figure 3.43**, it is assumed that it is not threaded too far and that the section in shear is an unthreaded part.

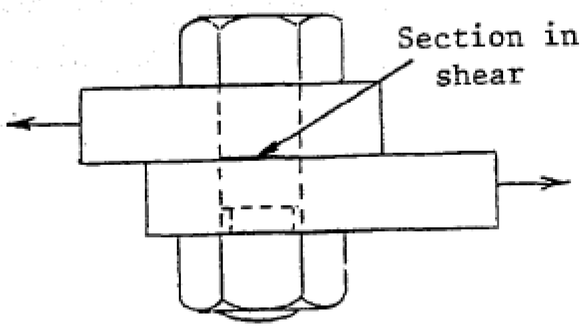
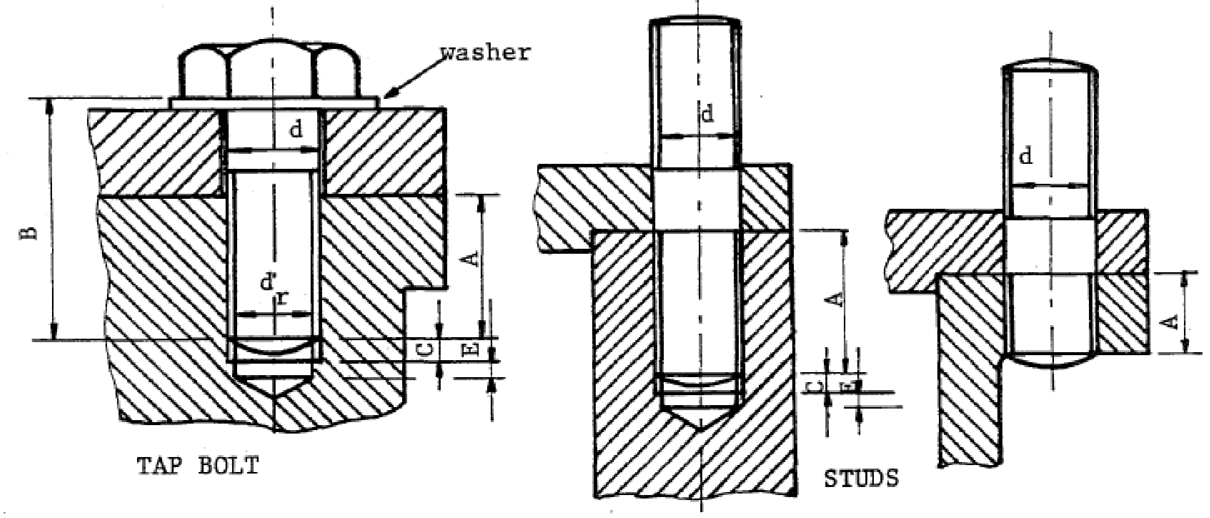


Figure 3.43

3.11.7 Length of threaded part on studs an tap bolts



A = Minimum distance threaded fastener must be screwed into threaded hole.

B = Standard length of tap bolt.

d = Nominal diameter of fastener

d_r = Root diameter

C + E = $\frac{1}{2}$d minimum

A for Steel = d

A for cast-iron, brass, bronze = 1,5d

A for aluminium = 2d

Figure 3.44

3.11.8 Covers for steam engine cylinders

The curved wall and circular cover of a steam engine cylinder are made out of a special type of cast-iron with a very fine grain, or of cast steel.

Covers may be flat or spherical, and are bolted to the end of the cylinder, using studs or tap bolts. In the cover through which the piston rod passes, provision is made for a stuffing box and gland.

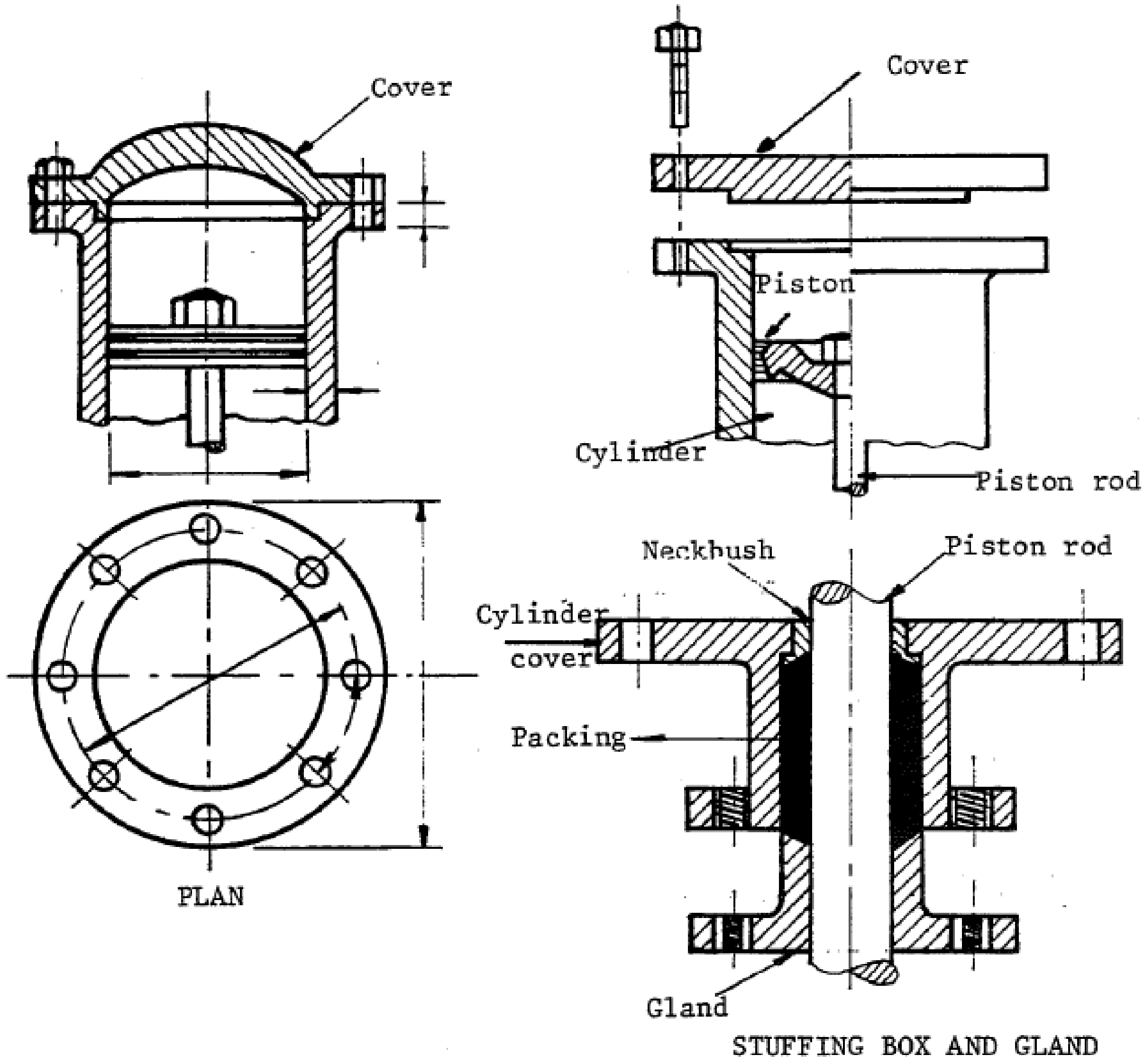


Figure 3.45 Covers for steam engine cylinders

3.11.9 Covers for inspection holes

Boilers used for power stations, factories and locomotives, are large, with numerous parts fitted inside them.

Entrance to the inside of the boiler is in certain cases necessary. Inspection must be carried out on parts fitted inside the boiler or repairs, or replacement to be done inside the boiler.

Inspection holes are covered with special covers made of cast iron, cast steel or mild steel. They may be circular, with a diameter of $\pm$400 mm or have an oval form, 400 mm x 300 mm, or any other special form.

Note:
Fixing of covers is carried out by means of tap bolts or clamps.

The finishing of the bearing areas of the cover and cylinder flange or boiler flange assures tight sealing.

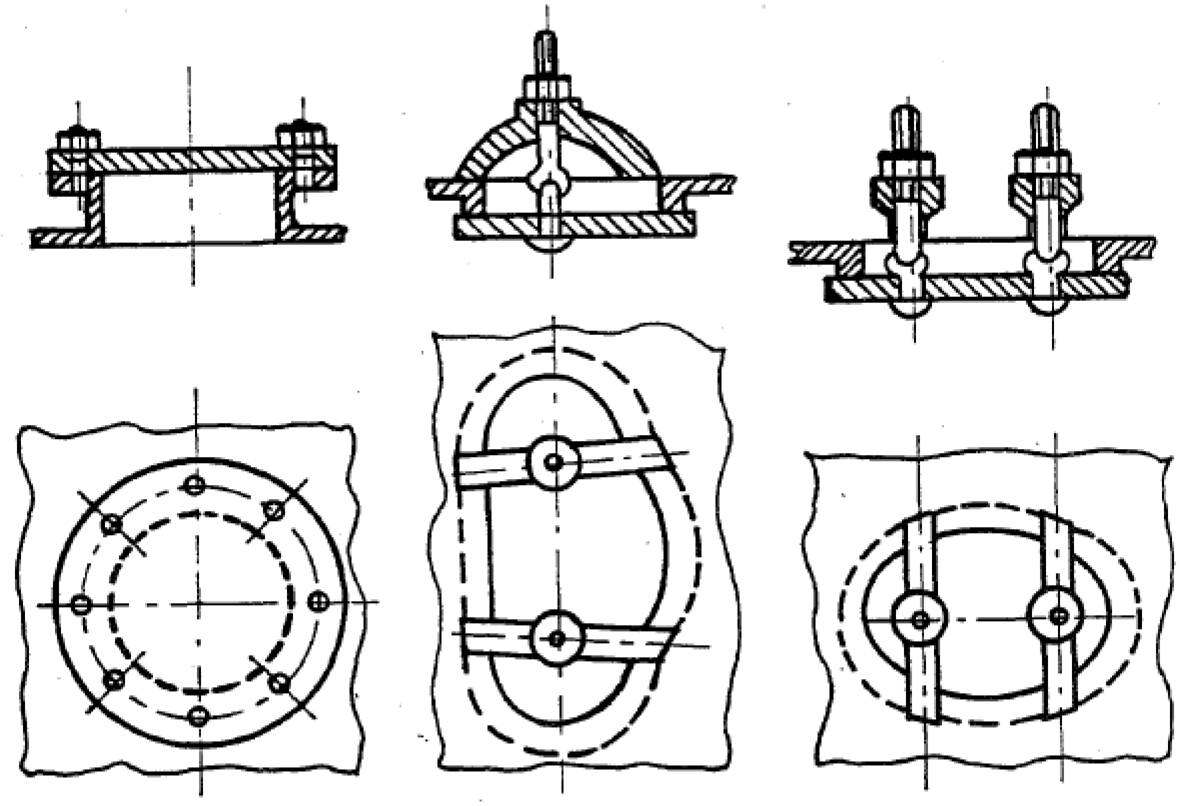


Figure 3.46 Various types of inspection cover

3.11.10 Design of bolts for fixing covers to steam engine cylinder and manholes
In designing the bolts or studs for cylinder and manhole covers etc, the procedure will depend to some extent on the information given in the problem, but there are a number of points to be taken into consideration in each case.

3.11.10.1 Number of studs for strength
The steam (or gas or water) pressure in the cylinder (or boiler or tank) acts on an area (circular, rectangular, etc.) and causes a total force on the cover of pressure times area.

$$P = P_i \times \frac{\pi D^2}{4} \text{ circular}$$
$$P = P_i \times L \times B \text{ rectangular}$$
$$P = P_i \times \pi(a \times b) \text{ ellipse}$$

Where P = Total force on cover (newtons)
P_i = Internal pressure (pascals)
D = Diameter of packing circle (metre)
L and B = Overall dimensions of the packing (metre)

In some cases, as shown in **Figure 3.47**, when the spigot is a good fit in the cylinder, it may be assumed that the steam pressure acts over the cross-sectional area of the cylinder only.

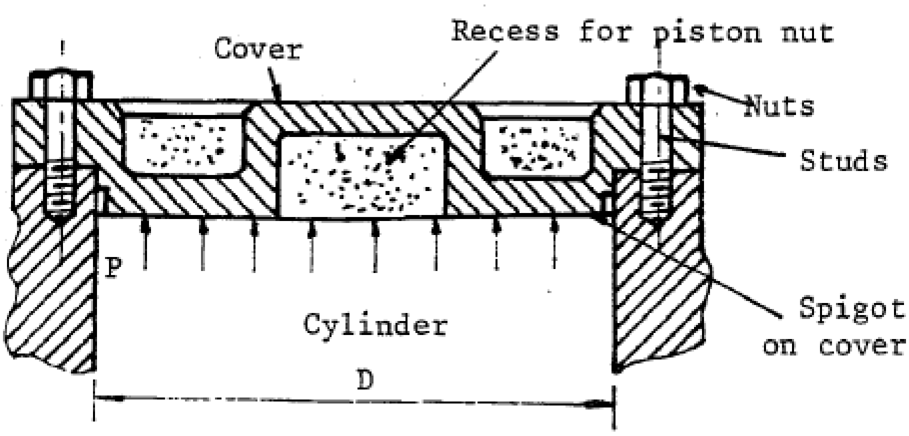


Figure 3.47 Cylinder and cover with spigot

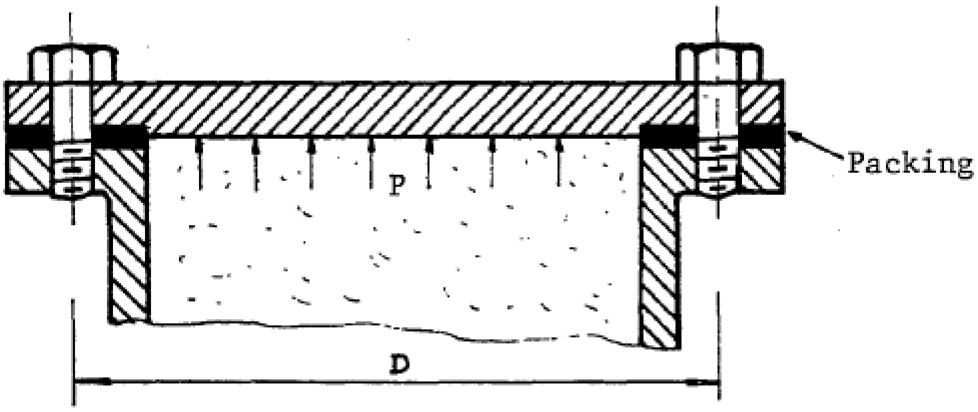


Figure 3.48 Cylinder and cover with cover packing

The total force OD the cover puts the studs into tension, so the core area must be used and the allowable load (force) per stud found from:

$$\frac{Total\ force}{Number\ of\ studs}$$

$$\therefore Alowable\ load\ per\ stud \quad = \frac{P}{N} = P_i \times \frac{\pi D^2}{4} \times \frac{1}{N}$$

Also

Allowable load per stud = core area of stud x allowable tensile stress

$$= \frac{\pi {d_r}^2}{4} \times \sigma t$$

Therefore $\quad P_i \times \frac{\pi D^2}{4} \times \frac{1}{N} = \frac{\pi {d_r}^2}{4} \times \sigma t$

$$N = P_i \times \frac{\pi D^2}{4} \times \frac{4}{\pi \times {d_r}^2 \times t}$$

$$N = \frac{P_i \times D^2}{{d_r}^2 \times \sigma t}$$

or when core diameter has to be found.

$$d_r^{\,2} = \frac{P_i}{N} \times \frac{\pi D^2}{4} \times \frac{4}{\pi \times \sigma t}$$

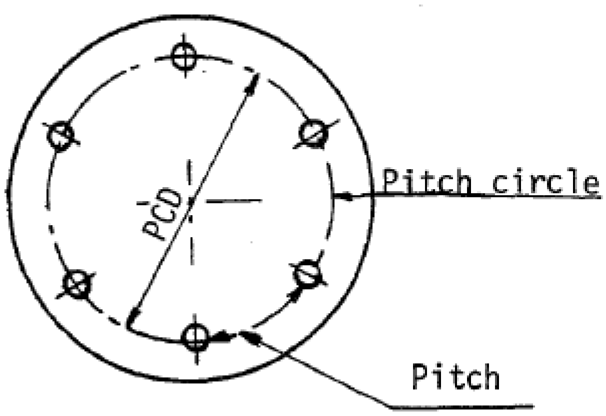
$$d_r = \sqrt{\frac{P_i}{N} \times \frac{\pi D^2}{\sigma t}}$$

The working stress used for the studs will be very low (ie a high factor of safety) to allow for the tightening of the nuts which puts a considerable and unknown load on them.

3.11.10.2 Number of studs for steam tightness

It is not sufficient for the studs or bolts to be strong enough, they must also ensure steam tightness, and in order to do so, they must be close enough to one another.

No hard and fast rule for this can be laid down, but the pitch of the studs or bolts (ie the distance between their centres measured on the pitch circle) should be from $3d$ for high pressures to $6\,d$ for low pressures, where d is the nominal diameter of the stud or bolt.



PCD = PITCH CIRCLE DIAMETER

Figure 3.49

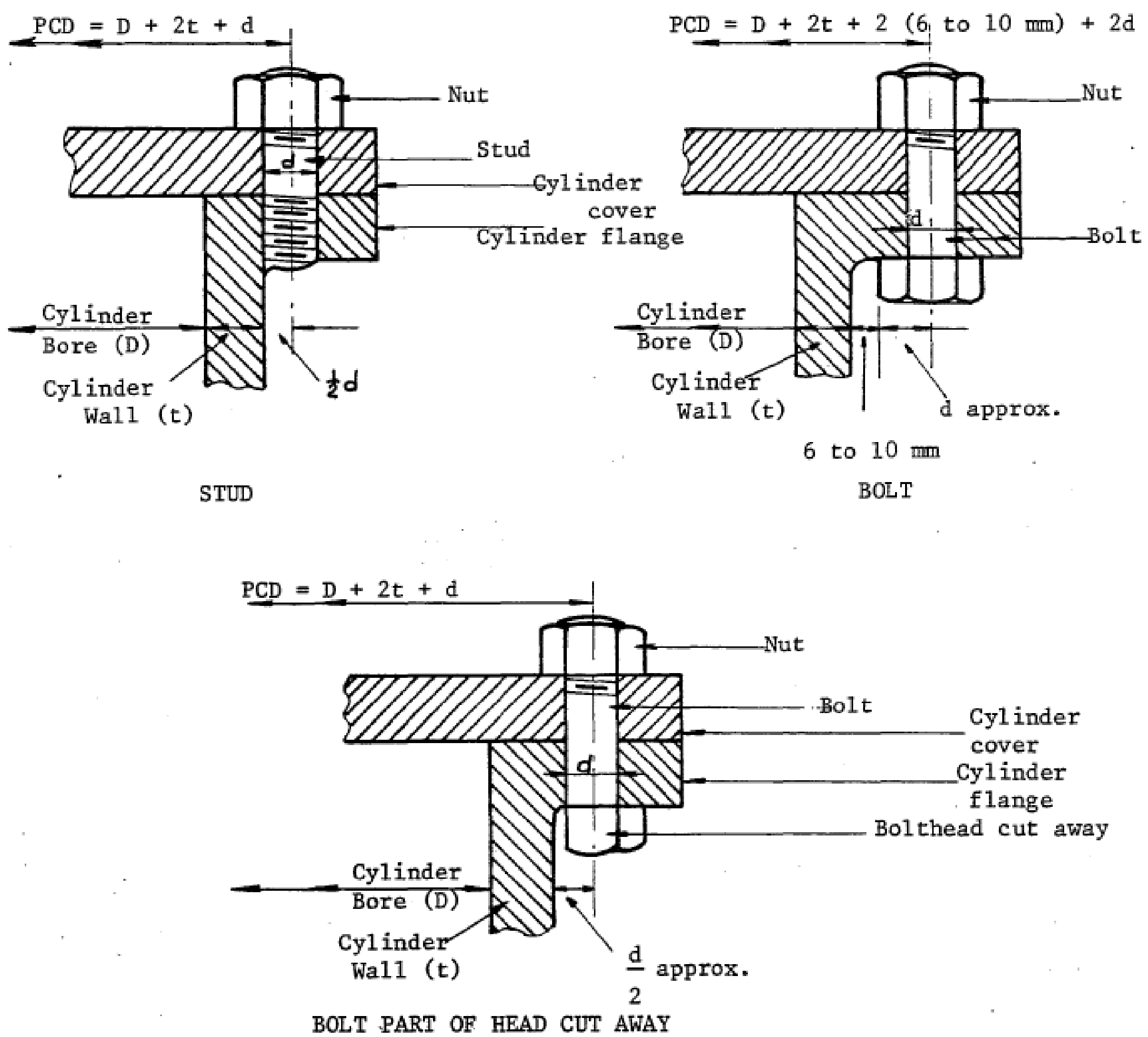


Figure 3.50

$$Pitch = \frac{Circumference\ of\ pitch\ circle}{Number\ of\ studs\ or\ bolts}$$

$$N = \frac{\pi \times PCD}{P}$$

3.11.10.3 Thickness of cylinder walls

The thickness (t) of metal in castings is in excess of that required for strength. This applies to cylinders that must be rigid and, in cases where no liner is used, allows for re-boring.

$$t = \frac{P_i N/m^2 \times D\ m}{20{,}6 \times 10^6\ N/m^2} + 0{,}008\ m$$

Where

P_i = internal pressure in pascals

D = internal cylinder diameter in metres

3.11.10.4 Thickness of cylinder covers

The thickness of cover (t_1) varies from $t + 5\,mm$ for small cylinders to $t + 14\,mm$ for larger cylinders.

3.11.10.5 Thickness of cylinder flanges

The thickness of cylinder flanges (A) should not be less than d for steel, 1,5 d for cast iron, and 2 d for aluminium.

3.11.10.6 Flange width

The distance from the centre of the fastener to the edge of the flange should not be less than 1 ,5 d. This gives a flange width of not less than $3d$.

3.11.10.7 Diameter of studs or bolts

In most problems of this type it is first necessary to decide on a suitable bolt or stud diameter, and in doing so, a few practical points must be taken into consideration.

In order to obtain steam tightness at the joint, the reels must be well tightened up. Unless a high tensile steel is used, studs or bolts of under M16 may very easily be overstressed in the lightening-up process and are liable to fail in use. They should, therefore, be avoided, except in very small assemblies.

Note:
From M16 to M24 there is little danger of over-tightening, and these are the most practical sizes.

Over M24 are clumsy, cannot be lightened to full advantage with an ordinary spanner, and should be avoided, except for large, high-pressure cylinders.

The size of studs or tap bolts (not bolts and nuts) is also fixed to some extent by the thickness of the cylinder flange into which they are screwed. A stud or tap bolt must screw in sufficiently far to ensure that there is no danger of it pulling out of the metal by stripping the thread. This depends on the materials of which the stud and cylinder are made.

For mild-steel studs into steel flanges, the maximum value of $d = A$, and for mild steel studs into cast iron, cast steel, brass or bronze flanges $d = \frac{A}{1,5}$ and for mild steel studs into aluminium flanges $d = \frac{A}{2}$. (See **Figure 3.51**).

Worked Example 3.15

Determine the tensile stress in an M22 diameter bolt carrying a tensile load of 22,3 kN.

Solution:

$$Tensile\ stress = \frac{Tensile\ load}{Core\ area}$$
$$= \frac{22{,}3 \times 10^3\ N}{282\ mm^2} \quad \text{(see Table 9.1)}$$
$$= 79{,}1\ N/mm^2$$
$$= 79{,}1 \times 10^6\ N/m^2$$
$$= 79{,}1\ MPa$$

Worked Example 3.16

What size bolt should be used to carry a tensile pull of 33,4 kN if the stress in the material is not to exceed 62 MPa?

Solution:

$$Tensile\ stress = \frac{Tensile\ load}{Root\ area}$$
$$Root\ area = \frac{Tensile\ load}{Tensile\ stress}$$
$$= \frac{33{,}4 \times 10^3\ N}{62 \times 10^6\ N/m^2}$$
$$= 0{,}00053871\ m^2$$
$$= 5{,}3871 \times 10^{-4}\ 10^6\ mm^2$$
$$= 538{,}71\ mm^2$$

Turn to **Table 3.1**, find the nearest higher core area in the table and read the nominal-size diameter from the left-hand column.

The nearest higher core area for coarse-threaded bolts equals 647 mm^2, which is an M33 diameter bolt.

The nearest higher core area for fine-threaded bolts equals 586 mm^2, which is an M30 x 2 diameter bolt.

Thus use an M33 coarse-threaded bolt or an M30 x 2 fine-threaded bolt.

3.12 Number, size and pitch of studs or bolts for cylinders, covers and manhole doors

Worked Example 3.17

A steam engine cylinder has a 200 mm bore and 22 mm thick walls. The maximum steam pressure is 1,38 MPa and the stress in the cover bolts is not to exceed 16,5 MPa. Determine the number of M24 coarse-threaded bolts required. Assume a cover with spigot.

Solution:

$$P = P_i \times \frac{\pi D^2}{4}$$
$$= 1{,}38 \times 10^6\ N/m^2 \times \frac{\pi \times (0{,}2)^2}{4}$$
$$= 43{,}35\ kN$$

This force has to be resisted by the bolts which are in tension.

$$Allowable\ load\ per\ bolt = \frac{Total\ force\ or\ cover}{Number\ of\ bolts}$$
$$= \frac{P}{N}$$
$$= \frac{43{,}35 \times 10^3\ newtons}{N} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

Also, $allowable\ load\ per\ bolt$ $= Core\ area\ of\ bolt \times Allowable\ tensile\ stress$
$= 324 \times 10^{-6}\ m^2 \times 16{,}5 \times 10^6\ N/m^2$
$= 5346\ newtons$

Equating this answer to equation 1, we get

$$\frac{43{,}35 \times 10^3}{N} newtons = 5346\ newtons$$
$$\therefore N = \frac{43{,}35 \times 10^3 newtons}{5346\ newtons}$$
$$N = 8{,}12$$

Using 9 bolts.

Now to check for steam tightness

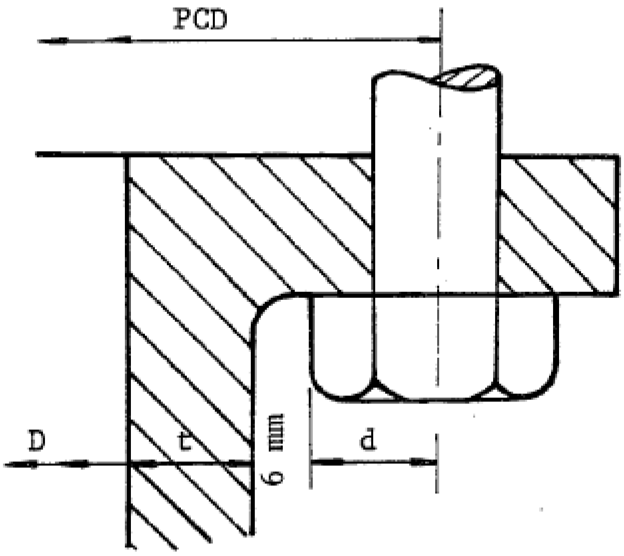


Figure 3.51

$$Pitch\ circle\ diameter = D + 2t + 2(6\ mm) + 2d$$
$$= 200\ mm + 2(22\ mm) + 2\ (6mm) + 2\ (24\ mm)$$
$$= 304\ mm$$

$$Pitch = \frac{Circumference\ of\ pitch\ circle}{Number\ of\ bolts}$$

$= \frac{\pi \times PCD}{N}$

$= \frac{\pi \times 304\ mm}{9}$

$= 106{,}12\ mm$

Since the bolts are 24 mm in diameter, and the pitch for steam tightness must be between 3d and 6d, then

3d to 6d

$= 3 \times 24\ mm\ to\ 6 \times 24\ mm$

$= 72\ mm\ to\ 144\ mm$

Thus a pitch of 106,12 mm will ensure a joint that is steamtight.

Worked Example 3.18

A steam engine cylinder is 0,45 m diameter and uses steam at 1,24 MPa.
Calculate:

1. The thickness of the cylinder walls, which are made of cast iron.
2. The thickness of the flanges.
3. The thickness of the cylinder cover.
4. Number, size and pitch (which should be approximately 4d) of mild steel studs for the cylinder cover.
5. Width of flange.

Tensile stress of steel = 464 MPa.
Factor of safety = 8.
A cover with a spigot is used.

Solution:

1. Thickness of cylinder walls $= \frac{P_i \times D}{20{,}6 \times 10^6\ N/m^2} + 0{,}008\ m$

$= \frac{1{,}24 \times 10^6 N/m^2 \times 0{,}45 m}{20{,}6 \times 10^6\ N/m^2} + 0{,}008\ m$

$= 0{,}035\ m$

$t = 35\ mm$

2. Thickness of the flanges $= 1{,}2\ t\ to\ 1{,}5\ t$

$= 1{,}2 \times 35\ mm\ to\ 1{,}5 \times 35\ mm$

$A = 42\ mm\ to\ 52{,}5\ mm$

Make it, say, 45 mm.

3. Thickness of cylinder cover $= t + 10\ mm$

$= 35\ mm + 10\ mm$

$= 42\ mm$

4. Size of studs $= \frac{A}{1{,}5}$

$= \frac{45\ mm}{1,5}$
$= 30\ mm\ diameter$

$Total\ force\ on\ cover = Internal\ pressure \times Area\ of\ cylinder$

$= P_i \times \frac{\pi D^2}{4}$
$= 1,24 \times 10^6 \times \frac{\pi \times (0,45)^2}{4}$
$= 197,21\ kN$

This force has to be resisted by the studs which are in tension.

$Allowable\ load\ per\ stud = \frac{Total\ force\ on\ cover}{Number\ of\ studs}$
$= \frac{197,21 \times 10^3}{N} newtons$ .... (1)

Also,
$allowable\ load\ per\ stud = Core\ area\ of\ stud \times Allowable\ tensile\ stress$
$= 519 \times 10^{-6}\ m^2 \times \frac{464 \times 10^6\ N/m^2}{8}$
$= 30\ 102\ newtons$

Note: coarse-thread studs are assumed.

Equating this answer to equation 1, we get
$30\ 102\ newtons = \frac{197,21 \times 10^3}{N} newtons$
$N = \frac{197,21 \times 10^3 newtons}{30\ 102\ newtons}$
$= 6,55$ (say 7)

$PCD = D + 2t + d$
$= 450\ mm + (2 \times 35\ mm) + 30\ mm$
$= 550\ mm$

$Pitch = \frac{Circumference\ of\ pitch\ circle}{Number\ of\ studs}$
$= \frac{\pi \times D}{N}$
$= \frac{\pi \times 550\ mm}{7}$
$= 246,84\ mm$

Pitch to be approximately $4 \times d$
$= 4 \times 30\ mm$
$= 120\ mm$

Design is unsatisfactory.

Our pitch is too large, thus we must use a smaller-diameter stud so that there can be more studs.

Let us try a M16 coarse-threaded stud.

$$Allowable\ load\ per\ stud = Core\ area\ of\ stud \times Allowable\ tensile\ stress$$

$$= 144 \times 10^{-6}\ m^2 \times \frac{464 \times 10^6}{8}$$
$$= 8352\ newtons$$

Equating this answer to equation 1, we get

$$8352\ newtons = \frac{197{,}21 \times 10^3}{N} newtons$$
$$N = \frac{197{,}21 \times 10^3 newtons}{8352\ newtons}$$
$$N = 26{,}6$$

Say 27 studs

$$PCD = D + 2t + d$$
$$= 450\ mm + (2 \times 35\ mm) + 16\ mm$$
$$= 536\ mm$$

$$Pitch = \frac{Circumference\ of\ pitch\ circle}{Number\ of\ studs}$$
$$= \frac{\pi \times 536\ mm}{27}$$
$$= 62{,}37\ mm$$

Pitch to be approximately $4 \times d$

$$= 4 \times 16\ mm$$
$$= 64\ mm$$

Design is satisfactory.

5. Width of flange $= 3 \times d$

$$= 3 \times 16\ mm$$
$$= 48\ mm$$

Worked Example 3.19

A flat circular cover is used to close the end of a steam pipe, 230 mm inside diameter. The pipe metal is 22 mm thick and the steam pressure is 1,24 MPa. Find a suitable diameter, number and pitch of the bolts holding down the cover if the working stress is 27,6 MPa.

Solution:

$$Total\ force\ on\ cover = Internal\ pressure \times Area\ of\ pipe$$

$= P_i \times \frac{\pi D^2}{4}$
$= 1{,}24 \times 10^6\ N/m^2\ \times \frac{\pi \times (0{,}23)^2}{4}$
$= 51{,}52\ kN$

$Allowable\ load\ per\ bolt\ = \frac{Total\ force\ on\ cover}{Number\ of\ bolts}$
$= \frac{P}{N}$
$= \frac{51{,}52 \times 10^3}{N} newtons$ ………………… (1)

Also, $allowable\ load\ per\ bolt\ = Core\ area\ of\ bolt\ \times\ Allowable\ tensile\ stress$

Let us try an M18 coarse-threaded bolt, which has a core area of 175 mm²

$allowable\ load\ per\ bolt\ = 175\ \times 10^{-6}\ m^2 \times 27{,}6 \times 10^6\ N/m^2$
$= 4830\ newtons$

Equate this answer to equation 1, we get
$4830\ newtons\ = \frac{51{,}52\ \times 10^3}{N} newtons$
$N\ = \frac{51{,}52 \times 10^3}{N} newtons$
$= 10{,}66$ (say 11 bolts)

$PCD\ = D\ +\ 2t\ + 2(6mm) + 2d$
$= 230\ mm\ +\ 2(22\ mm) +\ 2(6\ mm) + 2(18\ mm)$
$= 322\ mm$

$Pitch\ = \frac{Circumference\ of\ pitch\ circle}{Number\ of\ bolts}$
$= \frac{\pi \times PCD}{N}$
$= \frac{\pi \times 332}{11}$
$= 91{,}96\ mm$

Check for steam tightness

Pitch $= xd$, which should be between $3d$ and $6d$
$x\ = \frac{pitch}{d}$
$x\ = \frac{91{,}96\ mm}{18\ mm}$
$= 5{,}1$ which is in the limits

A smaller diameter bolt, say M16, would have given a better result.
You are advised to re-calculate the previous example, using M16 coarse-threaded bolts.

Worked Example 3.20

Figure 3.52 shows a steam chest opening, 360 mm by 200 mm, which is to be closed by means of a cast iron flat cover. The steam pressure is 1 MPa, and the jointing material may be considered as extending to the bolts and to be subjected to the same pressure.

Design the flange and cover, using a stress of 42 MP for the bolts. The steam chest walls are 20 mm thick and the flange must be capable of taking either bolts or studs although we are considering bolts only here.

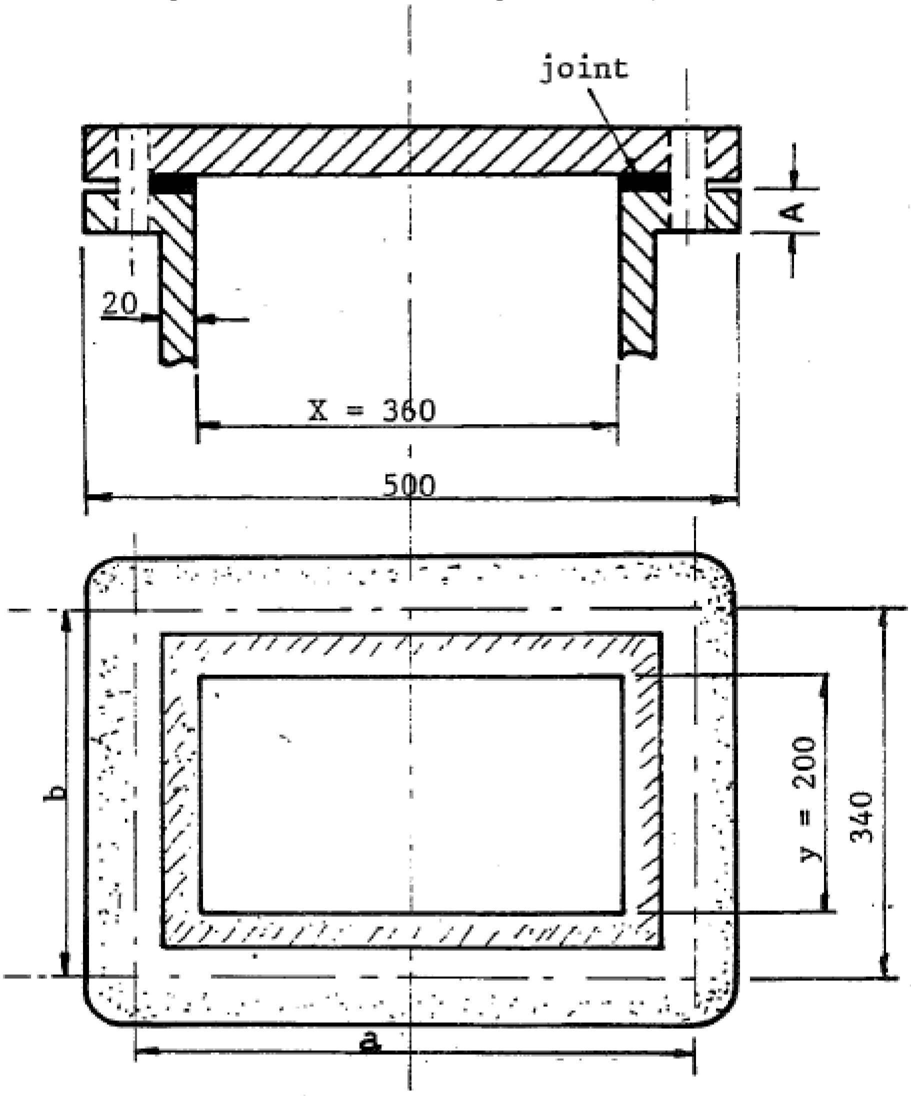


Figure 3.52

<u>Solution:</u>

$$Flange\ thickess = 1{,}5\ t$$
$$= 1{,}5\ t \times 20\ mm$$
$$A = 30\ mm$$

$Diameter\ of\ tap\ bolts = \frac{A}{1,5}$

$= \frac{30\ mm}{1,5}$

Try = 20 mm nominal diameter coarse-threaded bolts.

$Let\ a\ and\ b = overall\ pitch\ centres\ of\ the\ tap\ bolts$

$a = x + 2t + 2(8) + 2d$

$= 360 + (2 \times 20) + 16 + (2 \times 20)$

= 456 mm

$b = y + 2t + 2\,(8) + 2\,d$

$= 200 + (2 \times 20) + 16 + (2 \times 20)$

$= 296\ mm$

Effective joint area

The effective joint area should terminate at the inside of the bolts.

$L = a - d$

$= 456 - 20$

$= 436\ mm$

$B = b - d$

$= 296 - 20$

$= 276\ mm$

$Total\ force\ on\ cover = Internal\ pressure \times Area$

$P = P_i \times L \times B$

$= 1 \times 10^6 \frac{N}{m^2} \times 0,436\ m \times 0,276\ m$

$= 120,34\ kN$

This force has to be resisted by the bolts which are in tension.

$Allowable\ load\ per\ bolt = \frac{Total\ force\ on\ cover}{Number\ of\ bolts}$

$= \frac{P}{N}$

$= \frac{120,34 \times 10^3 newtons}{N}$ ………………… (1)

Also,

$allowable\ load\ per\ bolt = Core\ area\ of\ bolt \times Allowable\ tensile\ stress\ in\ bolts$

$allowable\ load\ per\ bolt = 225 \times 10^{-6}\ m^2 \times 42 \times 10^6\ N/m^2$

$= 9450\ newtons$

Equate this answer to equation (1), and we get

$\frac{120,34 \times 10^3}{N} = 9450$

$\therefore N = \frac{120,34 \times 10^3}{9450}$

= 12,73 (say 14 bolts)

Assume one bolt at each corner and the number of bolts along length a equal 5 and along b equal 4.

$$The\ pitch\ of\ the\ bolts\ along\ length\ a = \frac{456}{4} = 114\ mm$$

$$The\ pitch\ of\ the\ bolts\ along\ length\ b = \frac{296}{3} = 98{,}67\ mm$$

For steam tightness the pitches should be between $3d$ and $6d$.

$$3 \times 20\ mm\ to\ 6 \times 20\ mm$$
$$= 60\ mm\ to\ 120\ mm$$

The pitches of $114\ mm$ and $98{,}67\ mm$ are satisfactory.

Activity 3.1

1. Two flat bars, 200 mm wide by 20 mm thick, are to be joined by means of a butt joint with double cover straps. Conical head rivets at the bottom with countersunk heads at the top are to be used, with zig-zag riveting.
   There are three rows of rivets on each side of the butt joint (3 rivets in the first row, 2 rivets in the second row and 1 rivet in the third or outside row).
   Calculate:
   a) The diameter of the rivets.
   b) The thickness of the cover strap.
   c) The distance between the rows.
   d) The distance between the centre line of the rivet hole to the edge of the plates.
   e) The pitch.
   f) The diagonal pitch.
   g) The length of the cover strap.
   What is a joint like this called?
   Make a free-hand sectional front elevation sketch taken through the centre line of the bars, as well as a plan showing 12 rivets. Insert all dimensions.
2. Two lengths of 22 mm steel plate, each 280 mm wide, are to be joined by a single-riveted lap joint, using 4 rivets of 24 mm diameter. Give a fully dimensioned sketch of the joint. Determine the safe load that may be applied to the plates, and state how, and at what load, the joint would probably fail.
   Tensile stress of steel = 462 MPa
   Shear stress of steel = 384 MPa
   Crushing stress of steel = 618 MPa

Factor of safety 6.

3. Two lengths of tie-plate, 12 mm thick and 180 mm wide, are joined by a single-riveted lap joint with 3 rivets, 24 mm in diameter. When this joint is carrying a pull of 120 kN, determine:
   i) the tensile stress in the plates;
   ii) the shear stress in the rivets;
   iii) the crushing stress between the rivets and the plates.
   If the ultimate stresses of the materials are the same as in question 1, determine the efficiency of this joint.

Activity 3.2

1. Design a double-riveted lap joint with chain riveting for a tie-plate, 15 mm thick, to carry a pull of 280 kN.
   The ultimate stresses of steel in tension, shear and crushing are 465 MPa, 385 MPa and 620 MPa, respectively, and a factor of safety of 5 is to be used.
   Give a fully dimensioned sketch of the joint.
2. Calculate, by equating the tensile and shearing loads, a suitable pitch for a single-riveted lap joint for an 18 mm steel plate.
   Diameter of rivet = $6\sqrt{t}$
   Tensile stress of steel = 464 MPa
   Shearing stress of steel = 384 MPa
   Compressive stress of steel = 616 MPa
   Factor of safety = 8
   Calculate the failing loads per pitch length of your design for tension, shear and compression, and then determine the allowable load and the efficiency.
   Draw, freehand, two views of the joint, using flush countersunk heads on one side and standard snap heads on the other. Insert the dimensions.
3. Two steel plates, each 12 mm thick, are joined by a double-riveted lap joint, using snaphead rivets of diameter $d = 6\sqrt{t}$, using the nearest standard-size rivet.
   Design a suitable joint by equating tensile and shear strengths.
   Then calculate the safe loads per pitch width of the joint for tension, shear and crushing; deduce the efficiency of the joint.
   Tensile stress of plates, 492 MPa
   Shear stress of rivets, 402 MPa
   Crushing stress of rivets and plates, 648 MPa
   Factor of safety = 6
4. Two steel plates, each 15 mm thick, are joined by a double-riveted lap joint with rivets of diameter $d = 6\sqrt{t}$, using the nearest standard-size rivet.
   Determine a suitable pitch for the rivets, given:-
   Tensile stress of steel, 463 MPa
   Shear stress of steel, 386 MPa

Crushing stress of steel, 618 MPa.
Calculate the efficiencies relative to tearing, shear and crushing.
State the efficiency of the joint.

Activity 3.3

1. Two mild steel plates, each 15 mm thick, are connected by a double-riveted butt joint with two cover straps.
   If 20 mm diameter rivets are used, calculate the pitch of the rivets and the efficiency of the joint. Take the working stresses in shear as being 92 MPa and in tension as being 124 MPa. Calculate the bearing stress in the joint.
2. A tension member is to be made from two lengths of flat bar, 20 mm thick, and connected by means of a butt joint with two cover plates, the total load on the tie bar being 600 kN.
   Determine:
   (a) The width of the flat bar.
   (b) The number of 30 mm diameter rivets required.
   (c) The thickness of the cover plates.
   (d) The efficiency of the joint.
   Use the following safe stresses: tensile 124 MPa, shear 93 MPa, crushing 185 MPa. Sketch the lozenge joint, showing the arrangement of rivets with principal dimensions.

Activity 3.4

1. An air reservoir, 1 min diameter, is subjected to an internal pressure of 827 kPa. The longitudinal butt joint is double zig-zag riveting and has two cover-straps.
   a) Assume an efficiency of 75% in the longitudinal joint and determine the thickness of the shell plates.
   b) Find the diameter of the rivets to the nearest standard size.
   c) Find the pitch of the rivets to the nearest millimetre by
      (i) equating the shear strength of the rivets to the tensile strength of the plate;
      (ii) equating the bearing strength of the rivets to the tensile strength of the plate.
   d) Evaluate the efficiency of the joint for tearing, bearing and shear, using the smaller pitch. Use working stresses of 77 MPa in shear, 93 MPa in tension, and 154 MPa in bearing.

Activity 3.5

1. What size coarse-threaded metric bolt must be used to carry a tensile pull of 20 kN if the allowable stress is 55,2 MPa?
2. Determine the tensile stress in a fine-threaded stud having a core diameter of 12,16 mm carrying a pull of 15 kN.
3. Three mild-steel studs, each M20 diameter, have to be screwed into the following flanges:
   One in steel, one in cast iron and the other one into aluminium.
   Calculate the flange thickness for each case.
4. Calculate the number of 16 mm diameter coarse-threaded studs required from a strength point of view for the cover of a steam engine cylinder 356 mm in diameter, with maximum steam pressure 1,4 MPa, if the stress in the studs is not to exceed 62 MPa. Do you consider the number of studs you obtain to be suitable from a steam tightness point of view?
   Sketch an elevation of the cover, half in section, and insert dimensions.
   The cylinder wall is 30 mm thick and the flange 32 mm thick.
5. Determine the size, number and pitch of bolts for fixing a cover to the flanges of a steam engine of 380 mm bore. The thickness of the cast iron cylinder wall is 20 mm and the flanges are 25 mm thick. The steam pressure is 1 MPa. Allow a working stress in the bolts of 31 MPa.
6. A cast-steel manhole cover for a rectangular-shaped hole measuring 460 mm by 300 mm, with the corners rounded at 76 mm radii, is fitted to the end of a tank. The pressure in the tank is 5,7 Mpa. The plate for the tank is 25 mm thick and the flange of the manhole cover is 28 mm thick, and, in addition, it has a spigot 6 mm deep and several reinforcing ribs 20 mm thick on its back.
   Calculate the number and diameter of the studs, whose centres should be about 3d apart. The centres of the studs should not be nearer than 1½d to the edge of the tank plate and the manhole cover outer edge.
   Tensile strength of stud steel = 616 MPa. Factor of safety = 8.
   Sketch, about half-size, two views of your design with all main dimensions.

Self-Check

I am able to:	**Yes**	**No**
• Describe rivet heads		
• Describe types of riveted joints		
• Describe the design of riveted joints		
o Methods of failure of single-riveted lap joints		
o Working stress to be used if not given		
o Strength of solid plate		
o Efficiency of riveted joints		
• Describe the methods of failure of double-riveted lap joints		
• Describe butt joints		
• Describe lozenge joints		
• Describe riveted joints on high pressure cylinders		

o Thin cylinders		
o Joints for cylindrical pressure vessels, ie boilers and tanks		
• Describe fasteners		
o Threaded		
o Proportions of ISO thread		
o Designation of ISO screws		
o Specifications		
o Bolts and studs in tension		
o Bolts in shear		
o Length of threaded part on studs and tap bolts		
o Covers for steam engine cylinders		
o Covers for inspection holes		
o Design of bolts for fixing covers to steam engine cylinders and manholes		
• Describe studs or bolts used for steam cylinder covers and manhole doors		
o Number		
o Size		
o Pitch		
If you have answered 'no' to any of the outcomes listed above, then speak to your facilitator for guidance and further development.		

Module 4

Cotter and Knuckle Joints

Learning Outcomes
On the completion of this module the student must be able to:

- Describe knuckle joints
  - Standard proportions
- Describe cotter joints
  - Standard proportions
  - Design

4.1 Introduction

Knuckle joints are used to connect two shafts which are subjected to a load that moves in one plane and are found in valve mechanisms. Cotter joints are used where shafts are subjected to axial forces, eg at the connecting rod of a steam engine as well as at the crosshead connection.

In a cotter joint, the one end of the shaft is turned taper and fits into a taper socket on the end of the other shaft. When the taper end and socket are fitted, a hole is drilled through the fitted position and is secured by means of a cotter. The taper is ± 3 mm per 100 mm.

The clearance in the rod is on the opposite side to that of the clearance in the socket. The cotter is tapered on the one side only and the ends should be bevelled to prevent them from spreading when hit with a hammer. The deeper the cotter is driven into the joint, the more secure the joint becomes.

4.2 Knuckle joints

This joint is also known as the fork or pin joint, and is used to connect two bars, which require a small axial movement in one plane, and it is to be found in almost every branch of mechanical and some structural work: the tie bars of a roof truss or braced girder, links of a suspension bridge, valve mechanism of reciprocating engines, all are examples of its use.

As illustrated in **Figure 4.1**, it consists essentially of three parts, which are:
- the single eye forged with rod A;
- the double eye or fork forged with rod B;

- the pin C which connects the two parts of A and B at their junction with the eye.

Both rods have an octagonal form, which facilitates the manipulation of the joint and rods.

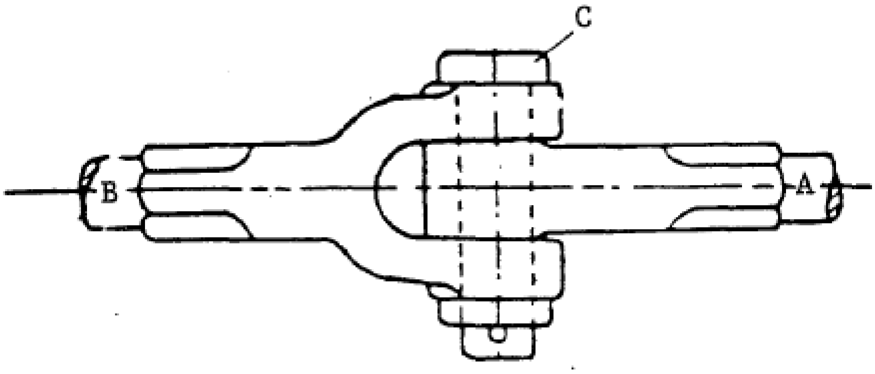


Figure 4.1

The joint will, as far as possible, be designed to have equal strength at its critical sections. The stress in any part of the joint is calculated from the knowledge of the load carried by the joint and the dimensions of the specific part.

Three forms of stresses are involved, namely, tensile, shear and crushing. In each case the stress is given by dividing the axial load by the cross-sectional area of the section.

$$Stress = \frac{Axial\ load}{cross-sectional\ area}$$

In order to design a knuckle joint, we must investigate the ways in which it can fail and the stresses induced in the various parts.

Failure of the joint may occur in the following ways:

- The solid rod of diameter d may tear

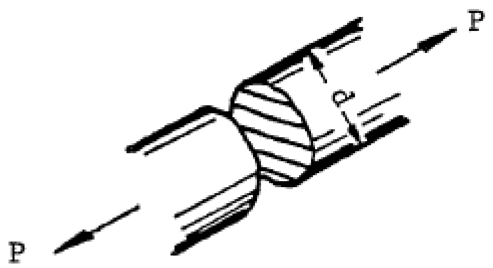


Figure 4.2

$$Axial\ load = Tensile\ stress \times Cross-sectional\ area$$

$$P = \sigma_t \times \frac{\pi d^2}{4}$$

- The single eye may tear

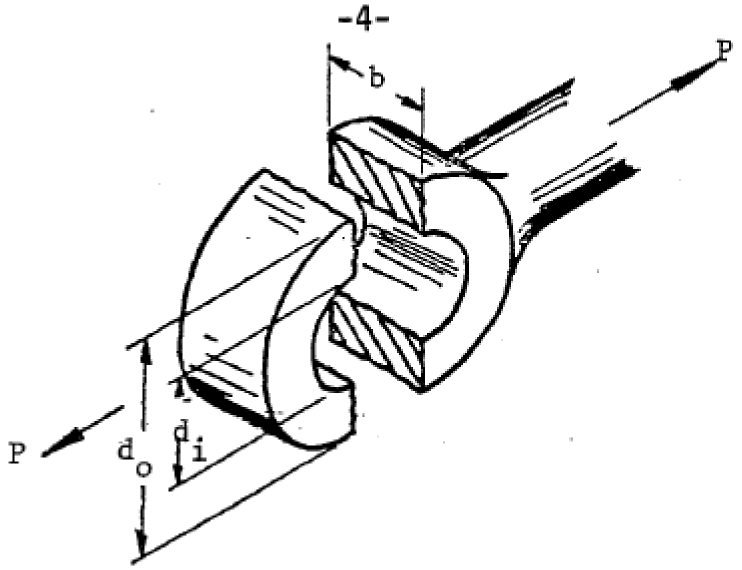


Figure 4.3

$$\begin{aligned} Axial\ load &= Tensile\ stress \times Cross - sectional\ area \\ P &= \sigma_t \times (d_o - d_i) \times b \end{aligned}$$

- The fork may tear

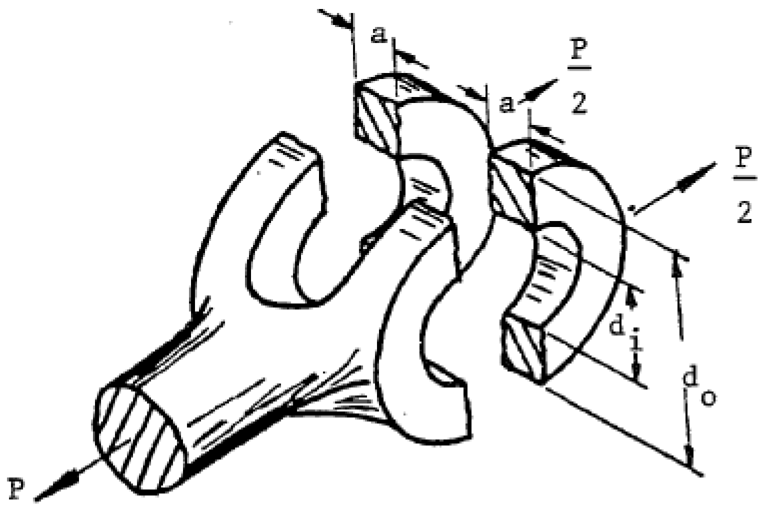


Figure 4.4

$$\begin{aligned} Axial\ load &= Tensile\ stress \times Cross - sectional\ area \\ P &= \sigma_t \times 2(d_o - d_i) \times a \end{aligned}$$

- The pin may shear

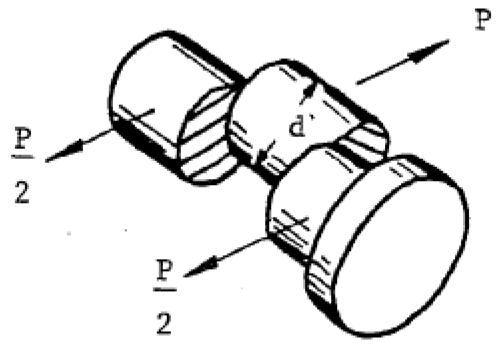


Figure 4.5

$$\begin{aligned} Axial\ load &= Shear\ stress \times Cross - sectional\ area \\ P &= \tau \times 2\frac{\pi {d_i}^2}{4} \end{aligned}$$

- The pin may be crushed against the single eye

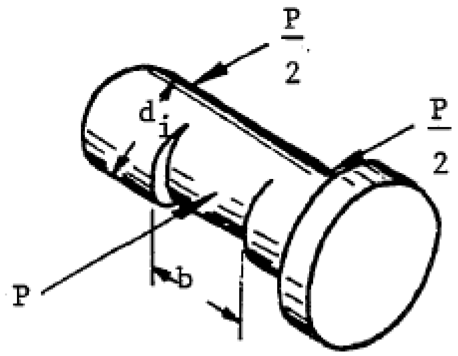


Figure 4.6

$$\begin{aligned} Axial\ load &= Crushing\ stress \times Projected\ area \\ P &= \sigma_t \times d_i \times b \end{aligned}$$

- The pin may be crushed against the fork

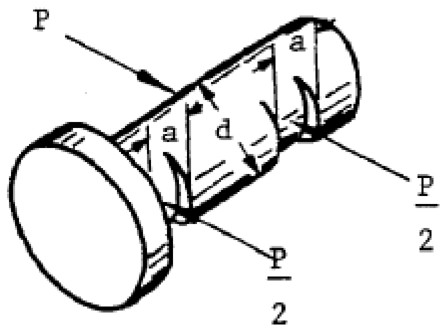


Figure 4.7

$$\begin{aligned} Axial\ load &= Crushing\ stress \times Projected\ area \\ P &= \sigma_t \times 2 \times d_i \times a \end{aligned}$$

Note:
The bending stress in the pin may be greater than the shear stress and it is for this reason that the pin diameter d; is usually made equal to the rod diameter d.

In each case:
- if P is the failing load, then stress is the ultimate stress
- if P is the safe working load, then stress is the safe working stress
- if P is the actual applied load, then stress is the actual induced stress

4.2.1 Standard proportions of the knuckle joint
These proportions should only be used when the rod diameter is known and calculations are not required.

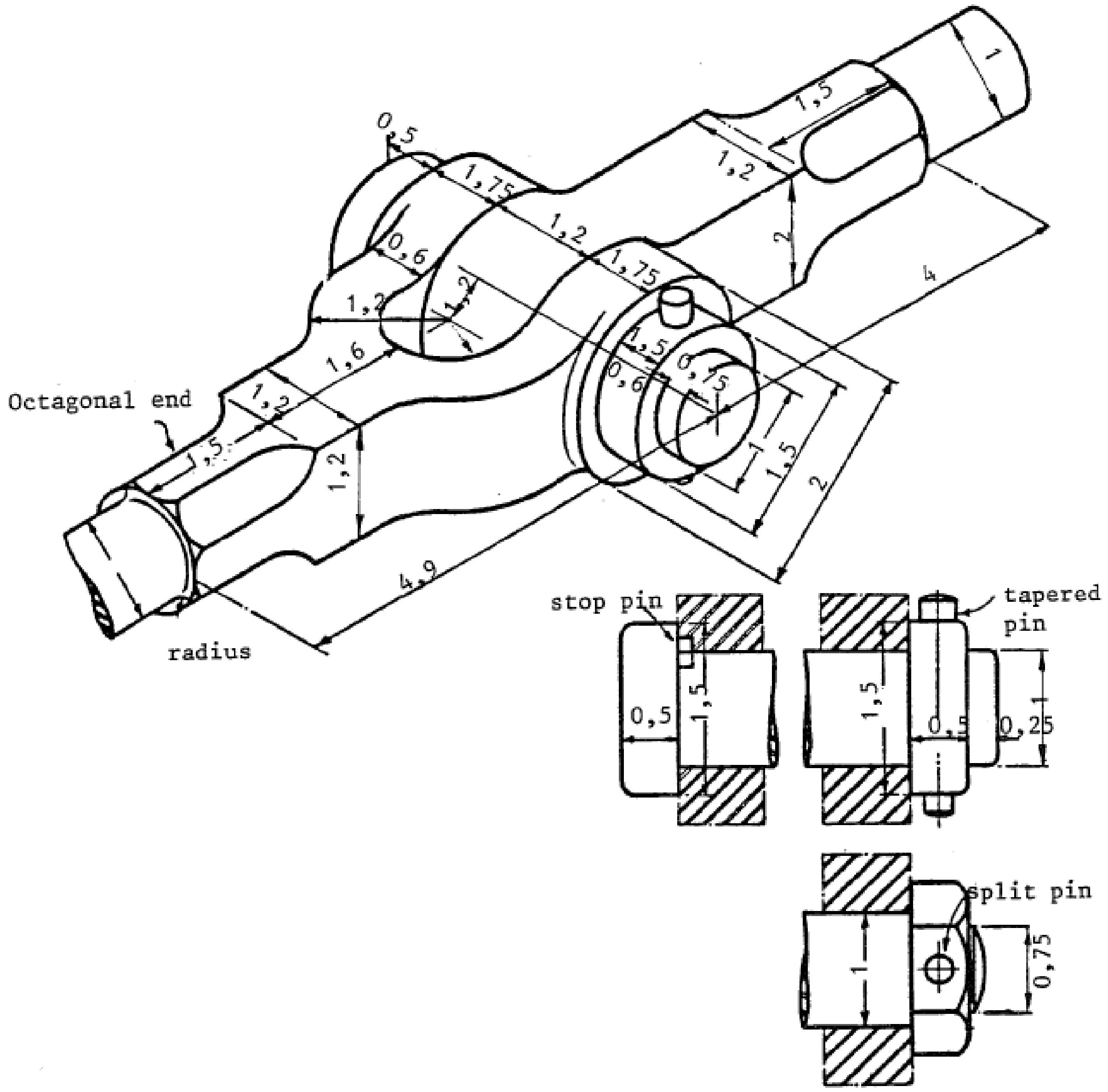


Figure 4.8

In engineering design, considerable use is made of what are known as "standard proportions". Text-books and handbooks have been published showing common parts and fitting such as knuckle joints, tottered joints, flanged couplings, etc, with all dimensions in good proportion.

These dimensions are the result partly of calculations and partly of experience. They will be found to vary somewhat in different handbooks, but will not differ very much in the main size.

Figure 4.8 shows an isometric sketch of a knuckle joint with standard proportions. It will be noted that the diameters of the rods and of the pin are each shown as one unit. If we wish to draw a joint for 20 mm diameter rods, then our unit will be 20 mm, and where a dimension of 1,2 is shown on the sketch, we will make it

$$= 1{,}2 \times 20\ mm$$
$$= 24\ mm$$

The other dimensions are worked out in a similar way.

Worked Example 4.1

A pin or knuckle joint connects the ends of two rods A and 8, each 24 mm in diameter and subjected to a pull of 30 kN. The end of rod A has an eye which is 48 mm in diameter and 30 mm thick bored to take the pin which is 24 mm in diameter. The forked end of rod B has two bearings, each with an outside diameter of 48 mm, 18 mm wide and bored 24 mm diameter for the pin.

Calculate:
1. Tensile stress in each rod away from the joint.
2. Shear stress in the pin.
3. Tensile stress in the metal of the eye.
4. Bearing stress between the pin and the fork bearings.

Solution:

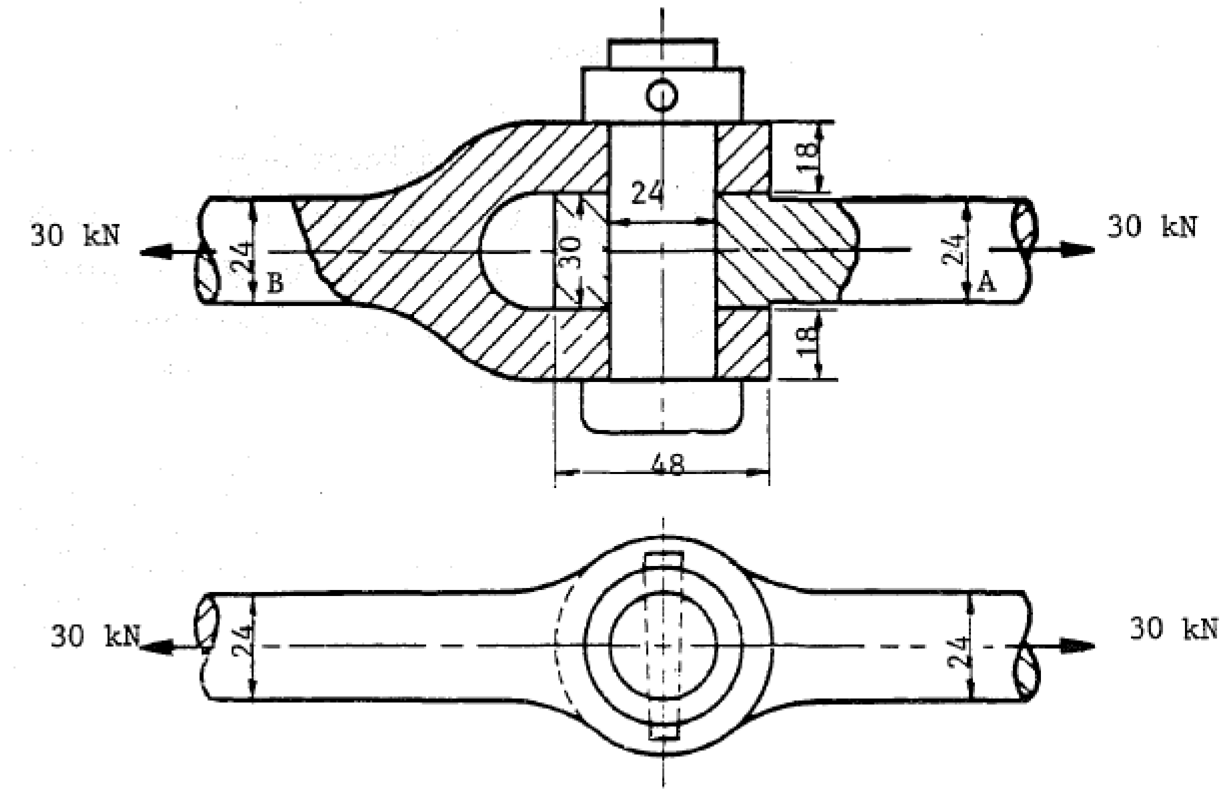


Figure 4.9

1. Tensile stress in each rod away from the joint

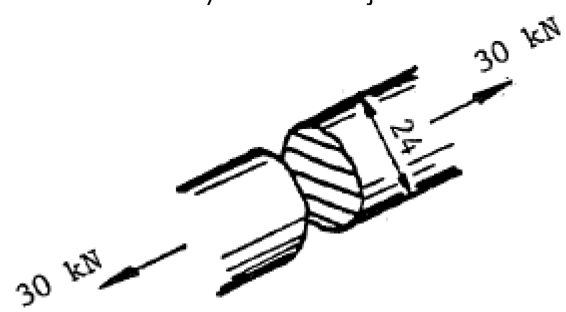


Figure 4.10

$$Axial\ load = Tensile\ stress \times Cross-sectional\ area$$

$$\therefore Tensile\ stress = \frac{Axial\ load}{Cross-sectional\ area}$$

$$\sigma_t = \frac{P}{\frac{\pi d^2}{4}}$$

$$= \frac{30 \times 10^3 N \times 4}{\pi \times (0,024\ m)^2}$$

$$= 66,32 \times 10^6\ N/m^2$$

$$= 66,32\ MPa$$

2. Shear stress in the pin

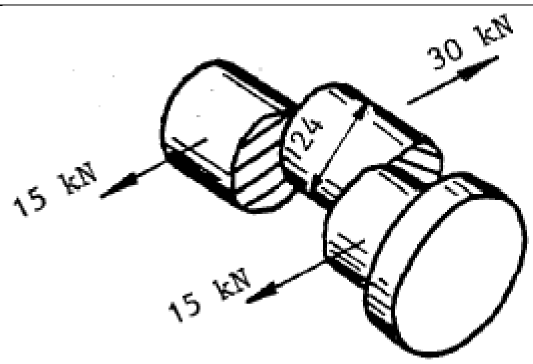


Figure 4.11

$$\text{Axial load} = \text{Shear stress} \times \text{Cross} - \text{sectional area}$$

$$\therefore \text{Shear stress} = \frac{\text{Axial load}}{\text{Cross-sectional area}}$$

$$\tau = \frac{P}{2 \times \frac{\pi d_i^2}{4}}$$

$$= \frac{30 \times 10^3 N \times 4}{2 \times \pi \times (0{,}024\ m)^2}$$

$$= 33{,}16 \times 10^6\ N/m^2$$

$$= 33{,}16\ MPa$$

3. Tensile stress in the metal of the eye

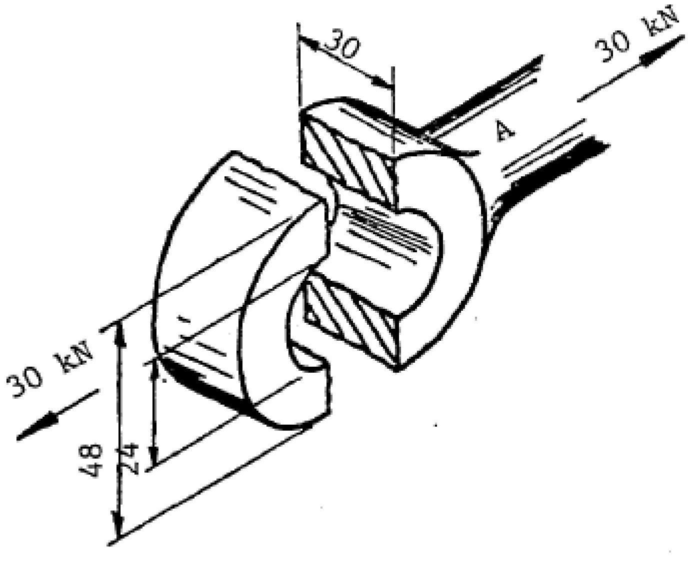


Figure 4.12

$$\text{Axial load} = \text{Tensile stress} \times \text{Cross} - \text{sectional area}$$

$$\therefore \text{Tensile stress} = \frac{\text{Axial load}}{\text{Cross-sectional area}}$$

$$\sigma_t = \frac{P}{(d_o - d_i) \times b}$$

$$= \frac{30 \times 10^3 N}{(0{,}048\ m - 0{,}024\ m) \times 0{,}03\ m}$$

$$= 41{,}67 \times 10^6\ N/m^2$$

$= 41{,}67\ MPa$

4. Bearing stress between the pin and the fork bearing

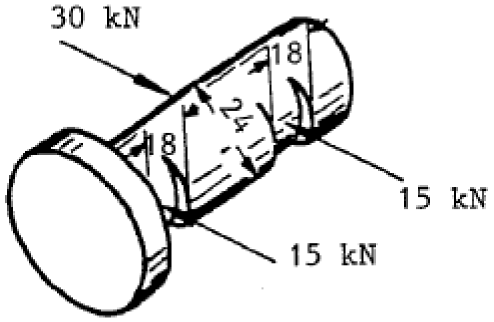


Figure 4.13

$$Axial\ load = Crushing\ stress \times Projected\ area$$

$$\therefore Crushing\ stress = \frac{Axial\ load}{Projected\ area}$$

$$\sigma_c = \frac{P}{2 \times d_i \times a}$$

$$= \frac{30 \times 10^3 N}{2 \times 0{,}024\ m \times 0{,}018\ m}$$

$$= 34{,}72\ \times 10^6\ N/m^2$$

$$= 34{,}72\ MPa$$

Worked Example 4.2

A knuckle joint for two round rods is required to carry a load of 50 kN. The working stresses in tension, shear and bearing must not exceed 124, 62 and 93 MPa, respectively.

Calculate the necessary diameters of the rods and pin.

Using the standard proportions given in **Figure 4.8**, determine the outside diameter and the widths of the eye and fork.

Check the tensile and bearing stresses in the eye and fork.

<u>Solution:</u>
Rods

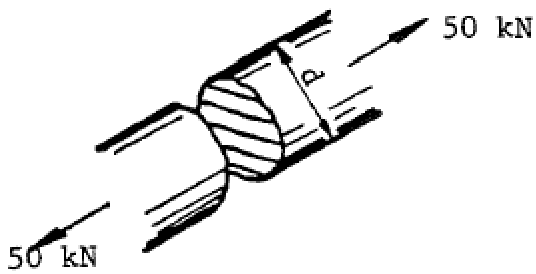


Figure 4.14

$$Axial\ load = Tensile\ stress \times Cross - sectional\ area$$

$$Cross - sectional\ area = \frac{Axial\ load}{Tensile\ stress}$$

$$\frac{\pi d^2}{4} = \frac{P}{\sigma_t}$$

$$d = \sqrt{\frac{P \times 4}{\sigma_t \times \pi}}$$

$$= \sqrt{\frac{30 \times 10^3 N \times 4}{124 \times 10^6\ N/m^2 \times \pi}}$$

$$= \sqrt{5{,}134\ \times 10^{-4}\ m^2}$$

$$= 0{,}0227\ m$$

Say 23 mm diameter rod.

Pin

$$Axial\ load = Shear\ stress \times Cross - sectional\ area$$

$$Cross - sectional\ area = \frac{Axial\ load}{Shear\ stress}$$

$$2 \times \frac{\pi {d_i}^2}{4} = \frac{P}{\tau}$$

$$d_i = \sqrt{\frac{P \times 4}{\tau \times 2 \times \pi}}$$

$$= \sqrt{\frac{50 \times 10^3 N \times 4}{62 \times 10^6\ N/m^2 \times 2 \times \pi}}$$

$$= \sqrt{5{,}134\ \times 10^{-4}\ m^2}$$

$$= 0{,}0227\ m$$

Say 23 mm diameter pin.

Standard proportions

Outside diameter of eye and fork = 2 x d
= 2 x 23 mm
= 46 mm

Width of eye = 1,2 X d
= 1,2 x 23 mm
= 27,6 mm

Say = 28 mm

Width of forks = 0,75 X d
= 0,75 x 23 mm
= 17,25 mm

Say = 18 mm

Tensile stress in eye

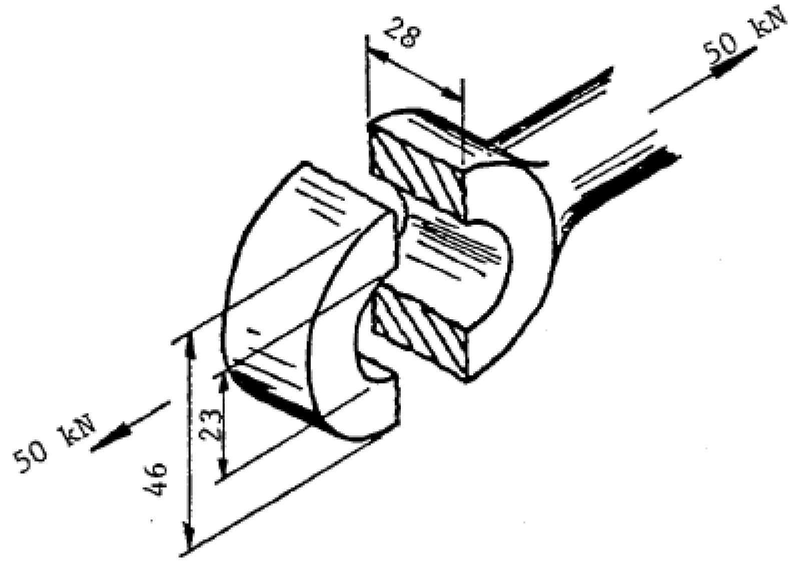


Figure 4.15

$$Axial\ load = Tensile\ stress \times Cross - sectional\ area$$

$$\therefore Tensile\ stress = \frac{Axial\ load}{Cross\text{-}sectional\ area}$$

$$\sigma_t = \frac{P}{(d_o - d_i) \times b}$$

$$= \frac{50 \times 10^3 N}{(0{,}046\ m - 0{,}023\ m) \times 0{,}028\ m}$$

$$= 77{,}64 \times 10^6\ N/m^2$$

$$= 77{,}64\ MPa\ which\ is\ less\ than\ the\ allowable\ 124\ MPa$$

Tensile stress in fork

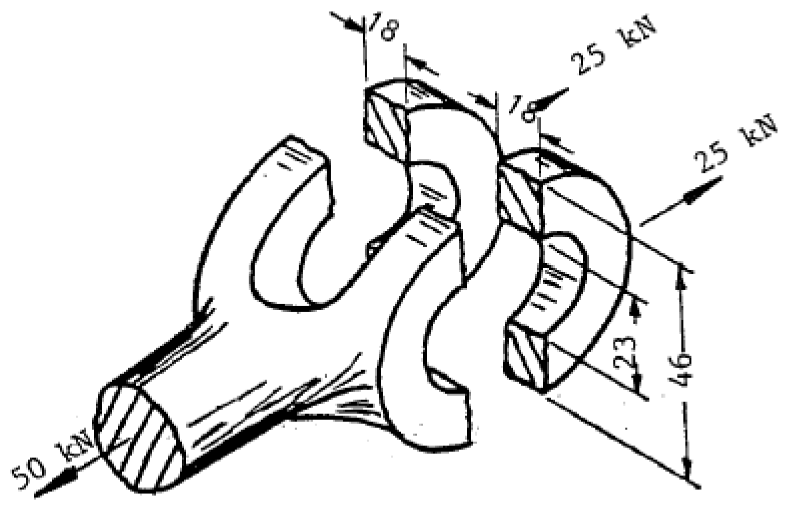


Figure 4.16

$$Axial\ load = Tensile\ stress \times Cross - sectional\ area$$

$$\therefore Tensile\ stress = \frac{Axial\ load}{Cross\text{-}sectional\ area}$$

$$\sigma_t = \frac{P}{2(d_o - d_i) \times a}$$

$$= \frac{50 \times 10^3 N}{2(0{,}046\ m - 0{,}023\ m) \times 0{,}018\ m}$$

$$= 60{,}39 \times 10^6\ N/m^2$$

$$= 60{,}39\ MPa\ which\ is\ less\ than\ the\ allowable\ 124\ MPa$$

Bearing stress in eye

This is the same as the bearing stress in the pin which is crushed against the eye.

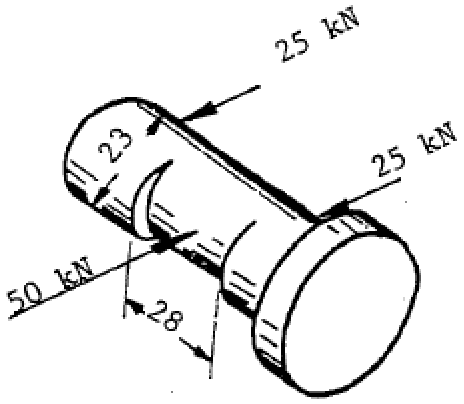


Figure 4.17

$$Axial\ load = Bearing\ stress \times Projected\ area$$

$$\therefore Bearing\ stress = \frac{Axial\ load}{Projected\ area}$$

$$\sigma_c = \frac{P}{d_i \times b}$$

$$= \frac{50 \times 10^3 N}{0{,}023\ m \times 0{,}028\ m}$$

$$= 77{,}64 \times 10^6\ N/m^2$$

$$= 77{,}64\ MPa,\ which\ is\ less\ tha\ the\ allowable\ 93\ MPa$$

Bearing stress in fork

This is the same as the bearing stress in the pin which is crushed against the fork.

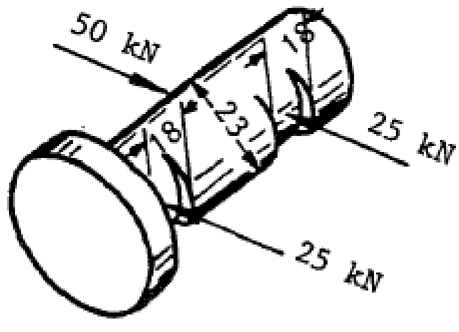


Figure 4.18

$$Axial\ load = Bearing\ stress \times Projected\ area$$

$$\therefore Bearing\ stress = \frac{Axial\ load}{Projected\ area}$$

$$\sigma_c = \frac{P}{2 \times d_i \times a}$$

$$= \frac{50 \times 10^3 N}{2 \times 0{,}023\ m \times 0{,}018\ m}$$

$$= 60{,}39\ \times 10^6\ N/m^2$$

$$= 60{,}39\ MPa\ which\ is\ less\ than\ the\ allowable\ 3\ MPa$$

Worked Example 4.3

A pin in a knuckle joint is subjected to an axial load of 90 kN. Assume that the thickness of the eye is to be 1,5 times the diameter of the pin. The allowable stress of the material in tension and compression as a result of bending is 62 MPa and the allowable stress in shear is 31 MPa. The allowable bearing stress is 21 MPa. Determine the required pin diameter.

Solution:

Check the pin for (a) bending (b) shear and (c) bearing.

(a) Bending

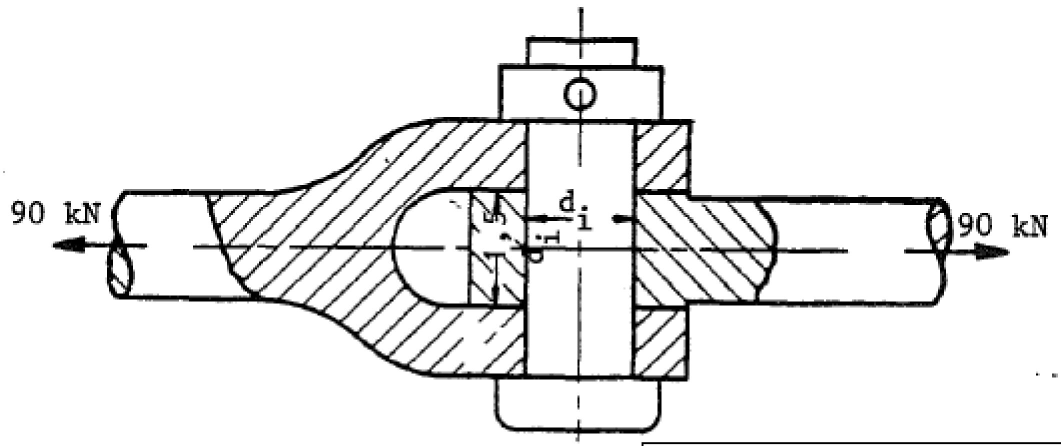


Figure 4.19

Where $M = bending\ moment$
$P = axial\ load$
$L = thickness\ of\ eye$
$\sigma_b = bending\ stress$
$Z_e = elastic\ modulus\ of\ section = \frac{\pi d_i^{\,3}}{32}$

$$M = \frac{P \times L}{8}$$

$$= \frac{90 \times 10^3 N \times 1{,}5 \times d_i}{8}$$

$$= 16875\ d_i N$$

Also $M = \sigma_b Z_e$

$$16875 \times d_i N = 62 \times 10^6 N/m^2 \times \frac{\pi \times d_i^{\,3}}{32}$$

$$16875 \times d_i N = 6{,}09 \times 10^6 \times d_i^{\,3} N/m^2$$

$$d_i^{\,2} = \frac{16875\ N}{6{,}09 \times 10^6 N/m^2}$$

$$d_i^{\,2} = 2{,}771 \times 10^{-3} m^2$$

$$d_i = 0{,}0526\ m$$

Say $= 53\ mm$

b) Shear

$$Axial\ load = Shear\ stress \times Cross - sectional\ area$$

$$Cross - sectional\ area = \frac{Axial\ load}{Shear\ stress}$$

$$\frac{2 \times \pi d_i^{\,2}}{4} = \frac{P}{\tau}$$

$$d_i^{\,2} = \frac{P \times 4}{\tau \times 2 \times \pi}$$

$$= \frac{90 \times 10^3 N \times 4}{31 \times 10^6\ N/m^2 \times 2 \times \pi}$$

$$d_i = \sqrt{1{,}85\ \times 10^{-3}\ m^2}$$

$$= 0{,}043\ m$$

Say $= 43\ mm$

b) Bearing

$$Axial\ load = Bearing\ stress \times Projected\ area$$

$$Projected\ area = \frac{Axial\ load}{Bearing\ stress}$$

$$d_i \times b = \frac{P}{\sigma_c}$$

$$d_i \times 1{,}5\ d_i = \frac{P}{\sigma_c}$$

$$= \sqrt{\frac{P}{\sigma_c \times 1{,}5}}$$

$$d_i = \sqrt{\frac{90\ \times 10^3\ N}{21 \times 10^6 N/m^2 \times 1{,}5}}$$

$$= 0{,}0535\ m$$

$$= 54\ mm$$

Say
A pin of 54 mm diameter should be used.

Worked Example 4.4

Two mild steel rods transmit a pull of 60 kN, and are to be connected by a knuckle joint. Calculate the diameter of the rods, and design the joint so that the ultimate tensile stress will not exceed 460 MPa, the ultimate shearing stress 420 MPa and the ultimate compressive stress - 835 MPa. Use a safety factor of 5.

<u>**Solution:**</u>

Working stresses

$$\sigma_t = \frac{460\times10^6 N/m^2}{5}$$
$$= 92\ MPa$$
$$\tau = \frac{420\times10^6 N/m^2}{5}$$
$$= 84\ MPa$$
$$\tau = \frac{835\times10^6 N/m^2}{5}$$
$$= 167\ MPa$$

The rod may fail in tension

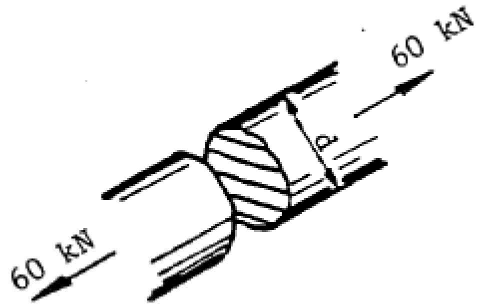


Figure 4.20

$$Axial\ load = Tensile\ stress \times Cross - sectional\ area$$

$$Cross - sectional\ area = \frac{Axial\ load}{Tensile\ stress}$$
$$\frac{\pi d^2}{4} = \frac{P}{\sigma_t}$$
$$d = \sqrt{\frac{P\times4}{\sigma_t\times\pi}}$$
$$= \sqrt{\frac{60\times10^3 N\times4}{92\times10^6\ N/m^2\times\pi}}$$
$$= \sqrt{0{,}0288\ \times 10^{-4}\,m^2}$$

Say $= 30\ mm\ diameter\ rod$

Pin may fail in double shear

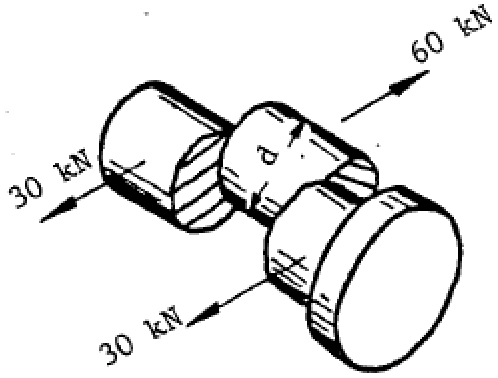


Figure 4.21

$$Axial\ load = Shear\ stress \times Cross - sectional\ area$$

$$Cross - sectional\ area = \frac{Axial\ load}{Shear\ stress}$$

$$\frac{2 \times \pi d_i^2}{4} = \frac{P}{\tau}$$

$$d_i = \sqrt{\frac{P \times 4}{\tau \times 2 \times \pi}}$$

$$= \sqrt{\frac{60 \times 10^3 N \times 4}{84 \times 10^6\ N/m^2 \times 2 \times \pi}}$$

$$= 0{,}021\ m$$

Say $= 21\ mm$

Since additional stresses as a result of bending in the pin must be allowed for, make the pin the same size as the rod, namely 30 mm in diameter.

Single eye on rod

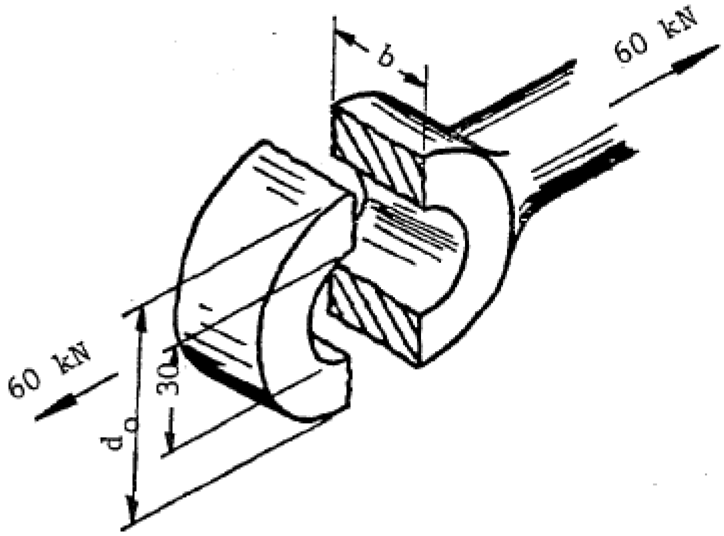


Figure 4.22

$$Axial\ load = Tensile\ stress \times Cross - sectional\ area$$

$$Cross - sectional\ area = \frac{Axial\ load}{Tensile\ stress}$$

$$(d_o - d_i)b = \frac{P}{\sigma_t}$$

$$b = \frac{P}{\sigma_t(d_o - d_i)}$$

$$Assume\ d_o = 2 \times d_i\ from\ standard\ proportions$$

$$= 2\ \times 30\ mm$$

$$= 60\ mm$$

$$Now\ b = \frac{60\ \times 10^3 N}{92\ \times 10^6\ N/m^2 \times (006\ m - 0{,}03\ m)}$$

$$= 0{,}0217\ m$$

Say $= 22\ mm$

Check the bearing stress in the eye

$$Axial\ load = Bearing\ stress\ x\ Projected\ area$$

$$Bearing\ stress = \frac{Axial\ load}{Projected\ area}$$

$$\sigma_c = \frac{P}{d_i \times b}$$

$$= \frac{60 \times 10^3\ N}{0{,}03\ m \times 0{,}022\ m}$$

$$= 90{,}9\ MPa$$

Which is less than the allowable Double eye on rod bearing stress of 167 MPa, therefore well within the limit allowed.

Double eye on rod

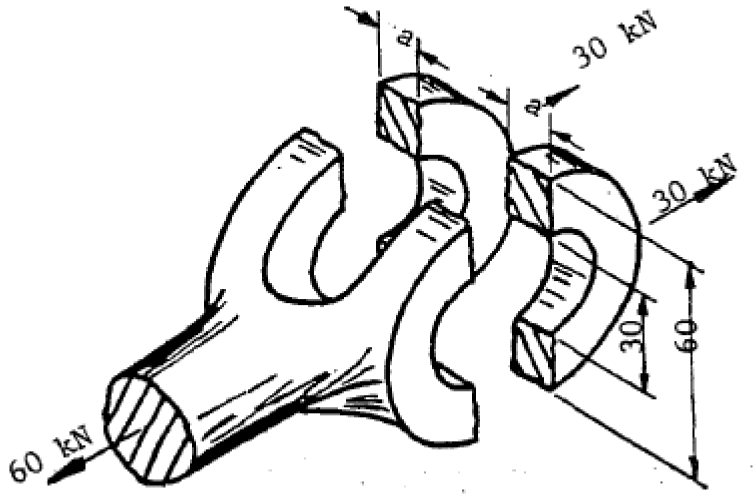


Figure 4.23

$$Axial\ load = Tensile\ stress \times Cross - sectional\ area$$

$$Cross - sectional\ area = \frac{Axial\ load}{Tensile\ stress}$$

$$2 \times (d_o - d_i)a = \frac{P}{\sigma_t}$$

$$a = \frac{P}{\sigma_t (d_o - d_i) \times 2}$$

$$= \frac{60 \times 10^3 N}{92 \times 10^6\ N/m^2 \times (006\ m - 0{,}03\ m) \times 2}$$

$$= 0{,}0109\ MPa$$

Say $12\ mm\ thick$

Check the bearing stress in the fork

$$Axial\ load = Tensile\ stress \times Projected\ area$$

$$Bearing\ stress = \frac{Axial\ load}{Projected\ area}$$

$$\sigma_c = 2(d_o - d_i)a$$

$$= \frac{60 \times 10^3 N}{2\ (0{,}06\ m - 0{,}03\ m)\ 0{,}012\ m}$$

$= 83{,}33\ MPa$

which is less than the allowable bearing stress of 167 MPa, therefore well within the limit allowed.

Thickness at curved section of fork

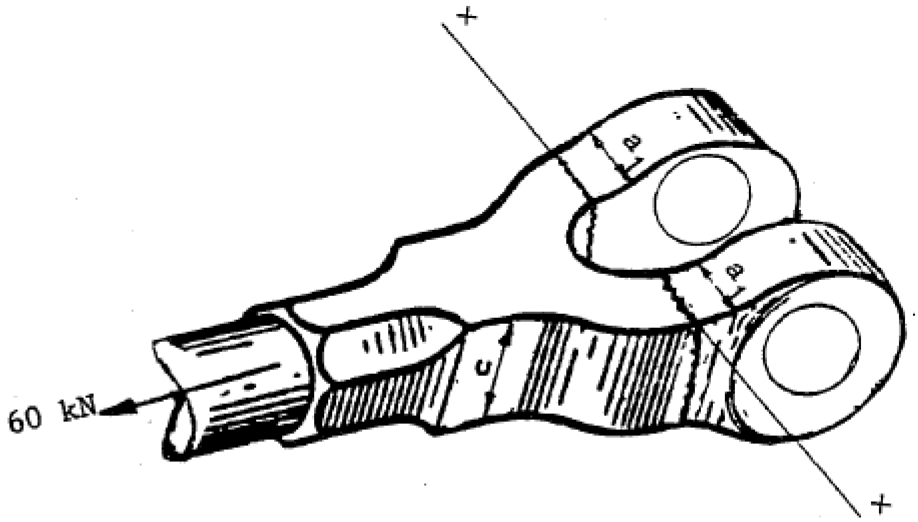


Figure 4.24

At this section $(x - x)$ the resisting area consists of two rectangles,

Where area $= 2 \times a_1 \times c$

Assume c $= 1{,}2 \times d$ and $a_1 = 0{,}6 \times d$
(from standard proportions)

$\therefore c$ $= 1{,}2 \times 30\ mm$
$= 36\ mm$ and
$a_1 = 0{,}6 \times 30\ mm$
$= 18\ mm$

Check tensile stress at this section

$$Axial\ load = Tensile\ stress \times Cross - sectional\ area$$

$$Tensile\ stress = \frac{Axial\ load}{Cross{-}sectional\ area}$$

$$\sigma_t = \frac{P}{2 \times a_1 \times c}$$

$$= \frac{60 \times 10^3 N}{2 \times 0{,}018\ m\ 0{,}036\ m}$$

$$= 46{,}3\ MPa, which\ is\ well\ below\ the\ tensile\ stress$$

All other dimensions are obtained by using standard proportions.

4.3 Cotter joints

Cotter joints are used chiefly to connect rods, etc, which are subjected to axial forces only; that is to say, when it is desired to connect two rods in the direction of their lengths.

For instance, a valve spindle may be connected to a valve rod, a piston rod may be connected to a cross-head, or two separate lengths of rods for a deep-well pump may be joint together.

⚠	**Note:** In most cases the load which is applied to the joint is tensile during the stroke in one direction, and compressive during the reverse stroke.

The cotter joint comprises of three main portions, namely
1. the rod which enters the socket **Figure 4.25a**;
2. the socket **Figure 4.25b**;
3. the cotter **Figure 4.25c**.

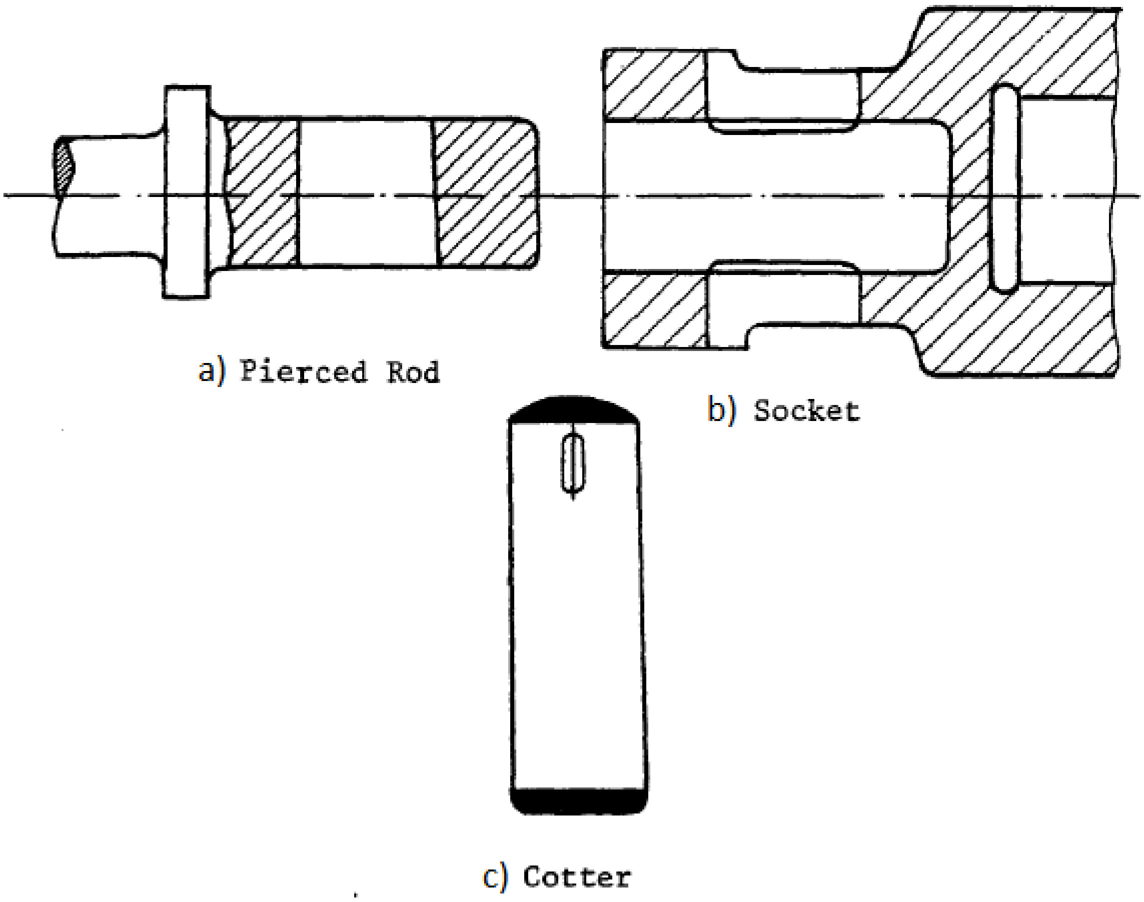


Figure 4.25

The cotter joint has numerous forms, some of which are shown in **Figure 4.26**.

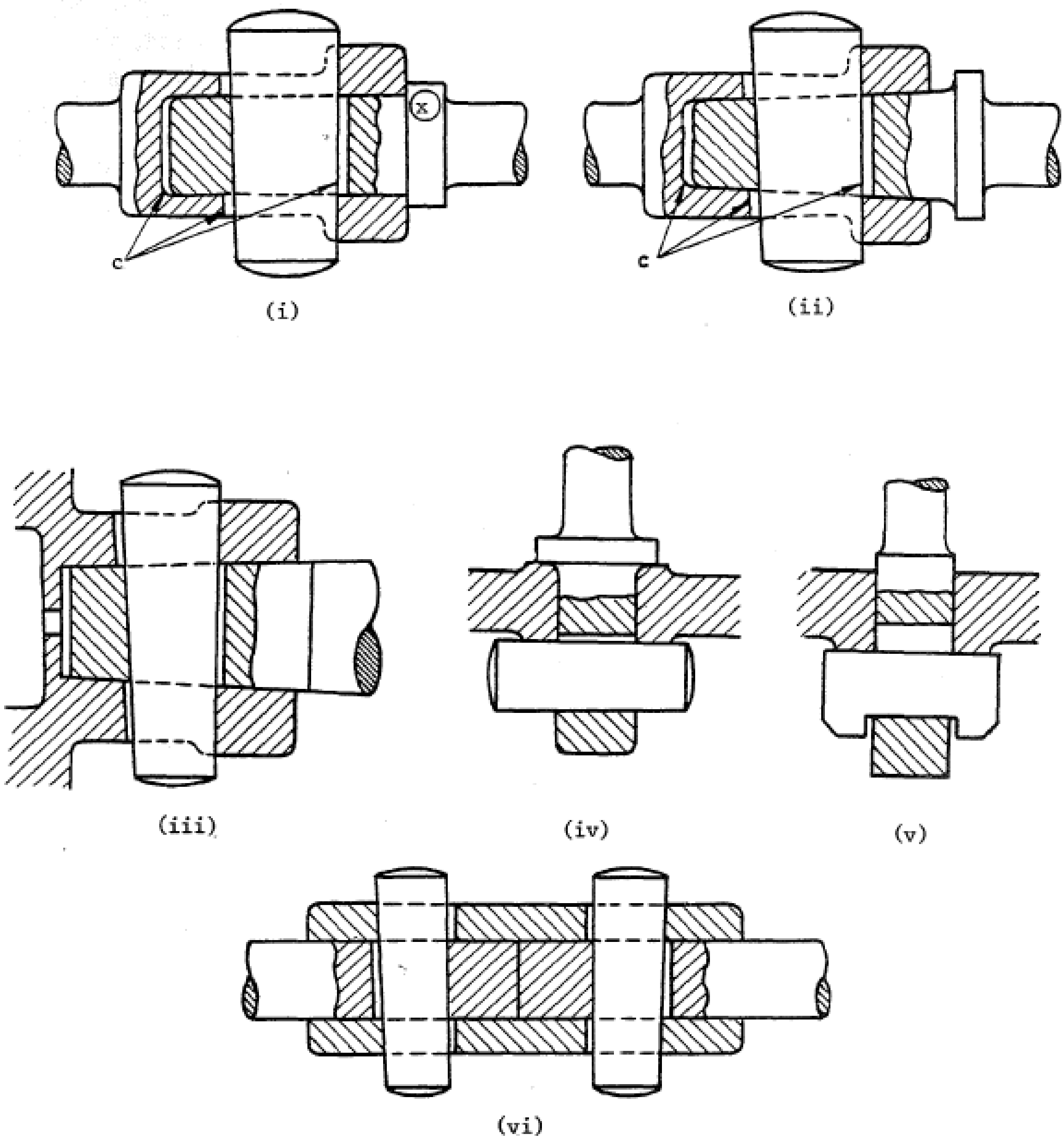


Figure 4.26

Figure 4.26(i) and **Figure 4.26(ii)**. These are the standard cottered joints for joining two rods. **Figure 4.26(i)** has a parallel rod end and socket, the rod being drawn up tightly against the collar “x" by driving in the tapper cotter.

Figure 4.26(ii) has a tapered rod end which is drawn tightly into the tapered socket by driving in the cotter. The collar is not essential in this case but is provided to help in removing the rod from the socket if it should stick.

In both cases note the clearances at “c". These are essential for the cotter to be able to draw the joint up tightly. The clearance in the rod is on the opposite side to the clearance in the socket, the clearance being approximately 3 mm.

Figure 4.26(iii) shows a piston rod attached to a cross-head by means of a cottered joint.

Figure 4.26(iv) and **Figure 4.26(v)** show cotters as used on foundation bolts, etc. Note the large clearance in **Figure 4.26(v)** to enable the cotter to be inserted. **Figure 4.26(vi)** . Two rods joined by a tubular sleeve and cotters.

In each case where the rod end is tapered the taper should be 1 in 16 or 1 mm per 16 mm on the diameter (ie the diameter decreases by 1 mm for 16 mm length of taper).

The taper on the cotters is 1 in 32 or 1 mm per 32 mm length except when some retaining device such as a grub-screw is provided to prevent the cotter working loose. In that case the taper may be considerably more.

Note:
The cotter is tapered only on one of its long sides, and that its cross-section is of rectangular shape.

We will always speak of the cotter being of breadth b and thickness t; so far as the breadth b is concerned, it is the mean breadth of the tapered cotter which is meant. t is of constant thickness, as is shown in **Figure 4.27**.

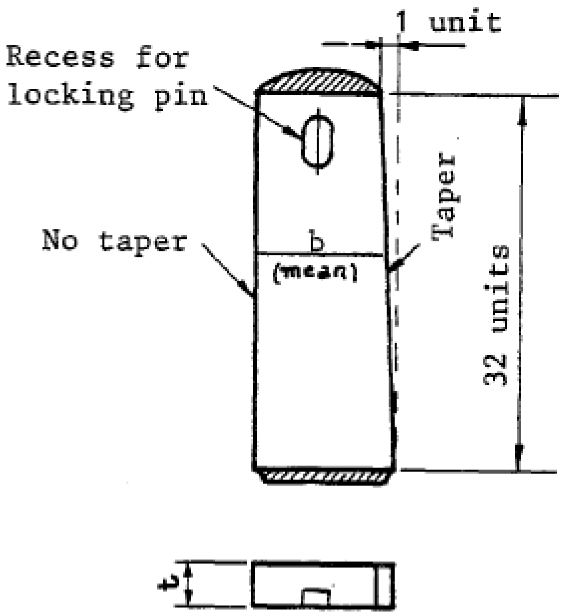


Figure 4.27

On giving consideration to **Figure 4.28**, it will be seen that when the load being carried by the joint is acting towards the right arrow P, the compressive load is being carried by the collar x; when the load acts in the reverse direction, the joint is subjected to tensile force, and this tensile load is carried by the cotter, which latter, obviously, tends to shear through in two places.

> **Note:**
> The slot which goes right through the socket, penetrates partly through the enlarged diameter D, and partly through the smaller diameter D_1.

This enlarged diameter, D, offers resistance to the cotter when the rod is being subjected to tension, that is, when the load is being carried and movement is in a direction opposite to that indicated by the arrow P.

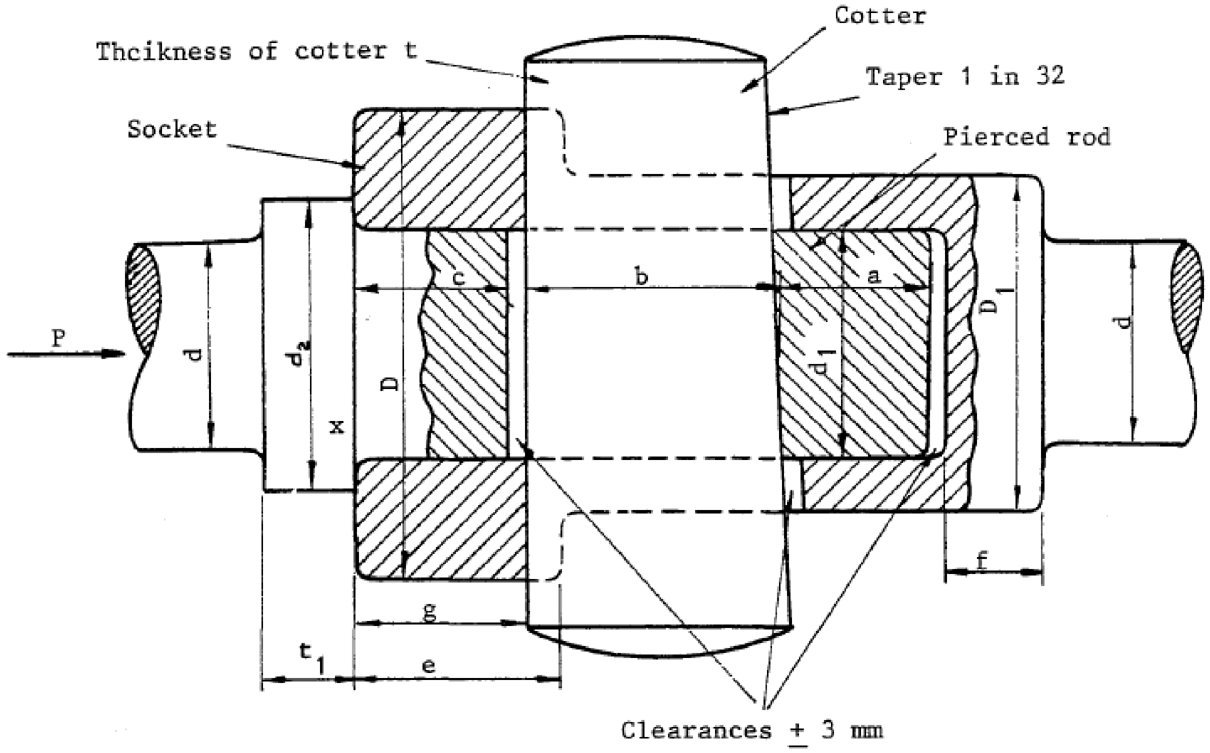


Figure 4.28

4.3.1 Standard proportions

The following standard proportions can be used for a cotter joint. All letters should refer to **Figure 4.28**.

$D = 2{,}4\,d$	$a = g = 0{,}75d$	$e = d$
$D_1 = 1{,}75d$	$b = 1{,}3d$	$f = 0{,}5d$
$d_1 = 1{,}21\,d$	$t = 0{,}31d$	$c = g - 3\,mm$
$d_2 = 1{,}5d$	$t_1 = 0{,}45d$	

Standard proportions only to be used when calculations are not required.

4.3.2 Design of cottered joints

In order to design cottered joints we must investigate all the possible ways in which they can fail. The symbols used are the same as those in **Figure 4.8**.

Suppose the joint is subjected to a fluctuating load of ±P newtons and that the safe tensile, shear and compressive stresses are σ_t, τ and σ_c respectively. For mild steel it is always safe to assume that the ratio of the safe tensile stress, to the safe shear stress to the safe crushing stress is as

1 is to 0,8 is to 1,6 ie :
$\sigma_t : \tau : \sigma_c$ is as 1 : 0,8 : 1,6

Referring to **Figure 4.29** we see that when the load is a compressive one, ie acting in the direction of arrow P the rod of diameter d is in compression, and that the thrust is carried by the collar having the diameter d_2. Failure may occur in the following ways.

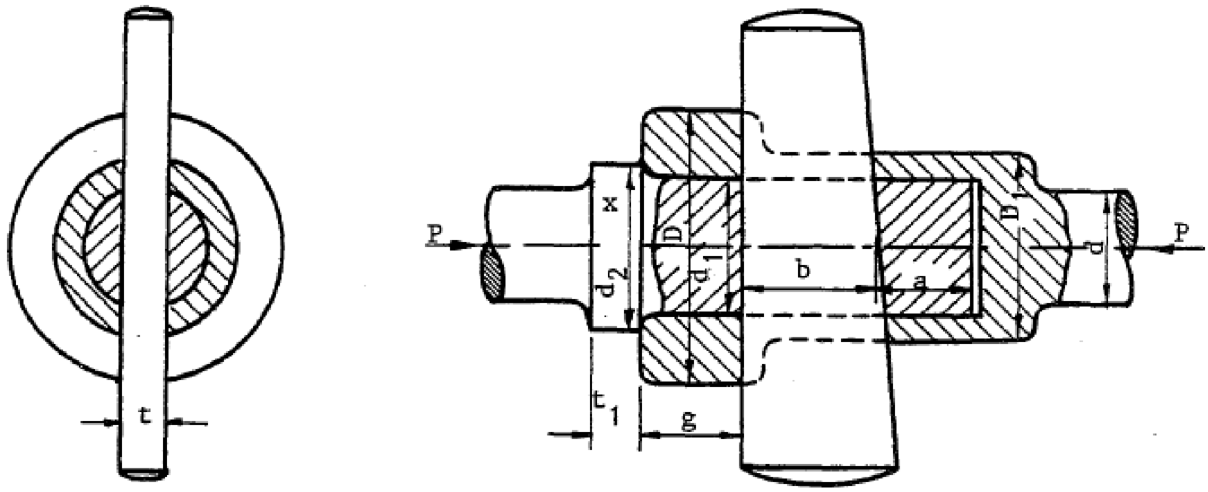


Figure 4.29

- The collar may be sheared so that it slides over-the solid shaft

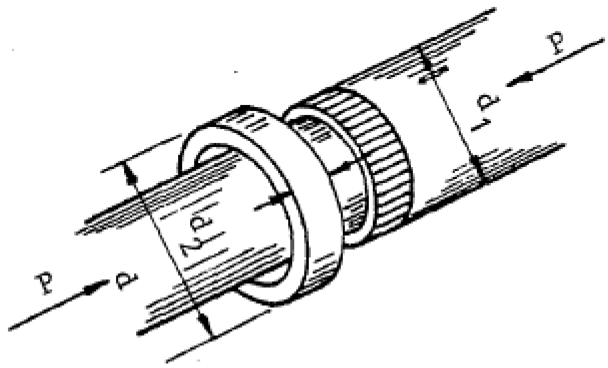


Figure 4.30

$$Shear\ load = Shear\ stress \times Shear\ area$$
$$P = \tau \times \pi d_1 \times t_1 \ \ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$

- The collar may be crushed against the Socket

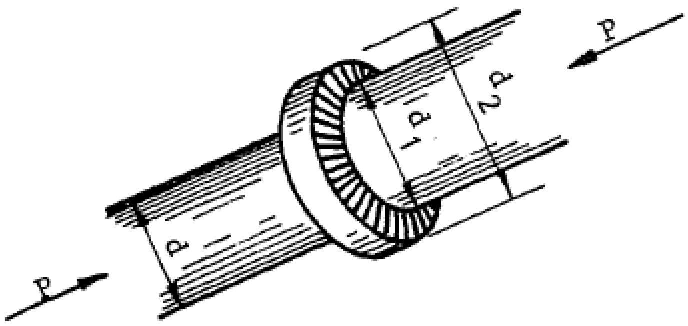


Figure 4.31

$$Crushing\ load = Crushing\ stress \times Projected\ area$$
$$P = \sigma_t \times \frac{\pi}{4}(d_2^2 - d_1^2) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

When the load is in the other direction, a tensile force acts along the rods, and failure may occur in the following ways.

- The solid rod of diameter d may tear

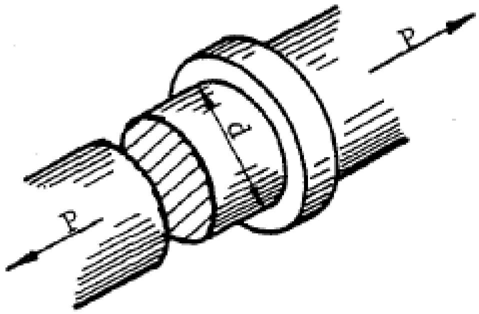


Figure 4.32

$$Tensile\ load = Tensile\ stress \times Cross - sectional\ area$$
$$P = \sigma_t \times \frac{\pi d^4}{4} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

- The pierced rod may tear

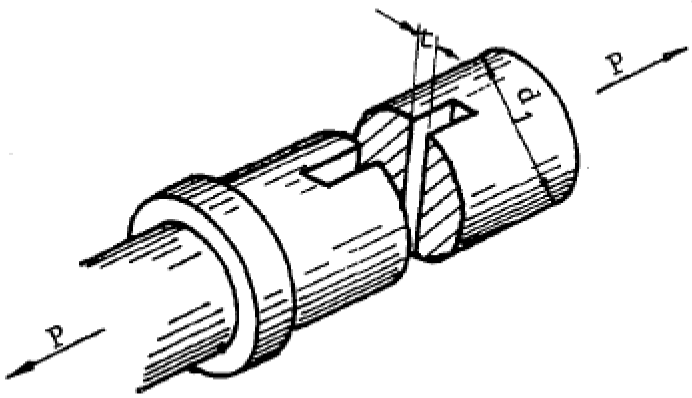


Figure 4.33

$$Tensile\ load = Tensile\ stress \times Cross - sectional\ area$$
$$P = \sigma_t \times \left(\frac{\pi d_1^2}{4} - d_1 \times t\right) \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Note:
If the pierced rod is tapered the diameter d_1 will be the diameter on the smaller portion of the taper.

- The socket may tear at the cotter hole

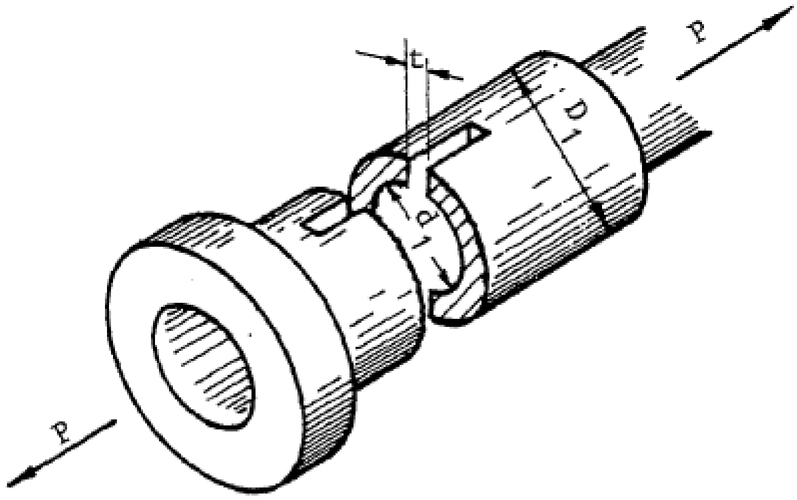


Figure 4.34

$$Tensile\ load = Tensile\ stress \times Cross - sectional\ area$$
$$P = \sigma_t \times \left\{\frac{\pi}{4}(D_1^2 - d_1^2) - t(D_1 - d_1)\right\} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

- The cotter may shear

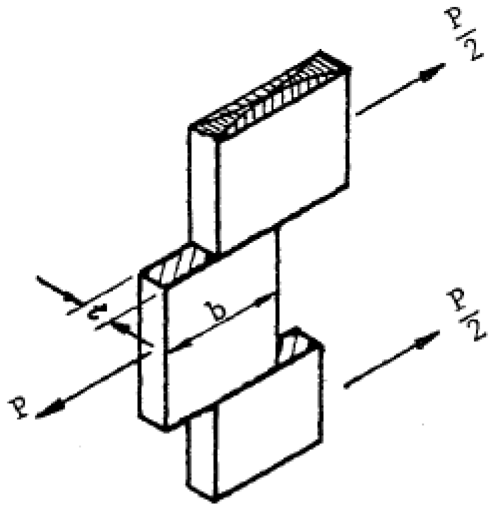


Figure 4.35

$$Shear\ load = Shear\ stress \times Cross - sectional\ area$$
$$P = \tau \times 2 \times t \times b \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Cotter is in double shear.

- The end of the pierced rod may shear

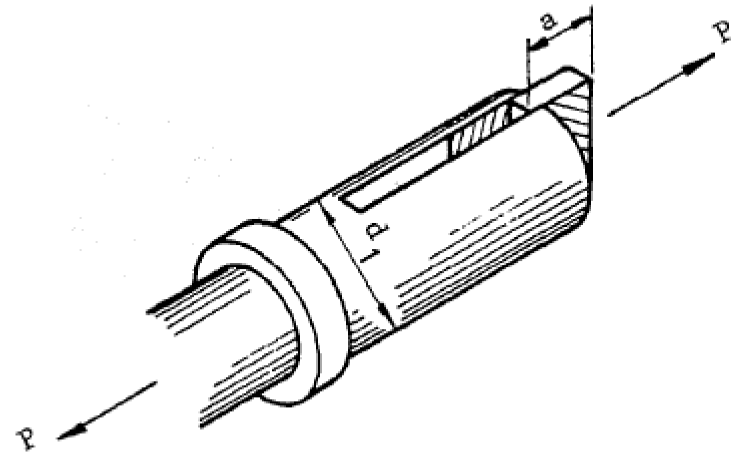


Figure 4.36

$$\begin{aligned} Shear\ load &= Shear\ stress \times Cross - sectional\ area \\ P &= \tau \times 2 \times d_1 \times a \ldots\ldots\ldots\ldots\ldots\ldots \ (7) \end{aligned}$$

- The end of the socket may shear

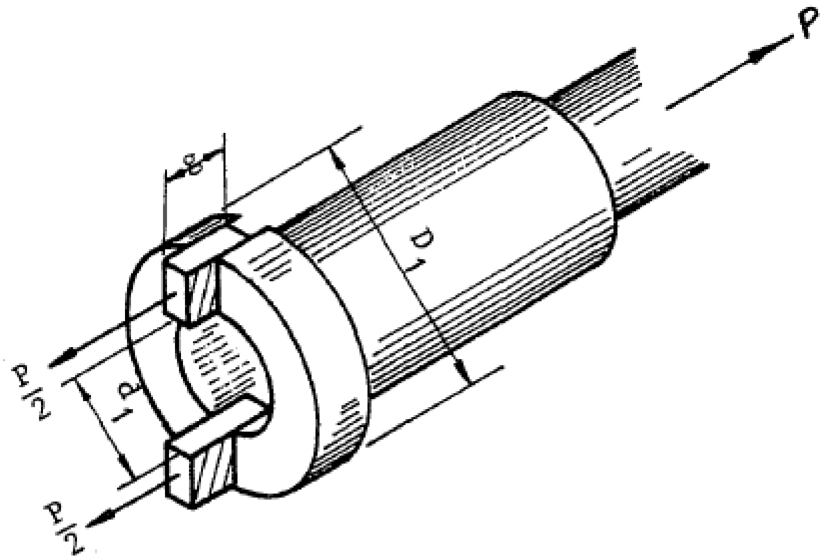


Figure 4.37

$$\begin{aligned} Shear\ load &= Shear\ stress \times Cross - sectional\ area \\ P &= \tau \times 2(D - d_1) \times g \ldots\ldots\ldots\ldots\ldots\ldots \ (8) \end{aligned}$$

- The cotter may be crushed against the rod

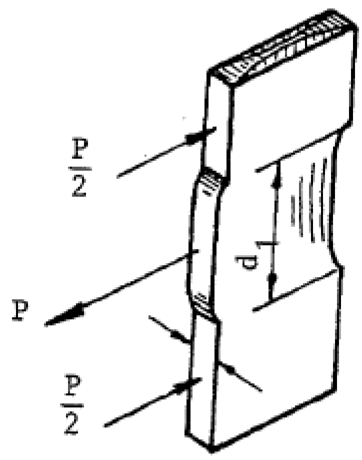


Figure 4.38

$$Crushing\ load = Crushing\ stress \times Projected\ area$$
$$P = \sigma_c \times d_1 \times t \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

- The cotter may be crushed against the socket

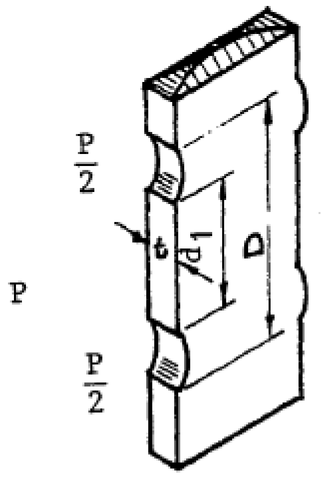


Figure 4.39

$$Crushing\ load = Crushing\ stress \times Projected\ area$$
$$P = \sigma_c \times (D - d_1)t \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

When the various sizes of the parts of a cotter joint are given and it is required to calculate the allowable load carried by the joint, or the various stresses induced in the various parts of the joint, equation (1) to (10) are used in the calculations.

For equal strength of all parts of the cotter joint, equations (1) to (10) are equal to each other, and can be used to calculate the various sizes for the parts of a joint.

Therefore, when the allowable tensile stress, shear stress and crushing stress as well as the allowable load carried by the joint are given and it is required to obtain suitable sizes for the parts of the cotter joint it is necessary to equate some of the equations (1) to (10) to each other as follows:

- To find the breadth (b) of the cotter

Equate equation (6) to equation (9).

$$\begin{aligned} Shear\ load\ in\ cotter &= Crushing\ load\ in\ cotter\ against\ rod \\ \tau \times 2 \times t \times b &= \sigma_c \times d_1 \times t \\ t &= \frac{\sigma_c \times d_1}{\tau \times 2} \ \ldots\ldots\ldots\ldots\ldots\ldots\ (11) \end{aligned}$$

- To find the thickness (t) of the cotter

Equate equation (4) to equation (9).

$$\begin{aligned} Tensile\ load\ in\ pierced\ rod &= Crushing\ load\ in\ cotter\ against\ rod \\ \sigma_t \times \left(\frac{\pi d_2^1}{4} - d_1 \times t\right) &= \sigma_c \times d_1 \times t \\ t &= \frac{\sigma_t \times \pi \times d_1}{4(\sigma_c + \sigma_t)} \ \ldots\ldots\ldots\ldots\ldots\ldots\ (12) \end{aligned}$$

- To find the diameter (d_1) of the pierced rod

Substitute the value of t from equation (12) in equation (9) and we have

$$\begin{aligned} P &= \sigma_c \times d_1 \times t \ \ldots\ldots\ldots\ldots\ldots\ldots\ (9) \\ P &= \sigma_c \times d_1 \times \frac{\sigma_t \times \pi \times d_1}{4(\sigma_c + \sigma_t)} \\ \therefore d_1 &= \sqrt{\frac{4 \times P(\sigma_c + \sigma_t)}{\pi(\sigma_c + \sigma_t)}} \end{aligned}$$

$$\frac{To\ find\ outside\ diameter\ (d_2)\ of\ collar}{Equate\ equation\ (2)\ to\ equation\ (9)}$$

$$\begin{aligned} Crushing\ load\ in\ collar &= Crushing\ load\ in\ cotter\ against\ rod \\ \sigma_t \times \frac{\pi}{4}(d_2^2 - d_1^2) &= \sigma_c \times d_1 \times t \\ d_2 &= \sqrt{\frac{4}{\pi}(d_1 \times t) + d_1^2} \ \ldots\ldots\ldots\ldots\ldots\ (14) \end{aligned}$$

Always adjust calculated diameters of the collar joint to correspond to the next upper standard size round bar.

Table 4.1 shows the standard diameters of round steel bars, in millimeters:

Standard diameters of round steel bars (mm)			
6	20	50	100
8	22	55	110
10	25	60	130
12	30	65	150
14	35	70	170
16	40	80	
18	45	90	

Table 4.1

$$\frac{To\ find\ the\ thickness\ (t_1)\ of\ the\ collar}{Equate\ equation\ (1)\ to\ equation\ (6)}$$

$$Shear\ load\ in\ collar = Shear\ load\ in\ cotter$$

$$\tau \times \pi d_1 \times t_1 = \tau \times 2 \times t \times b$$

$$t = \frac{2 \times t \times b}{\pi \times d_1} \text{ .... } (15)$$

- To find the rod diameter (d)

Equate equation (3) to equation (4).

$$Tensile\ load\ in\ rod = Tensile\ load\ in\ pierced\ rod$$

$$\sigma_t \times \frac{\pi d^2}{4} = \sigma_t \left(\frac{\pi d_1^2}{4} - d_1 \times t\right)$$

$$d = d_1^2 - \frac{4 d_1 \times t}{\pi} \text{ .... } (16)$$

- To find the length (a) at the end of the pierced rod

Equate equation (7) to equation (6).

$$Shear\ load\ in\ end\ of\ pierced\ rod = Shear\ load\ in\ cotter$$

$$\tau \times 2 \times d_1 \times a = \tau \times 2 \times t \times b$$

$$a = \frac{t \times b}{d_1} \text{ .... } (17)$$

- To find the enlarged diameter (D) of the socket

Equate equation (9) to equation (10).

$$Crushing\ load\ in\ cotter\ against\ rod = Crushing\ load\ in\ cotter\ against\ socket$$

$$\sigma_c \times d_1 \times t = \sigma_t \times (D - d_1)t$$

$$D = 2d_1 \ldots\ldots\ldots\ldots\ldots (18)$$

- To find the diameter (D_1) of the pierced socket

Equate equation (5) to equation (4).

$$\sigma_t \times \left\{\frac{\pi}{4}(D_1^2 - d_1^2) - t(D_1 - d_1)\right\} = \sigma_t\left(\frac{\pi d_1^2}{4} - d_1 \times t\right)$$

$$0{,}785D_1^2 - 1{,}57d_1^2 - D_1t + 2d_1t = 0 \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Equation (19) is a quadratic equation and can be solved by using the formula

$$D_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- To find the length (g) at the end of the enlarged end of the socket

Equate equation (8) to equation (6).

$$Shear\ load\ in\ enlarged\ end\ of\ socket = Shear\ load\ in\ cotter$$

$$\tau \times 2(D - d_1)g = \tau \times 2 \times t \times b$$

$$g = \frac{t \times b}{D_1 - d_1} \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

The head of a cotter joint may also be in the form of a Stirrup. (Rectangular). **Figure 4.40**.

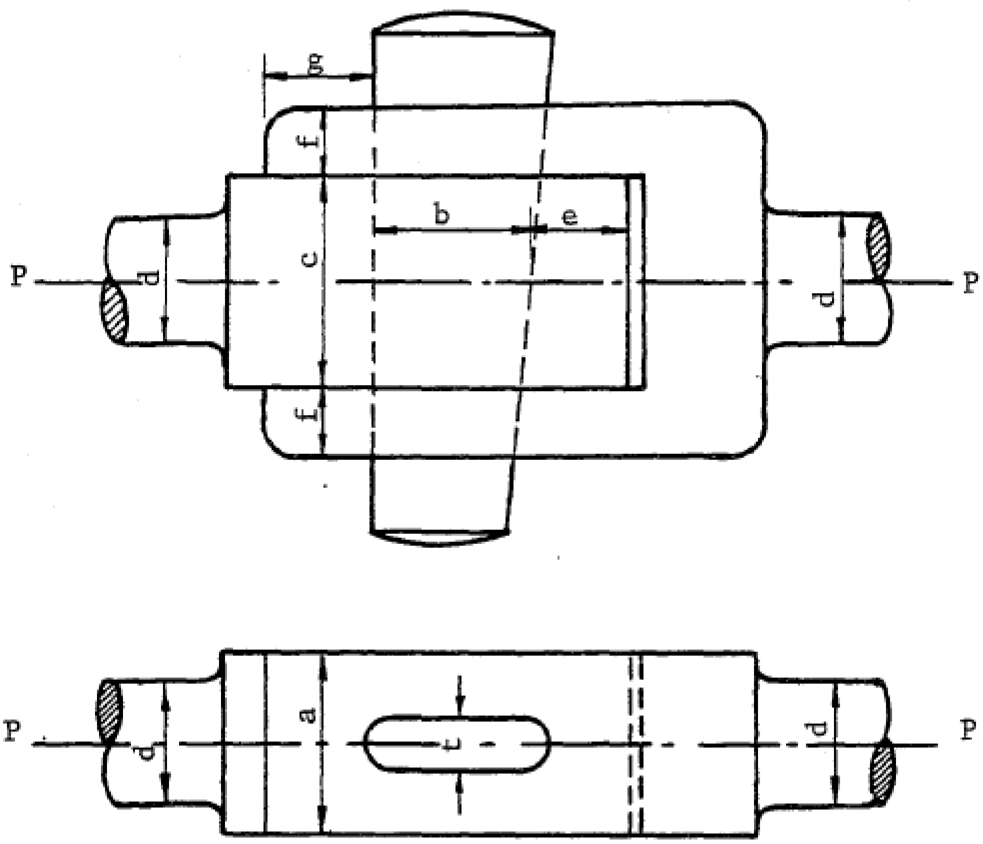


Figure 4.40

The joint may fail in any of the following ways:

i. The solid rod may tear

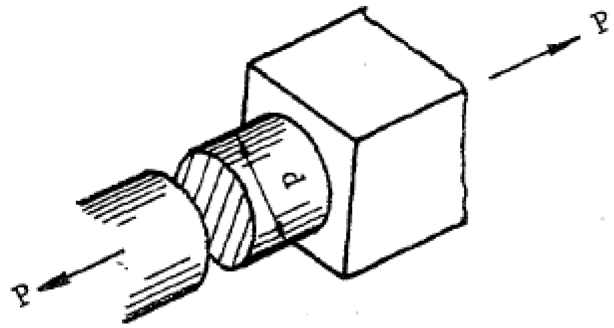


Figure 4.41

$$Tensile\ load = Tensile\ stress \times Cross - sectional\ area$$

$$P = \sigma_t \times \frac{\pi d^2}{4} \ \ldots\ldots\ldots\ldots\ldots\ldots \text{(a)}$$

ii. The square head may tear at the cotter hole

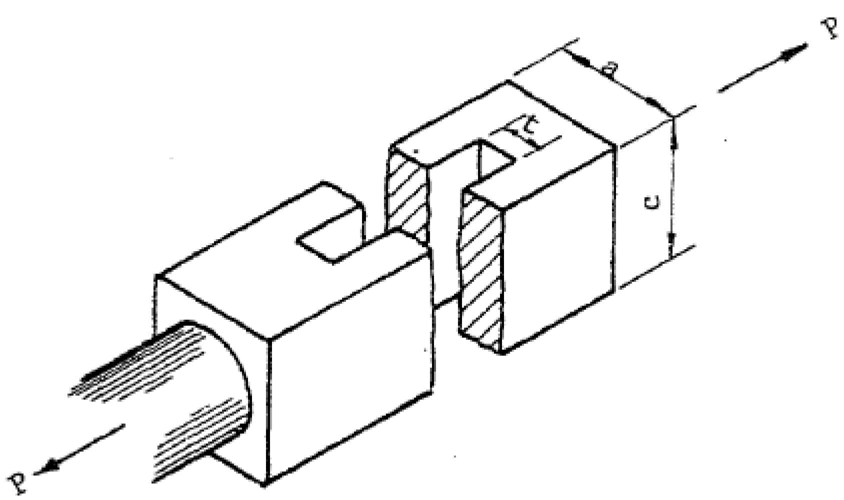


Figure 4.42

$$Tensile\ load = Tensile\ stress \times Cross - sectional\ area$$

$$P = \sigma_t \times c(a - t) \ \ldots\ldots\ldots\ldots\ldots\ldots \text{(b)}$$

Again, try to use a standard size square steel bar for the pierced rod. **Table 4.3** shows the standard width across flats in millimeters of square steel bar:

Standard diameters of round steel bars (mm)			
10	16	25	40
12	18	30	45
14	20	35	50

Table 4.2

iii. The forked head-may tear at the cotter hole

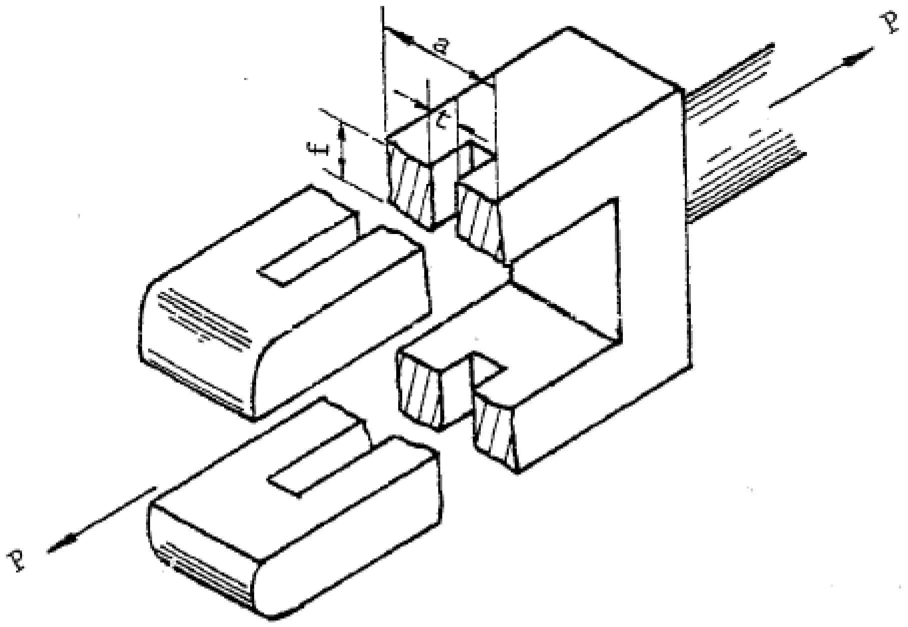


Figure 4.43

$$Tensile\ load = Tensile\ stress \times Cross - sectional\ area$$

$$P = \sigma_t \times 2 \times f \times (a - t) \quad \dots\dots\dots\dots\dots\dots \text{(c)}$$

iv. The cotter may shear

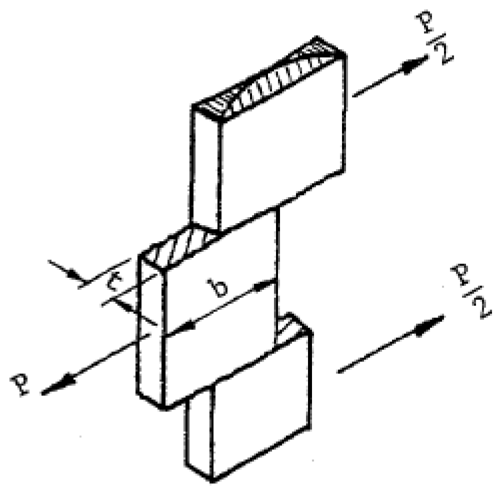


Figure 4.44

$$Shear\ load = Shear\ stress \times Cross-sectional\ area$$

$$P = \tau \times 2 \times b \times t \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(d)}$$

v. Shearing of the rectangular head

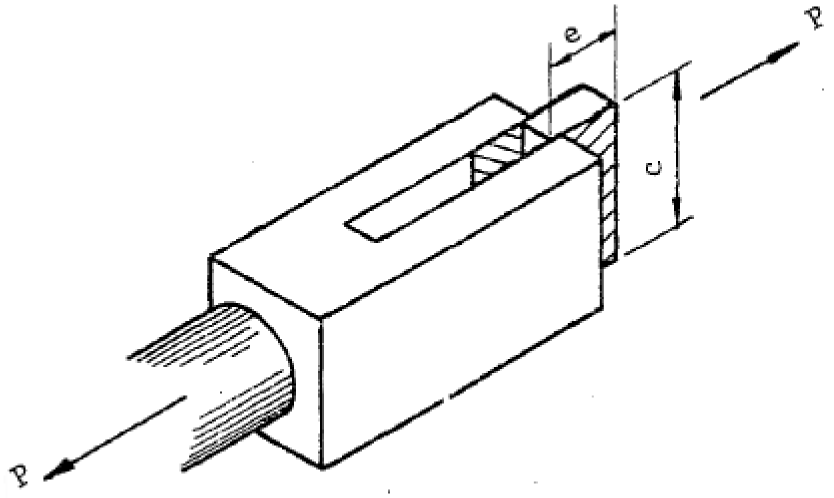


Figure 4.45

$$Shear\ load = Shear\ stress \times Cross-sectional\ area$$

$$P = \tau \times 2 \times c \times e \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(e)}$$

vi. Shearing of the forked ends

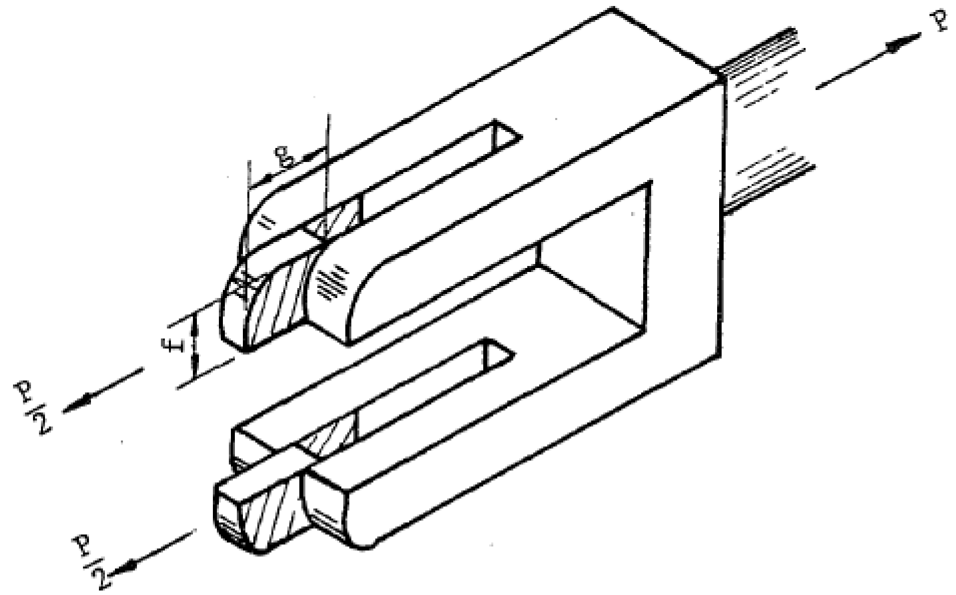


Figure 4.46

$$Shear\ load = Shear\ stress \times Cross-sectional\ area$$

$$P = \tau \times 4 \times f \times g \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(f)}$$

vii. Cotter may be crushed against the square head

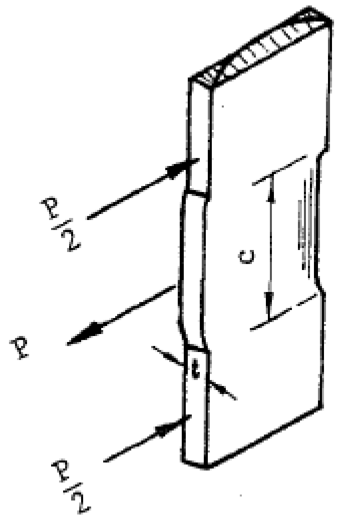


Figure 4.47

$$Crushing\ load = Crushing\ stress \times Projected\ area$$

$$P = \sigma_C \times c \times t \ \ldots\ldots\ldots\ldots\ldots\ldots \text{(g)}$$

viii. Cotter may be crushed against the forked end

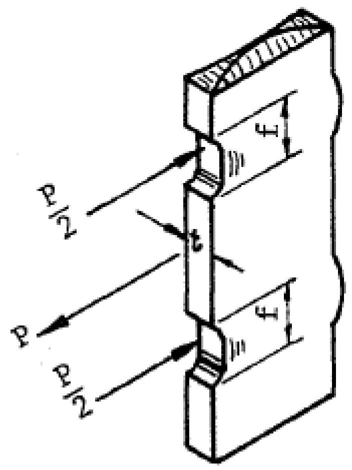


Figure 4.48

$$Crushing\ load = Crushing\ stress \times Projected\ area$$

$$P = \sigma_C \times 2 \times f \times t \ \ldots\ldots\ldots\ldots\ldots\ldots \text{(g)}$$

Again, for equal strength of all parts of the joint, equations (a) to (g) are equal to each other, and can be used to calculate the various sizes for the parts of such a joint.

Worked Example 4.5

A cottered bolt of the type shown in **Figure 4.26(iv)** has to be designed to carry a pull of 130 kN and the working stresses are not to exceed 124 MPa, 93 MPa and 216 MPa for tension, shear and crushing respectively.

Design the bolt and the cotter.

Solution:

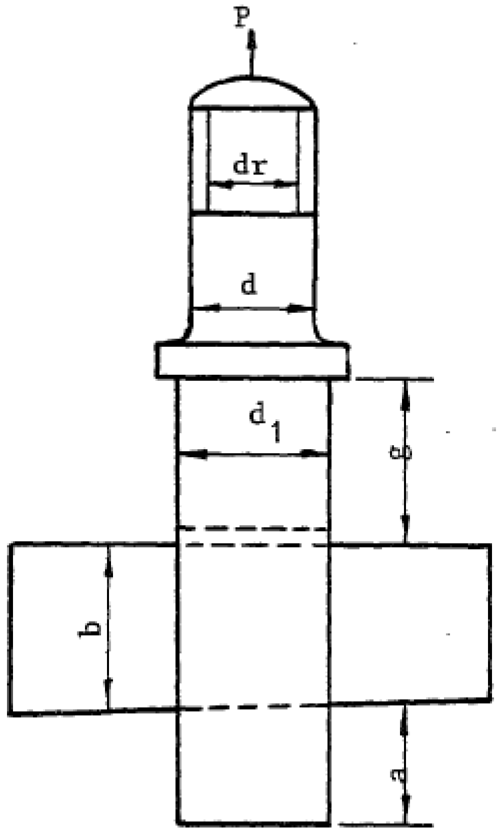


Figure 4.49

Find diameter d_1 pierced section

$$d_1 = \sqrt{\frac{4 \times P(\sigma_c + \sigma_t)}{\pi(\sigma_c + \sigma_t)}}$$

$$= \sqrt{\frac{4 \times 130 \times 10^3\ (216 \times 10^6 + 124 \times 10^6)}{\pi(216 \times 10^6 + 124 \times 10^6)}}$$

$$= \sqrt{2,10\ \times\ 10^{-3}}$$

$$d_1 = 0,0458\ m$$

Say 50 mm standard size round bar.

Cotter breadth (b)

$$Shear\ load\ in\ cotter = Crushing\ load\ in\ cotter\ against\ pierced\ section$$

$$\tau \times 2 \times t \times b = \sigma_c \times d_1 \times t$$

$$b = \frac{\sigma_c \times d_1}{\tau \times 2}$$

$$= \frac{216 \times 10^6 N/m^2 \times 0{,}050\ m}{93 \times 10^6 N/m^2 \times 2}$$

$$b = 0{,}0581 \text{ Say 58 mm}$$

Cotter thickness (t)

$$Tensile\ load\ in\ pierced\ section = Crushing\ load\ in\ cotter\ against\ pierced\ section$$

$$\left(\frac{\pi d_1^2}{4} - d_1 \times t\right) = \sigma_t \times d_1 \times t$$

$$t = \frac{\sigma_t \times \pi \times d_1}{4(\sigma_c + \sigma_t)}$$

$$= \frac{124 \times 10^6 N/m^2 \times \pi \times 0{,}050\ m}{4(216 \times 10^6 N/m^2 \times 124 \times 10^6 N/m^2)}$$

$$t = 0{,}0143 \text{ Say 14 mm}$$

Root diameter (d_r) of bolt

$$Tensile\ load\ in\ bolt = Tensile\ load\ in\ pierced\ section$$

$$\sigma_t \times \frac{\pi d_r^2}{4} = \sigma_t\left(\frac{\pi d_1^2}{4} - d_1 \times t\right)$$

$$d_r = \sqrt{d_1^2 - \frac{4 d_1 \times t}{\pi}}$$

$$= \sqrt{(0{,}050\ m)^2 - \frac{4 \times 0{,}050\ m \times 0{,}014\ m}{\pi}}$$

$$= \sqrt{1{,}609 \times 10^{-3}}$$

$$= 0{,}040\ m$$

Use M48 nominale diameter standard size bolt.

Dimension (a) at end of pierced section

$$Shear\ load\ in\ pierced\ section\ end = Shear\ load\ in\ cotter$$

$$\tau \times 2 \times d_1 \times a = \tau \times 2 \times t \times b$$

$$a = \frac{t \times b}{d}$$

$$= \frac{0{,}014\ m \times 0{,}058\ m}{0{,}050\ m}$$

$$= 0{,}0162\ m \quad \text{Say } 17\ mm$$

Dimension (g)

This depends on where the bolt has to fit and does not affect the strength.

Collar

This does not carry the load as the bolt is in tension so any reasonable dimension will do.
Make it say 60 mm diameter and 6 mm thick.

Worked Example 4.6

Design a cottered joint to transmit 60 kN. Use ultimate shear stress= 417 MN/m², ultimate tensile stress= 460 MN/m² and ultimate crushing stress = 834 MN/m². Use a factor of safety of 5.

Solution:

Allowable stresses

$$\text{Shear stress} = \frac{417 \times 10^6 N/m^2}{5} = 83{,}4\ MPa$$

$$\text{Tensile stress} = \frac{460 \times 10^6 N/m^2}{5} = 92\ MPa$$

$$\text{Crushing stress} = \frac{834 \times 10^6 N/m^2}{5} = 166{,}8\ MPa$$

Diameter of pierced rod d_1

$$d_1 = \sqrt{\frac{4 \times P(\sigma_c + \sigma_t)}{\pi(\sigma_c + \sigma_t)}}$$

$$= \sqrt{\frac{4 \times 60 \times 10^3\ (166{,}8 \times 10^6 + 92 \times 10^6)}{\pi(166{,}8 \times 10^6 + 92 \times 10^6)}}$$

$$= \sqrt{1{,}2884 \times 10^{-3}}$$

$$d_1 = 0{,}0359\ m$$

Say $40\ mm$ diameter which correspond to a standard size round bar.

Breadth of cotter

$$Shear\ load\ in\ cotter = Crushing\ load\ in\ cotter\ against\ rod$$

$$\tau \times 2 \times t \times b = \sigma_c \times d_1 \times t$$

$$b = \frac{\sigma_c \times d_1}{\tau \times 2}$$

$$= \frac{166{,}8 \times 10^6 N/m^2 \times 0{,}04\ m}{83{,}4 \times 10^6 N/m^2 \times 2}$$

$$= 0{,}04\ m$$

$$b = 40\ m$$

Cotter thickness (t)

$$Tensile\ load\ in\ pierced\ rod = Crushing\ load\ in\ cotter\ against\ rod$$

$$\left(\frac{\pi d_1^2}{4} - d_1 \times t\right) = \sigma_t \times d_1 \times t$$

$$t = \frac{\sigma_t \times \pi \times d_1}{4(\sigma_c + \sigma_t)}$$

$$= \frac{92 \times 10^6 N/m^2 \times \pi \times 0{,}04\ m}{4(166{,}8 \times 10^6 N/m^2 + 92 \times 10^6 N/m^2)}$$

t = 0,0111 Say 11 mm

Outside diameter "d_2" of collar

$$\begin{aligned}
Crushing\ load\ in\ collar &= Crushing\ load\ in\ cotter\ against\ rod \\
\sigma_t \times \frac{\pi}{4}(d_2^2 - d_1^2) &= \sigma_c \times d_1 \times t \\
d_2 &= \sqrt{\frac{4}{\pi}(d_1 \times t) + d_1^2} \\
&= \sqrt{\frac{4}{\pi}(0{,}04\ m \times 0{,}011\ m) + (0{,}04\ m)^2} \\
&= \sqrt{2{,}16 \times 10^{-3}}
\end{aligned}$$

$d_2 = 0{,}0465\ \mathrm{m}$. Say 50 mm diameter which correspond to a standard size round bar.

Thickness t_1 of the collar

$$Shear\ load\ in\ collar = Shear\ load\ in\ cotter$$

$$\begin{aligned}
\tau \times \pi d_1 \times t_1 &= \tau \times 2 \times t \times b \\
t &= \frac{2 \times t \times b}{\pi \times d_1} \\
&= \frac{2 \times 0{,}011\ m \times 0{,}04\ m}{\pi \times 0{,}04\ m} \\
&= 0{,}007\ m \\
t_1 &= 7\ mm
\end{aligned}$$

Diameter of rod "d"

$$Tensile\ load\ in\ rod = Tensile\ load\ in\ pierced\ rod$$

$$\begin{aligned}
\sigma_t \times \frac{\pi d^2}{4} &= \sigma_t \left(\frac{\pi d_1^2}{4} - d_1 \times t\right) \\
d &= \sqrt{d_1^2 - \frac{4}{\pi}\ d_1 \times t} \\
&= \sqrt{(0{,}04\ m)^2 - \frac{4}{\pi} \times 0{,}04\ m \times 0{,}011\ m} \\
&= \sqrt{1{,}0398 \times 10^{-3}} \\
d &= 0{,}0322
\end{aligned}$$

Say 35 mm diameter which correspond to a standard size round bar.

Length "a" at the end of the pierced rod

$$Shear\ load\ in\ end\ of\ pierced\ rod = Shear\ load\ in\ cotter$$

$$\begin{aligned}
\tau \times 2 \times d_1 \times a &= \tau \times 2 \times t \times b \\
a &= \frac{t \times b}{d_1} \\
&= \frac{0{,}011\ m \times 0{,}040\ m}{0{,}040\ m} \\
a &= 0{,}011\ m
\end{aligned}$$

Enlarge diameter "D" of the socket

Equate equation (9) to equation (10).

$$\begin{aligned}
Crushing\ load\ in\ cotter\ against\ rod &= Crushing\ load\ in\ cotter\ against\ socket \\
\sigma_C \times d_1 \times t &= \sigma_C \times (D - d_1)t \\
D &= 2 \times d_1 \\
&= 02 \times 40\ mm \\
D &= 80\ mm
\end{aligned}$$

correspond to a standard size round bar.

Diameter "D_1" of pierced socket

$$\begin{aligned}
\sigma_t \times \left\{\frac{\pi}{4}(D_1^2 - d_1^2) - t(D_1 - d_1)\right\} &= \sigma_t\left(\frac{\pi d_1^2}{4} - d_1 \times t\right) \\
0{,}785D_1^2 - 1{,}57d_1^2 - D_1 t + 2d_1 t &= 0 \\
0{,}785D_1^2 - 1{,}57 \times 0{,}04^2 - D_1 \times 0{,}011 + 2 \times 0{,}04 \times 0{,}011 &= 0 \\
0{,}785D_1^2 - 0{,}0025 - 0{,}011D_1 + 0{,}00088 &= 0 \\
D_1^2 - 0{,}041D_1 - 0{,}0021 &= 0 \\
D_1 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{+0{,}014 \pm \sqrt{+0{,}000196 + 0{,}0084}}{2} \\
&= \frac{+0{,}014 \pm \sqrt{+0{,}00896}}{2} \\
&= \frac{+0{,}014 \pm 0{,}093}{2} \\
&= 0{,}535\ m
\end{aligned}$$

Say 55 mm correspond to standard size round bar.

Length "g" at the end of the enlarged socket

$$Shear\ load\ in\ enlarged\ end\ of\ socket = Shear\ load\ in\ cotter$$

$$\begin{aligned}
\tau \times 2(D - d_1)g &= \tau \times 2 \times t \times b \\
g &= \frac{t \times b}{D - d_1} \\
&= \frac{0{,}011\ m \times 0{,}04\ m}{0{,}08\ m - 0{,}04\ m} \\
g &= 0{,}011\ m \\
&= 11\ m
\end{aligned}$$

Worked Example 4.7

Two lengths of 65 mm diameter steel tie-rods are joined together by a cylindrical sleeve 300 mm long, 65 mm inside diameter and 25 mm thick with two slightly tapered cotters 150 mm long and 50 mm by 20 mm mid-section inserted in prepared slots, midway on either side of the joint.

Calculate the failing loads in tension, shear and crushing and then state the allowable load on the joint.

Tensile stress of mild steel	= 495 MPa
Shearing stress of mild steel	= 386 MPa
Crushing stress of mild steel	= 618 MPa
Factor of safety	= 6

How could this joint be improved?

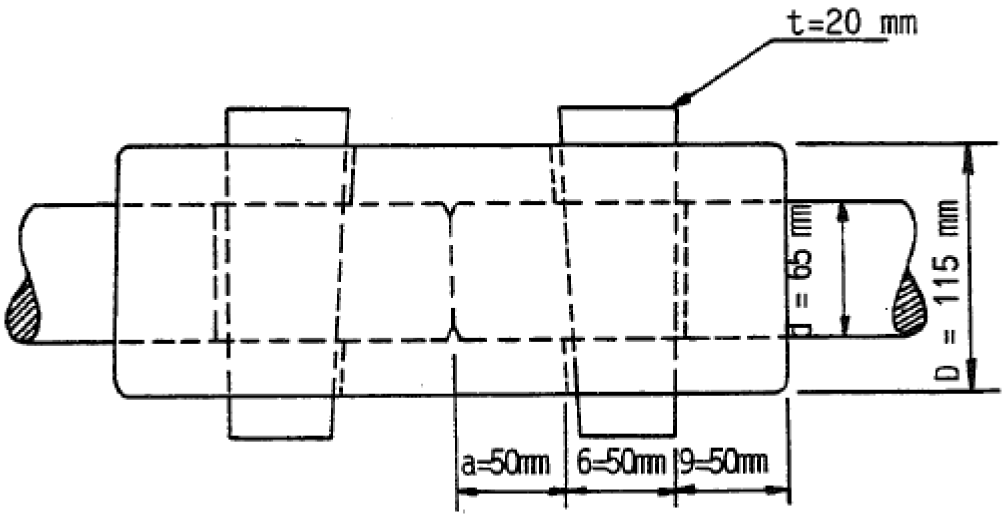


Figure 4.50

As failing loads are asked for, ultimate stresses must be used.
There is no need to consider the solid rod as it will definitely fail first at the cotter hole.

Note: The socket is of uniform thickness therefore $D_1 = D$ (see **Figure 4.28**).

Tensile load for rod at cotter hole

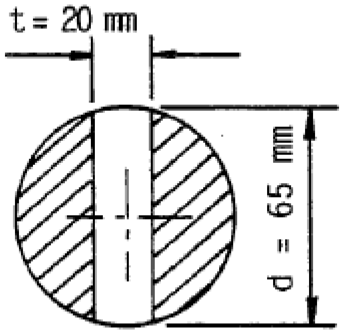


Figure 4.51

$$Tensile\ load \ = Tensile\ stress \times Cross - sectional\ area$$

$$P \ = \sigma_t \times \left(\frac{\pi d^2}{4} - d_1 \times t\right)$$
$$= 495 \times 10^6 N/m^2 \left(\frac{\pi \times (0{,}065\ m)^2}{4}\right) - 0{,}065\ m\ \times 0\ 02\ m$$
$$= 999{,}1\ kN$$

Tensile load for sleeve at cotter hole

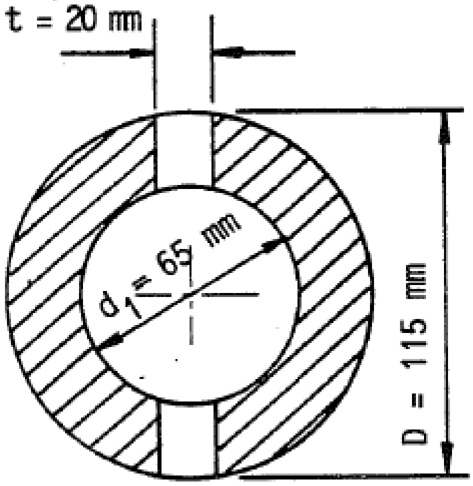


Figure 4.52

$$Tensile\ load \ = Tensile\ stress \times Cross - sectional\ area$$
$$P \ = \sigma_t \times \left\{\frac{\pi}{4}[D^2 - d_1^2] - [D - d_1]t\right\}$$
$$= 495 \times 10^6 N/m^2 \left\{\frac{\pi}{4}[(0{,}115\ m)^2 - (0{,}065 m)^2] - [0{,}115 m - 0{,}065 m]0{,}02 m\right\}$$
$$= 495 \times 10^6 N/m^2 [0{,}0077\ m^2 - 0{,}001\ m^2]$$
$$= 331{,}7\ kN$$

Shear Load for Cotter

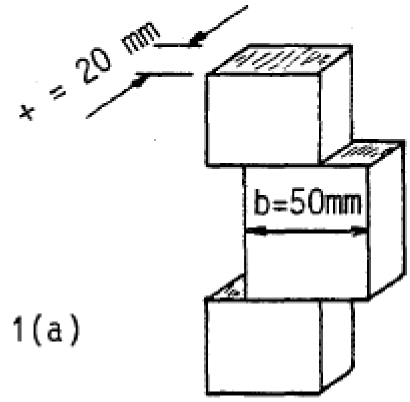


Figure 4.53

$$Shear\ load = Shear\ stress \times Cross - sectional\ area$$

$$P = \tau \times 2 \times t \times b$$
$$= 386 \times 10^6 N/m^2 \times 2 \times 0{,}02\ m \times 0{,}05\ m$$
$$= 772\ kN$$

Shear Load to shear end of pierced rod

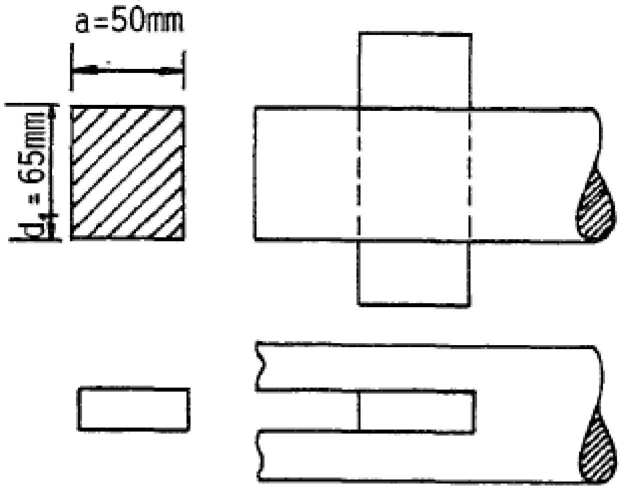


Figure 4.54

$$Shear\ load = Shear\ stress \times shear\ area$$

$$P = \tau \times 2 \times d_1 \times a$$
$$= 386 \times 10^6 N/m^2 \times 2 \times 0{,}065\ m \times 0{,}05\ m$$
$$= 2509\ kN$$

Shear Load to shear end of pierced sleeve

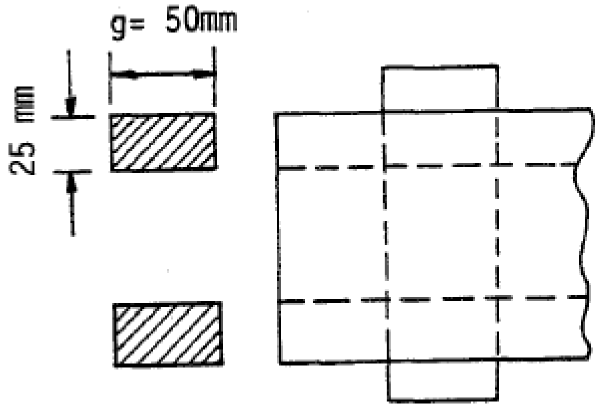


Figure 4.55

$$Shear\ load = Shear\ stress \times Cross - sectional\ area$$

$$P = \tau \times 4 \times g \times 0{,}025$$
$$= 386 \times 10^6 N/m^2 \times 4 \times 0{,}05\ m \times 0{,}025\ m$$
$$= 1920\ kN$$

Crushing cotter against rod

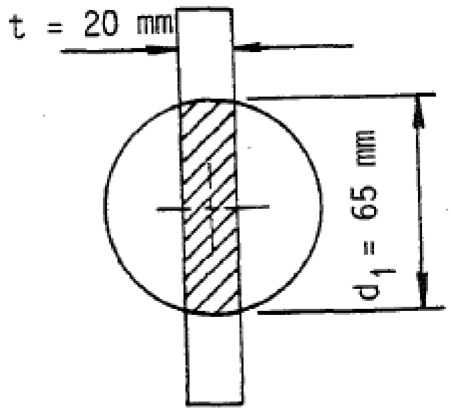


Figure 4.56

$$Crushing\ load = Crushing\ stress \times Projected\ area$$

$$P = \sigma_C \times d_1 \times t$$
$$= 618 \times 10^6 N/m^2 \times 0{,}065\ m \times 0{,}02\ m$$
$$= 803{,}4\ kN$$

Crushing cotter against sleeve

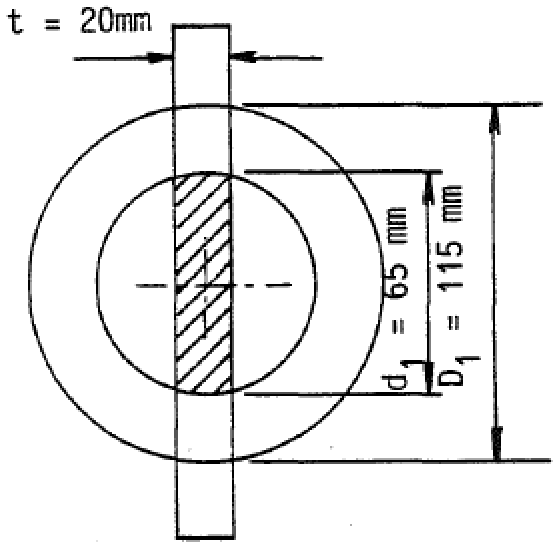


Figure 4.57

$$Crushing\ load = Crushing\ stress \times Projected\ area$$

$$P = \sigma_C \times (D - d_1)t$$
$$= 618 \times 10^6 N/m^2 \times (0{,}115\ m - 0{,}065\ m)0{,}02\ m$$
$$= 618\ kN$$

The failing load for the joint is therefore:

618 kN and the allowable load

$$= \frac{Failing\ load}{Factor\ of\ safety}$$
$$= \frac{618 \times 10^3 N}{6}$$
$$= 103\ kN$$

In order to improve this joint we must raise this value of 618 kN by increasing the contact area between the cotter and the sleeve. This can be done either by increasing the thickness of the cotter, or the outside diameter of the sleeve, or both.

Increasing the thickness of the cotter will decrease the 999,1 kN strength of the rod at the cotter hole and should, therefore, be avoided, but we notice that crushing the cotter on the rod is 803,4 kN and shear of the cotter 772 kN. Thus, an increase in cotter thickness is justified.

To obtain the most suitable joint we must design it for equal strength of all parts.

The rod diameter d_1 was given and we assume that it can not be altered.

Therefore d_1 = 65 mm.

Sizes of cotter

Breadth

$$Shear\ load\ in\ cotter = Crushing\ load\ in\ cotter\ against\ rod$$
$$\tau \times 2 \times t \times b = \sigma_c \times d_1 \times t$$
$$b = \frac{\sigma_c \times d_1}{\tau \times 2}$$
$$= \frac{618 \times 10^6 N/m^2 \times 0{,}065\ m}{386 \times 10^6 N/m^2 \times 2}$$
$$= 0{,}052\ m$$
$$b = 52\ m$$

Thickness

$$Tensile\ load\ in\ pierced\ rod = Crushing\ load\ in\ cotter\ against\ rod$$
$$\sigma_t\left(\frac{\pi d^2}{4} - d_1 \times t\right) = \sigma_c \times d_1 \times t$$
$$t = \frac{\sigma_t \times \pi \times d}{4(\sigma_c + \sigma_t)}$$
$$= \frac{495 \times 10^6 N/m^2 \times \pi \times 0{,}065\ m}{4(618 \times 10^6 N/m^2 + 495 \times 10^6 N/m^2)}$$
$$= 0{,}023\ m$$
$$= 23\ mm$$

Outside diameter of sleeve

$$Crushing\ load\ in\ cotter\ against\ rod = Crushing\ load\ in\ cotter\ against\ sleeve$$
$$\sigma_c \times d_1 \times t = \sigma_c \times (D - d_1)t$$
$$D = 2d_1$$
$$= 2 \times 0{,}065$$
$$= 0{,}130\ m$$
$$D = 130\ mm$$

Length"a" at end of rod

$$Shear\ load\ in\ pierced\ rod = Shear\ load\ in\ cotter$$
$$\tau \times 2 \times d_1 \times a = \tau \times 2 \times t \times b$$
$$a = \frac{t \times b}{d_1}$$
$$= \frac{0{,}023\ m \times 0{,}052\ m}{0{,}065\ m}$$
$$a = 0{,}0184\ m$$

say 19 mm

Length "g" at the end of the enlarged socket

$$Shear\ load\ in\ end\ of\ sleeve = Shear\ load\ in\ cotter$$
$$\tau \times 2(D - d_1)g = \tau \times 2 \times t \times b$$

$$g = \frac{t \times b}{D - d_1}$$
$$= \frac{0{,}023\ m \times 0{,}052\ m}{0{,}130\ m - 0{,}065\ m}$$
$$= 0{,}0184\ m$$

Say 19 mm

Improved cotter joint

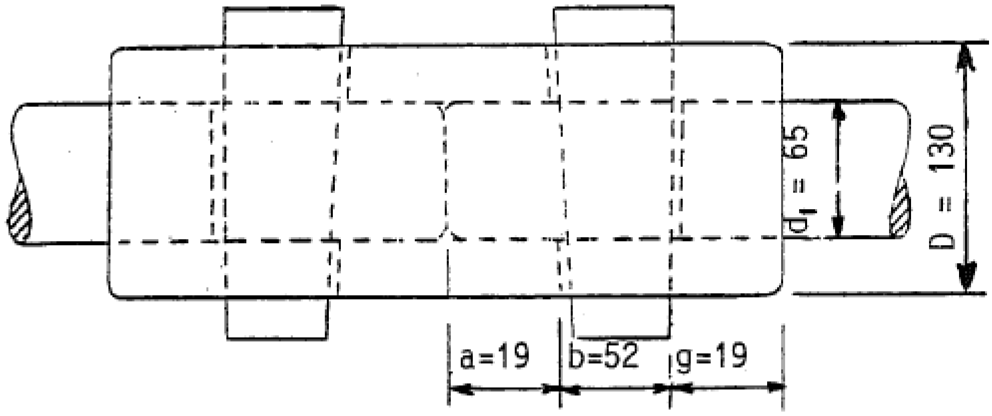


Figure 4.58

Failing load of this joint

Tensile load in rod at cotter hole

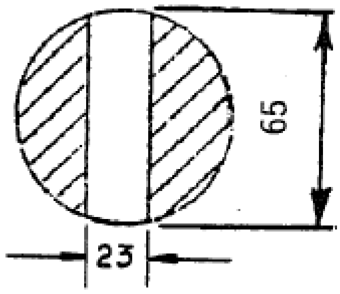


Figure 4.59

$$P = \sigma_t \times \left(\frac{\pi {d_1}^2}{4} - d_1 \times t\right)$$
$$= 495 \times 10^6 N/m^2 \left(\frac{\pi \times (0{,}065\ m)^2}{4} - 0{,}065\ m \times 0{,}023\ m\right)$$
$$= 902{,}5\ kN$$

Tensile load in sleeve at cotter hole

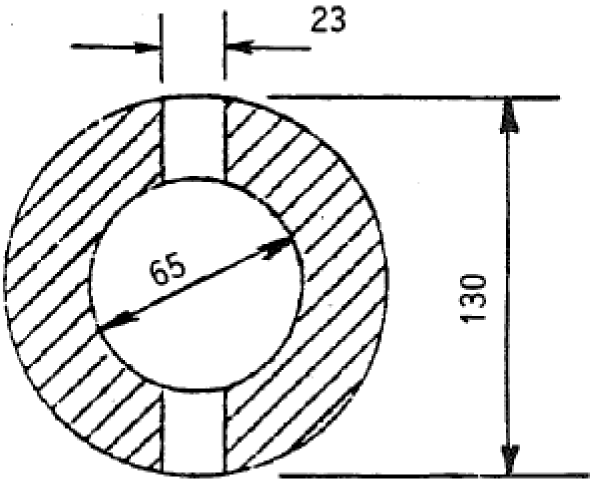


Figure 4.60

$$P = \sigma_t \times \left\{\frac{\pi}{4}[D^2 - d_1^2] - [D - d_1]t\right\}$$
$$= 495 \times 10^6 N/m^2 \left\{\frac{\pi}{4}[(0{,}13\ m)^2 - (0{,}065m)^2] - [0{,}13\ m - 0{,}065m]0{,}023m\right\}$$
$$= 4188\ kN$$

Shear load in cotter

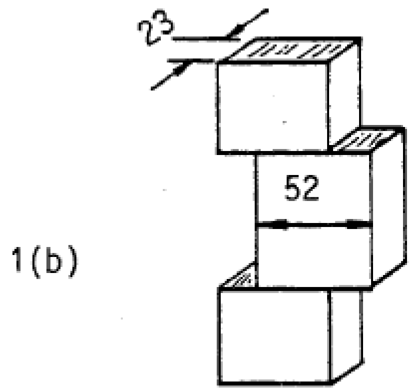


Figure 4.61

$$P = \tau \times 2 \times t \times b$$
$$=386 \times 10^6 N/m^2 \times 2 \times 0{,}023\ m \times 0{,}052\ m$$
$$= 923{,}3\ kN$$

Shear load to shear end of pierced rod

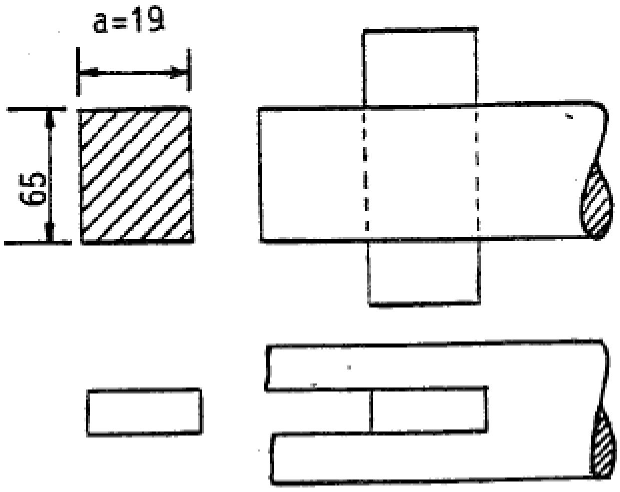


Figure 4.62

$$P = \tau \times 2 \times d_1 \times a$$
$$= 386 \times 10^6 N/m^2 \times 2 \times 0{,}065\ m \times 0{,}019\ m$$
$$a = 953{,}4\ kN$$

Shear load to shear end of pierced sleeve.

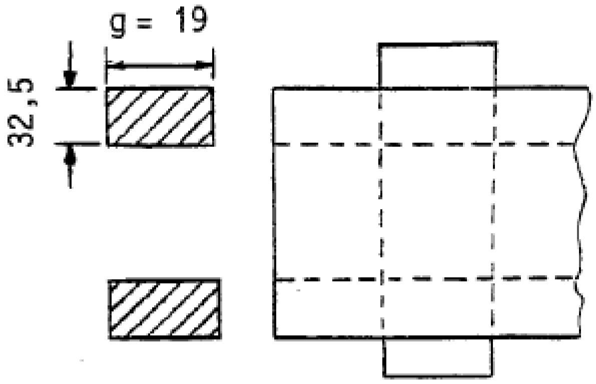


Figure 4.63

$$P = \tau \times 4 \times g \times 0{,}0325\ m$$
$$=386 \times 10^6 N/m^2 \times 4 \times 0{,}019\ m \times 0{,}0325\ m$$
$$= 953{,}4\ kN$$

Crushing load in cotter against rod.

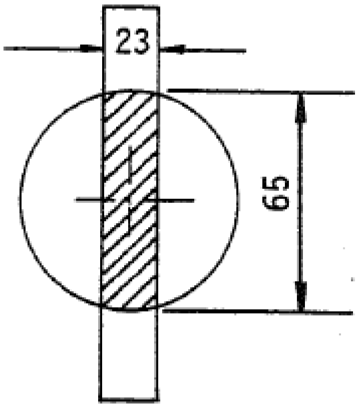


Figure 4.64

$$P = \sigma_C \times d_1 \times t$$
$$= 618 \times 10^6 N/m^2 \times 0{,}065\ m \times 0{,}023\ m$$
$$= 923{,}9\ kN$$

Crushing load in cotter against sleeve

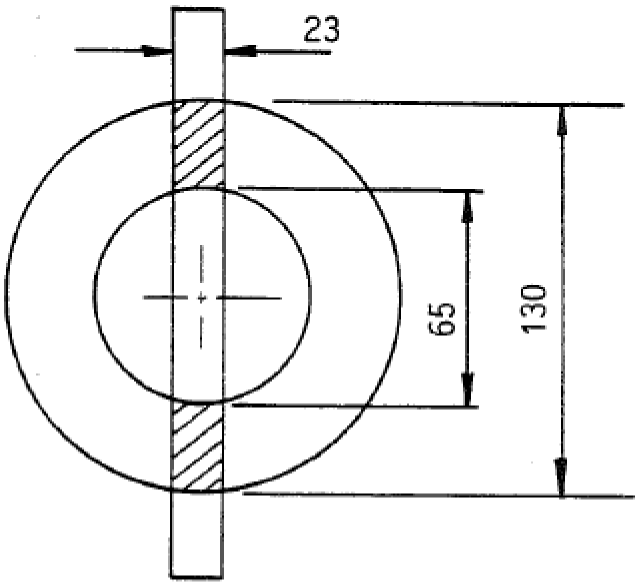


Figure 4.65

$$P = \sigma_C \times (D - d_1)t$$
$$= 618 \times 10^6 N/m^2 \times (0{,}13\ m - 0{,}065\ m)0{,}023\ m$$
$$= 923{,}9\ kN$$

The failing load for the improved joint is therefore 902,5 kN and the allowable load

$$= \frac{Failing\ load}{Factor\ of\ safety}$$
$$= \frac{902{,}5 \times 10^3 N}{6}$$
$= 150{,}42\ kN$ instead of $103\ kN$ a big improvement

Worked Example 4.8

A cottered joint joins two 45 mm diameter rods. The end of one rod is increased to 50 mm diameter and then tapered to 45 mm diameter over a length of 128 mm. The other rod has a socket 80 mm outside diameter bored to fit the tapered end of the first rod. A cotter 50 mm wide and 12 mm thick fitted midway in the taper portion joins the two rods.

Calculate for a pull of 22 kN in the rods:

(a) tensile stress in the rods away from the joint
(b) shear stress in the cotter
(c) tensile stress in the rod at the cotter hole
(d) tensile stress in the socket at the cotter hole
(e) crushing stress between cotter and rod-end
(f) crushing stress between cotter and socket

Criticise this joint.

Solution:

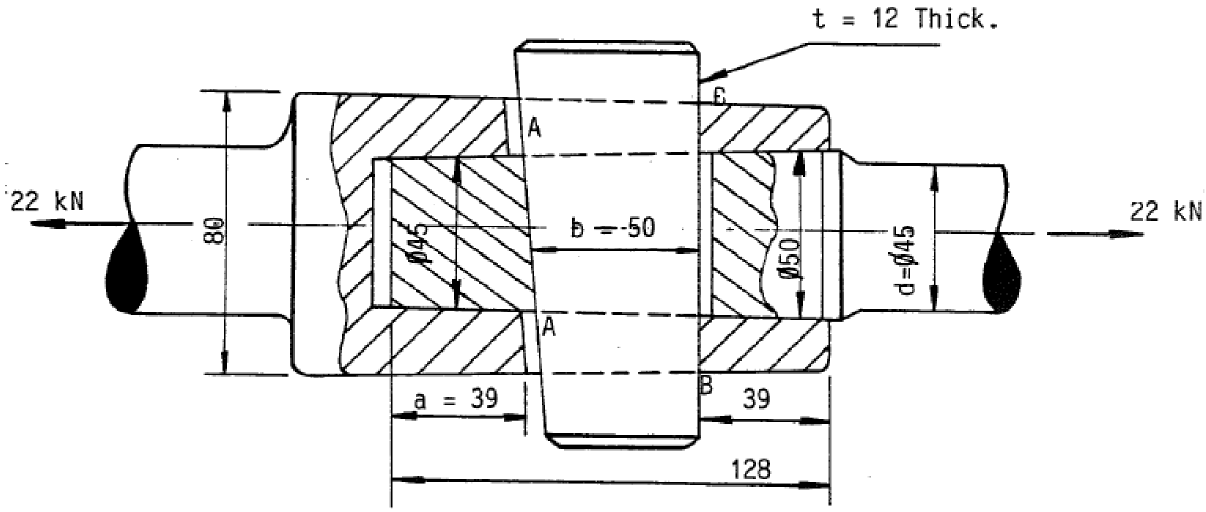


Figure 4.66

(a) Tensile stress in rods

$$Tensile\ load = Tensile\ stress \times Cross - sectional\ area$$
$$P = \sigma_t \times \frac{\pi d^2}{4}$$
$$\sigma_t = \frac{4P}{\pi d^2}$$
$$= \frac{4 \times 22 \times 10^3 N}{\pi \times (0{,}045\ m)^2}$$
$$= 13{,}83 \times 10^6\ N/m^2$$
$$= 13{,}83\ MPa$$

(b) Shear stress in the cotter

$$Shear\ load = Shear\ stress \times Cross - sectional\ area$$

$$P = \tau \times 2 \times t \times b$$
$$\tau = \frac{P}{2 \times t \times b}$$
$$= \frac{22 \times 10^3 N}{2 \times 0{,}012\ m^2 \times 0{,}05\ m}$$
$$= 18{,}33 \times 10^6\ N/m^2$$
$$= 18{,}33\ MPa$$

(c) Tensile stress in the pierced rod

$$Tensile\ load = Tensile\ stress \times Cross - sectional\ area$$
$$P = \sigma_t \times \left(\frac{\pi {d_1}^2}{4} - d_1 \times t\right)$$
$$\sigma_t = \frac{P}{\frac{\pi {d_1}^2}{4} - d_1 \times t}$$

The weakest section will be at AA and the diameter there may be obtained either by calculation or by measurement on a sketch drawn to scale.

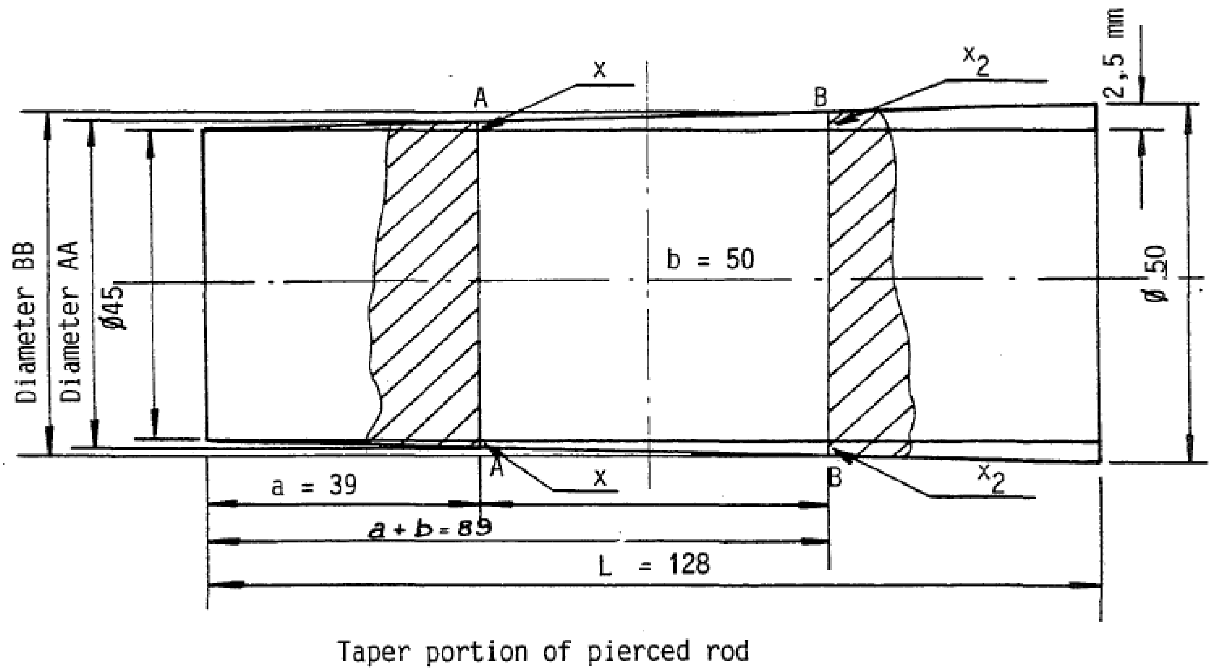


Figure 4.67

$$\frac{x}{a} = \frac{2{,}5\ mm}{L}$$
$$x = \frac{2{,}5\ mm}{L} \times a$$
$$= \frac{2{,}5\ mm \times 39\ mm}{128\ mm}$$
$$= 0{,}762\ mm$$

$$Diameter\ at\ AA = 45\ mm + 2\ x.$$

$$= 45\ mm\ +\ (2\ \times 0{,}762\ mm)$$
$$= 45\ mm\ +\ 1{,}52$$
$$= \mathit{46{,}52\ mm}$$

$$\sigma_t = \left\{\frac{\pi}{4} \times (0{,}04652\ m)^2 - 0{,}04652\ m \times 0{,}012\ m\right\}$$
$$= 19{,}27 \times 10^6\ N/m^2$$
$$= 19{,}27\ MPa$$

(d) Tensile stress in the socket at the cotter hole

The weakest section is at BB and the internal diameter must be calculated or measured from a sketch drawn to scale.

$$\frac{x_2}{a+b} = \frac{2{,}5\ mm}{4}$$
$$x^2 = \frac{2{,}5\ mm}{L} \times a + b$$
$$= \frac{2{,}5\ mm \times 89\ mm}{128\ mm}$$
$$= 0{,}74\ mm$$

$$Diameter\ at\ BB = 45\ mm\ +\ 2\ x_2$$
$$= 45\ mm\ +\ (2\ \times 1{,}74\ mm)$$
$$= \mathit{48{,}5\ mm}$$

$$Tensile\ load = Tensile\ stress \times Cross - sectional\ area$$
$$P = \sigma_t \times \left\{\frac{\pi}{4}[D_1^2 - d_1^2] - t[D_1 - d]\right\}$$
$$\sigma_t = \frac{P}{\sigma_t \times \left\{\frac{\pi}{4}[D_1^2 - d_1^2] - t[D_1 - d_1]\right\}}$$

$$= \frac{22 \times 10^3 N}{\sigma_t \times \left\{\frac{\pi}{4}[(0{,}08\ m)^2 - (0{,}0485\ m)^2] - 0{,}012\ m[0{,}08\ m - 0{,}0485\ m]\right\}}$$
$$= 7{,}85 \times 10^6\ N/m^2$$
$$= 7{,}85\ MPa$$

(e) Crushing stress between cotter and rod-end

$$Crushing\ load = Crushing\ stress \times Projected\ area$$

$$P = \sigma_C \times d_1 \times t$$
$$= \frac{P}{d_1 \times t}\ (d_1 = diameter\ at\ AA)$$
$$= \frac{22 \times 10^3 N}{0{,}04652\ m \times 0{,}012\ m}$$
$$= 39{,}41 \times 10^6 N/m^2$$
$$= 39{,}41\ MPa$$

(f) Crushing stress between cotter and socket

$$Crushing\ load = Crushing\ stress \times Projected\ area$$

$$P = \sigma_C \times (D - d_1)t$$
$$= \frac{P}{(D-d_1)t}\ (d_1 = diameter\ at\ BB)$$
$$= \frac{22 \times 10^3 N}{(0{,}08\ m \times 0{,}0485\ m) \times 0{,}012\ m}$$
$$= 58{,}2 \times 10^6 N/m^2$$
$$= 58{,}2\ MPa$$

Criticism

The first thing we notice is that the rods are very lightly loaded at 22 kN because the stress (a) 13,83 MPa is very low. Let us examine the other stresses.

The normal values for ultimate strengths of mild steel are:

Tension = 463 MPa
Shear = 386 MPa
Crushing = 618 MPa

In each of the six points considered we have therefore,

$$Factor\ of\ safety = \frac{Ultimate\ stress\ of}{Working\ stress}$$

a. $\frac{463\ MPa}{13{,}83\ MPa} = 33{,}5$ b. $\frac{386\ MPa}{18{,}33\ MPa} = 21{,}1$

c. $\frac{463\ MPa}{19{,}27\ MPa} = 24$ d. $\frac{463\ MPa}{7{,}85\ MPa} = 59$

e. $\frac{618\ MPa}{39{,}41\ MPa} = 15{,}7$ f. $\frac{618\ MPa}{58{,}2\ MPa} = 10{,}6$

This shows poor design as there is no use having a factor of safety of 59 for one part when the factor is only 10,6 for another. The factor of safety for the joint as a whole is only 10,6.

The use of a thicker cotter would raise b, e and f but it would decrease c, so a larger diameter rod-end is also needed. f, which is exceptionally low could also be raised by increasing the. outer diameter of the socket at its end.

Let us redesign this joint to carry a load of 22 kN using a factor of safety of say 30.

$$Allowable\ tensile\ stress = \frac{Ultimate\ tensile\ stress}{Factor\ of\ safety}$$
$$= \frac{463 \times 10^6\ N/m^2}{30}$$
$$\sigma_t = 15{,}43\ MPa$$

$$Allowable\ shear\ stress = \frac{Ultimate\ shear\ stress}{Factor\ of\ safety}$$
$$= \frac{386 \times 10^6\ N/m^2}{30}$$
$$\tau = 12{,}87\ MPa$$

$$Allowable\ crushing\ stress = \frac{Ultimate\ crushing\ stress}{Factor\ of\ safety}$$
$$= \frac{618 \times 10^6\ N/m^2}{30}$$
$$\sigma_c = 20{,}6\ MPa$$

To find the diameter d_1 of the pierced rod at AA

$$d_1 = \sqrt{\frac{4 \times P(\sigma_c + \sigma_t)}{\pi(\sigma_c + \sigma_t)}}$$
$$= \sqrt{\frac{4 \times 22 \times 10^3\ (20{,}6 \times 10^6 + 15{,}43 \times 10^6)}{\pi(20{,}6 \times 10^6 + 15{,}43 \times 10^6)}}$$
$$= \sqrt{3{,}175\ \times\ 10^{-3}}$$
$$= 0{,}056\ m$$
$$d_1 = 56\ m$$

To find the breadth (b) of the cotter

$$Shear\ load\ in\ cotter = Crushing\ load\ in\ cotter\ against\ rod$$
$$\tau \times 2 \times t \times b = \sigma_c \times d_1 \times t$$
$$b = \frac{\sigma_c \times d_1}{\tau \times 2}$$
$$= \frac{20{,}6 \times 10^6 N/m^2 \times 0{,}056\ m}{12{,}87 \times 10^6 N/m^2 \times 2}$$
$$= 0{,}045\ m$$
$$b = 45\ m$$

To find the thickness (t) of the cotter

$$Tensile\ load\ in\ pierced\ rod = Crushing\ load\ in\ cotter\ against\ rod$$
$$\sigma_t\left(\frac{\pi d_1^2}{4} - d_1 \times t\right) = \sigma_C \times d_1 \times t$$
$$t = \frac{\sigma_t \times \pi \times d_1}{4(\sigma_c + \sigma_t)}$$
$$= \frac{15{,}43 \times 10^6 N/m^2 \times \pi \times 0{,}056\ m}{4(20{,}6 \times 10^6 N/m^2 + 15{,}43 \times 10^6 N/m^2)}$$
$$= 0{,}0188$$
$$t = 19\ mm$$

To find the rod diameter (d)

$$Tensile\ load\ in\ rod = Tensile\ load\ in\ pierced\ rod$$

$$\sigma_t \times \frac{\pi d^2}{4} = \sigma_t\left(\frac{\pi d_1^2}{4} - d_1 \times t\right)$$

$$d = \sqrt{d_1^2 - \frac{4d_1 \times t}{\pi}}$$

$$= \sqrt{(0{,}056\ m)^2 - \frac{4 \times 0{,}56\ m \times 0{,}019\ m}{\pi}}$$

$$= \sqrt{1{,}78 \times 10^{-3}}$$

$$= 0{,}0422\ m$$

$$Say\ d = 42\ mm$$

To find the length"a" at the end of the pierced rod

$$Shear\ load\ in\ end\ of\ pierced\ rod = Shear\ load\ in\ cotter$$

$$\tau \times 2 \times d_1 \times a = \tau \times 2 \times t \times b$$

$$a = \frac{t \times b}{d_1}$$

Note d_1 should actually be the average diameter of the front end of the pierced rod as shown in **Figure 4.68**, but as the taper of the pierced rod is only 1 in 16, on the diameter (which is small), diameter at AA may be used.

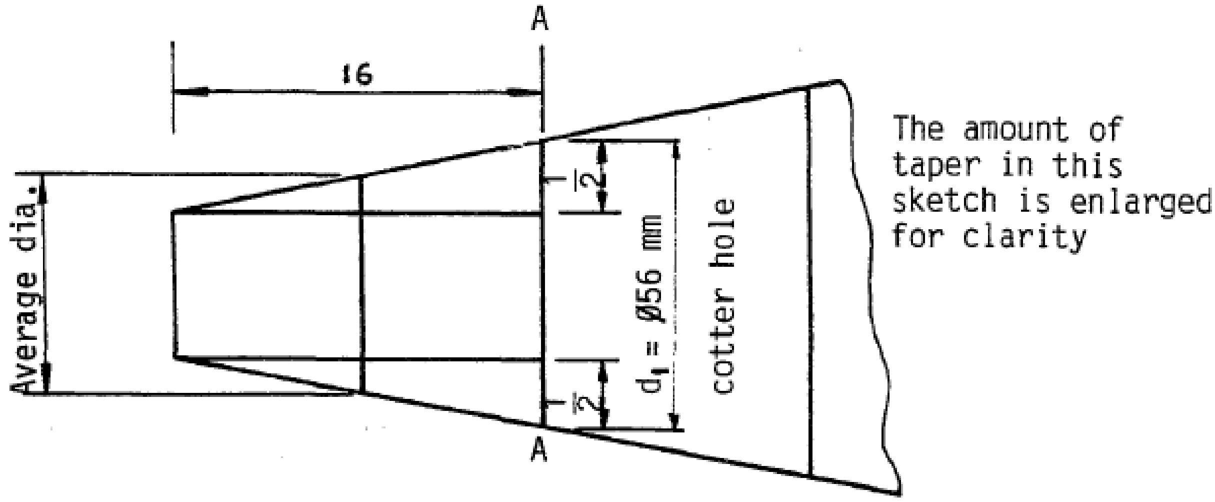


Figure 4.68

$$= \frac{0{,}019\ m \times 0{,}045\ m}{0{,}056\ m}$$

$$a = 0{,}0153\ m$$

To find the enlarged diameter (b) of the socket

$$Crushing\ load\ in\ cotter\ against\ rod = Crushing\ load\ in\ cotter\ against\ socket$$

$$\sigma_C \times d_1 \times t = \sigma_C \times (D - d_1)t$$

$$D = 2d_1$$

$$= 02 \times 0{,}056\ m$$

$$= 0{,}112\ mm$$

$$D = 112\ mm$$

To find the diameter (D_1) of the pierced socket

$$\sigma_t \times \left\{\frac{\pi}{4}(D_1^2 - d_1^2) - t(D_1 - d_1)\right\} = \sigma_t\left(\frac{\pi d_1^2}{4} - d_1 \times t\right)$$
$$0{,}785D_1^2 - 1{,}57d_1^2 - D_1t + 2d_1t = 0$$
$$0{,}785D_1^2 - 1{,}57 \times 0{,}056^2 - D_1 \times 0{,}019 + 2 \times 0{,}056 \times 0{,}019 = 0$$
$$0{,}785D_1^2 - 0{,}0049 - 0{,}019D_1 + 0{,}00213 = 0$$
$$D_1^2 - 0{,}0242D_1 - 0{,}0035 = 0$$
$$D_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{+0{,}024 \pm \sqrt{+0{,}000586 + 0{,}014}}{2}$$
$$= \frac{0{,}0242 \pm 0{,}121}{2}$$
$$= 0{,}0725$$

Say $D_1 = 73\ mm$ correspond to standard size round bar.

To find the length "g" at the end of the enlarged end of the socket

$$Shear\ load\ in\ enlarged\ end\ of\ socket = Shear\ load\ in\ cotter$$

$$\tau \times 2(D - d_1)g = \tau \times 2 \times t \times b$$
$$g = \frac{t \times b}{D - d_1}$$
$$= \frac{0{,}019\ m \times 0{,}045\ m}{0{,}112\ m - 0{,}056\ m}$$
$$= 0{,}0153\ m$$

Say $g = 16\ mm$

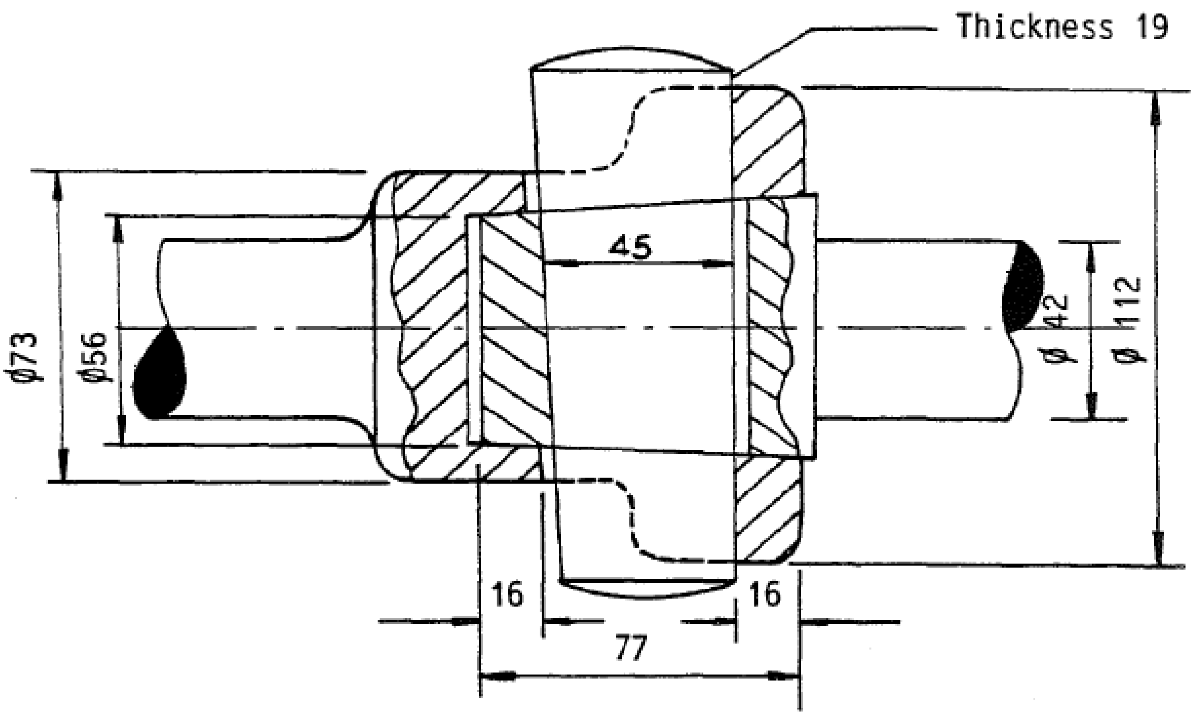


Figure 4.69

The above design gives the following stresses for the various parts:

a) Tensile stress in the rods = 15,85 MPa.
b) Shear stress in the *cotter* = 12,87 MPa.
c) Tensile stress in the pierced rod = 15,72 MPa.
d) Tensile stress in the socket at the cotter hole = 15,72 MPa.
e) Crushing stress between cotter and rod-end = 20,68 MPa.
f) Crushing stress between cotter and socket = 20,68 MPa.

The factor of safety in all six ways is approximately 30.

Worked Example 4.9

Find the diameter (d), the diameter (d_1) of the enlarged end of a cotter bolt. Also find the width (b) of the cotter, thickness 13 mm.

Given: $\tau = \frac{4}{5}\ \sigma_t : \sigma_C = 2\sigma_t$

<u>Solution:</u>

Diameter of pierced end

$$\begin{aligned}
Tensile\ load\ in\ pierced\ rod &= Crushing\ load\ in\ cotter\ against\ bolt \\
\sigma_t\left(\frac{\pi d_1^2}{4} - d_1 \times t\right) &= \sigma_C \times d_1 \times t \\
d_1 &= \frac{4(\sigma_c+\sigma_t)t}{\sigma_2 \times \pi} \\
&= \frac{4(2\sigma_c+\sigma_t)\times 0{,}013\ m}{\sigma_t \times \pi} \\
&= \frac{4\times 3\sigma_t \times 0{,}013\ m}{\sigma_t \times \pi} \\
&= 0{,}0496\ m
\end{aligned}$$

Say $d_1 = 50\ mm$ (diameter of pierced end)

Nominal diameter of bolt

$$\begin{aligned}
Tensile\ load\ in\ bolt &= Tensile\ load\ in\ pierced\ end \\
\sigma_t \times \frac{\pi d^2}{4} &= \sigma_1\left(\frac{\pi d_1^2}{4} - d_1 \times t\right) \\
d &= \sqrt{d_1^2 - \frac{4d_1 \times t}{\pi}} \\
&= \sqrt{0{,}05^2 - \frac{4\times 0{,}05\ \times 0{,}013}{\pi}} \\
&= \sqrt{0{,}00167} \\
&= 0{,}0409
\end{aligned}$$

Say d = 41 mm core diameter of bolt
Using a M48 diameter bolt (nominal size)

Width of cotter

$$\begin{aligned}
Shear\ load\ in\ cotter &= Crushing\ load\ in\ cotter\ against\ bolt \\
\tau \times 2 \times t \times b &= \sigma_c \times d_1 \times t \\
b &= \frac{\sigma_c \times d_1}{\tau \times 2} \\
&= \frac{2 \times \sigma_c \times d_1}{\frac{4}{5} \times \sigma_t \times 2} \\
&= \frac{5}{4} \times 0{,}05\ m \\
&= 0{,}0625\ m \\
\text{Say } b &= 63\ mm\ width
\end{aligned}$$

Say for instance that the core diameter of the bolt was given in the question, say 36 mm and the rest of the question was the same, the problem would be solved in the following manner:

$$\begin{aligned}
Tensile\ load\ in\ the\ bolt &= Tensile\ stress \times Core\ area \\
P &= \sigma_t \times \frac{\pi d^2}{4} \\
&= \sigma_t \times \frac{\pi \times (0{,}036\ m)^2}{4} \\
P &= 1{,}018 \times 10^{-3} \times \sigma_t
\end{aligned}$$

Diameter of Pierced end

$$\begin{aligned}
d_1 &= \sqrt{\frac{4 \times P(\sigma_c + \sigma_t)}{\pi(\sigma_c + \sigma_t)}} \\
&= \sqrt{\frac{4 \times 1{,}018 \times 10^{-3} \sigma_t\ (2\sigma_t + \sigma_t)}{\pi(2\sigma_t + \sigma_t)}} \\
&= \sqrt{1{,}944 \times 10^{-3}} \\
&= 0{,}0441\ m \\
\text{Say } d_1 &= 45\ mm
\end{aligned}$$

Width of cotter

$$\begin{aligned}
Shear\ load\ in\ cotter &= Crushing\ load\ in\ cotter\ against\ bolt \\
\tau \times 2 \times t \times b &= \sigma_c \times d_1 \times t \\
b &= \frac{\sigma_c \times d_1}{\tau \times 2} \\
&= \frac{2\ \sigma_c \times 0{,}045\ m}{\frac{4}{5} \sigma_t \times 2} \\
&= \frac{0{,}045\ m \times 5}{4} \\
&= 0{,}056\ m \\
b &= 56\ mm
\end{aligned}$$

Activity 4.1

1. Two steel rods, joined by a forked joint, take a working load of 30 kN, and the working tensile stress in the rods is 40 MPa. Calculate the required rod diameter (d) and sketch a suitable joint, using the following proportions: diameter of pin $(d_1) = d$; width of eye $(b) = 1{,}25\ d$; width of each fork bearing $(a) = 0{,}75\ d$; outside diameter of eye and fork $(d_0) = 2d$.
   Then calculate:
   1. shear stress in the pin
   2. bearing stress on the pin in the eye
   3. bearing stress on the pin in the fork
   4. tensile stress in the metal of the eye.
2. A round rod (D) sustains a pull of 100 kN. If the maximum tensile stress in the rod is not to exceed 62 MPa, find the diameter of the rod.
   If the above rod is connected to a knuckle joint, as shown in the figure, calculate the pin diameter (d) using a shear stress of 80 Mpa, a bearing stress of 120 MPa and a bending stress of 75 MPa.
   Use empirical values to determine the other dimensions of the joint, and sketch two views fully dimensioned.

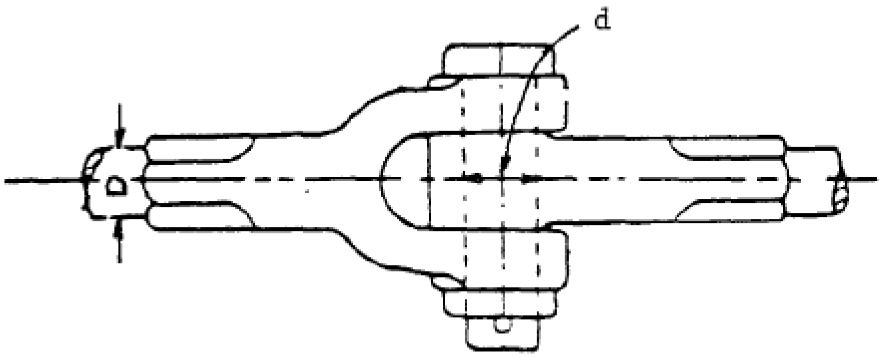


   Figure 4.70
3. Give the reason why, in a knuckle joint, the diameter of the pin is made equal to the diameter of the rod.

Activity 4.2

1. In a cottered foundation bolt, the diameter of the bolt through which the cotter passes is 40 mm and the thickness of the cotter is 12,5 mm.
   Find:
   (a) the allowable load for the bolt;
   (b) the necessary width of the cotter;
   (c) the bearing stress between the cotter and the bolt.
   Take shear stress as 40 MPa and tensile stress as 70 MPa.
2. Design a cottered bolt similar to that shown in **Figure 4.26 (iv)**.

The bolt is to carry a pull of 100 kN with working stresses of 124 MPa, 93 MPa and 200 MPa in tension, shear and crushing respectively. The end of the bolt where the cotter passes through is to be square in section.
Give a dimensioned sketch of your design.

3. Design a cottered joint for rods 70 mm in diameter. Use the following working stresses: Shear = 75 MPa, Crushing = 160 MPa and Tensile = 80 MPa.

Activity 4.3

1. Two round steel rods 40 mm in diameter are connected by a cottered joint which consists of a cylindrical sleeve into which the rods are pushed and each rod has a cotter passing through it and through the sleeve. Design the joint and make a detailed dimensional sketch of the joint. Assume tensile stress in rods = 77 MPa, shearing stress in the cotters = 70 MPa, and the bearing pressure on the cotter = 123 MPa.
2. Sketch a cottered joint for connecting the ends of two round steel rods 25 mm diameter using a cotter 6 mm thick and 25 mm wide. The end of one rod is increased to 30 mm diameter (Mean diameter of the tapered portion) where it fits into the socket of the other rod.
   The outside diameter of the socket is 60 mm.
   Calculate for a pull of 9 kN in the rods.
   a) tensile stress in the rods away from the joint.
   b) tensile stress in each rod at the cotter hole
   c) shear stress in the cotter
   d) crushing stresses on the cotter (i) in the rod (ii) in the socket.
   e) tensile stress in the socket
   Note: The length of the taper head is not given therefore d1, the mean diameter of the taper should be used in the calculations.
3. Using standard proportions sketch, full size, a sectional elevation and an outside plan of cottered joint for 30 nm diameter rods.
   Insert all dimensions.

Self-Check

I am able to:	Yes	No
• Describe knuckle joints		
o Standard proportions		
• Describe cotter joints		
o Standard proportions		
o Design		

If you have answered ‘no’ to any of the outcomes listed above, then speak to your facilitator for guidance and further development.

Module 5

Pistons

Learning Outcomes
On the completion of this module the student must be able to:

- Describe pistons
  - Rings
  - Rods
- Describe shafting
  - Torsion equation for shafts
  - Power developed by a torque
- Describe the design of shafts
- Describe the saving of mass
- Describe the standard sizes of bright steel bars
- Describe geared shafts
- Describe the forces which act on the parts of the driving mechanism of steam and similar engines
  - Crossheads
  - Design of the crankshaft
  - Internal combustion engine

5.1 Introduction

The piston rod is generally known as the coupling between the piston and the crankshaft in an engine, or between the cylinder head of a steam engine and the crosshead.

The crosshead is the link with the connecting rod mounted on the steam engine's crankshaft. The crosshead is mounted on a single or double slide bar. It receives power axially from the cylinder and transfers it to the turning crankshaft.

Very few, if any, theoretical calculations can be applied to the design of pistons. They are almost always designed from empirical formulae which have been derived from practical experience.

5.2 Steam engine pistons

5.2.1 Box-type pistons

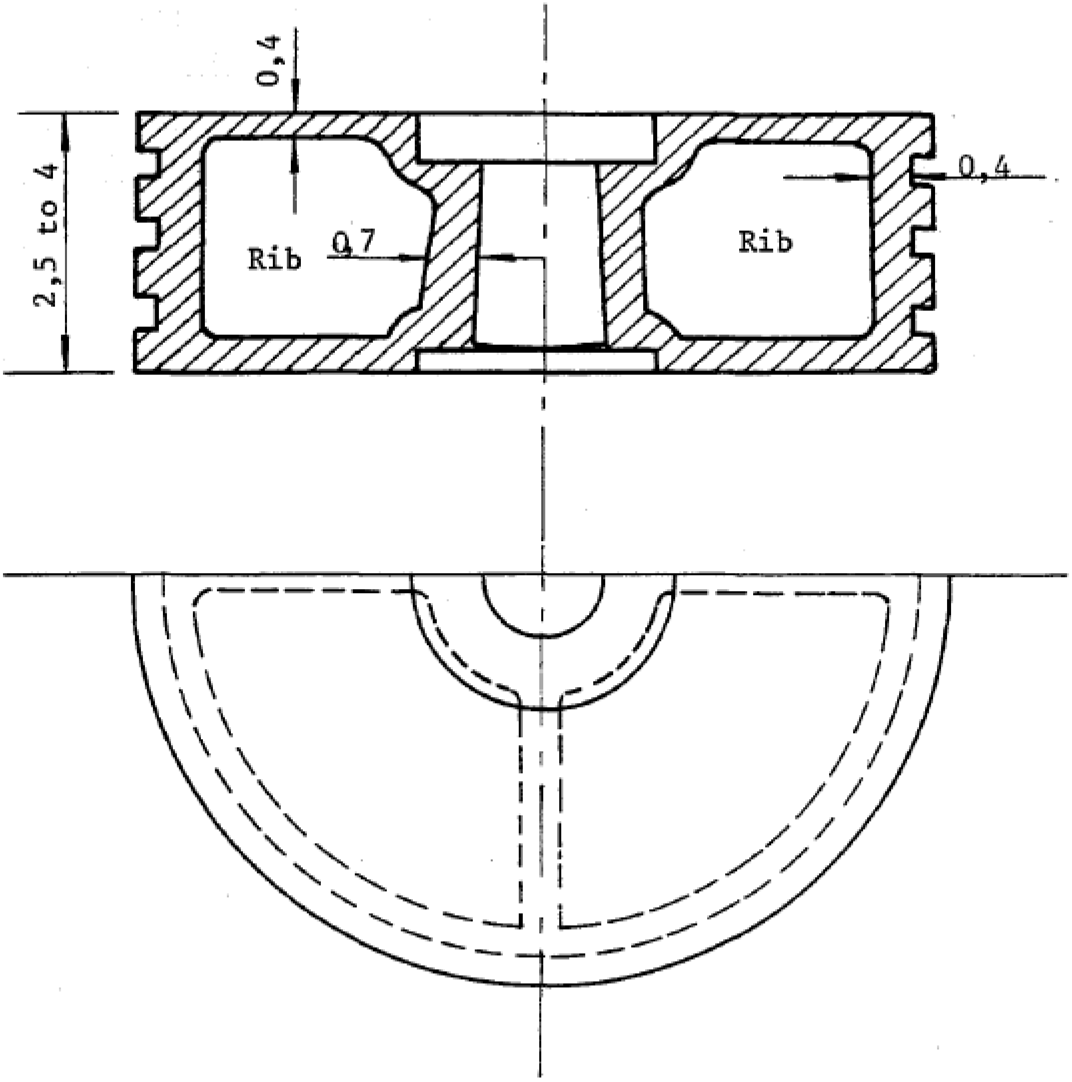


Figure 5.1

Pistons of this nature have to be made sufficiently strong to resist the greatest pressure to which they are likely to be subjected. This in many cases means that a pressure greater than the initial steam pressure acting has to be allowed for.

It is in pistons of this nature especially that calculations for their design cannot be made with any degree of complete accuracy and because of this it is usual to fix the various thicknesses of metal by means of some empirical rule or other.

For medium sized and fairly large cast iron box type pistons, the proportions may be as given in **Figure 5.1**. The proportions given in this figure are in terms of the unit $0{,}12D\sqrt{p}$ where D = cylinder diameter in metre and p = steam pressure in pascals.

Box-type pistons, unless of small size, are usually strengthened by means of radial ribs, as shown in **Figure 5.1**.

The number of these ribs is generally based on the diameter of the piston, although it should be realised that such number also depends to a certain extent upon the stiffness of the design of the body or (box) of the piston.

A rule for the approximate number of ribs is 40 + 2 where 0 is the piston diameter in metres. As far as is practicable make the thickness of these ribs the same as the thickness of the body metal of the pistons.

5.2.2 Conical type pistons

Perhaps the most commonly used type of high pressure piston is that of the type shown in **Figure 5.2**.

This is the conical type of piston, complete with junk ring, piston ring and spring, and an endeavour will be made to show how the principal sizes of such a piston are obtained.

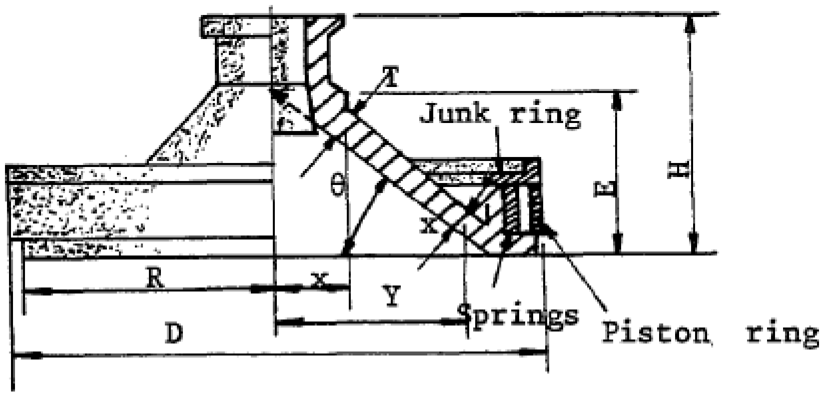


Figure 5.2

The outside diameter, D, is of course fixed from consideration of the power to be developed in the cylinder.

For the total depth H of the piston there is no hard and fast rule but some idea of this may be obtained from the fact that the height E (in metre) of the cone is usually made = $\frac{D}{6}$, again D should be in metre. For thicknesses T and t use

$$T = \frac{p}{2 \times \sigma \times \sin\theta} \times \frac{R^2 - x^2}{x}$$

$$\text{And } t = \frac{p \times y}{\sigma \times \sin\theta}$$

where T = the thickness at a point x metre distant from the axis of the piston rod.

t = thickness at a point y metre distant from the axis of the piston rod.

p = difference of pressure (pascal) between two sides of piston.

σ = maximum allowable stress in material (21 MPa for cast iron, and 62 MPa for steel).

θ = the base angle of the cone (approximately 45°).

The whole object in adopting the conical shape for a piston is to obtain the utmost rigidity and strength without having undue mass. A typical piston of this type is shown in **Figure 5.3**.

In this, the piston boss is bored with a fine taper to fit the piston rod, and in addition the piston bears against a collar on the rod. When compared with **Figure 5.2** it will be seen that the piston of **Figure 5.3** has three rings in place of the deep ring; furthermore, the piston in **Figure 5.3** is not provided with a junk ring.

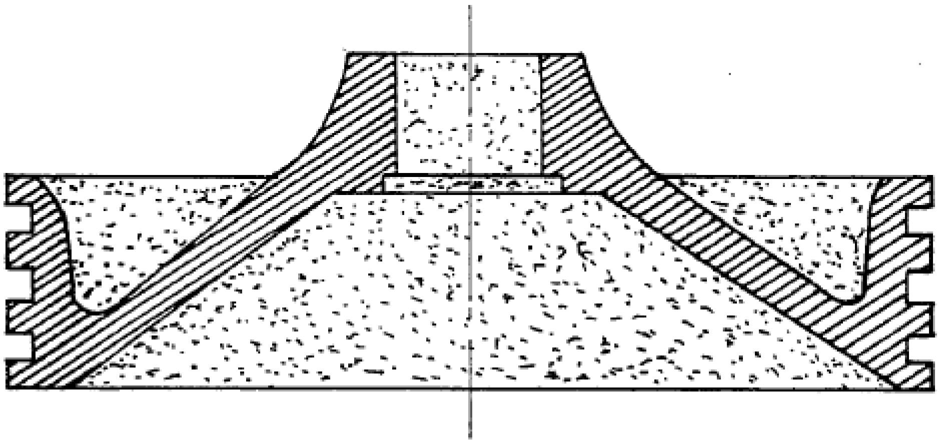


Figure 5.3

Conical pistons may be made of either cast steel, forged steel or of cast iron.

Sometimes a pressed steel conical body C is fitted with a cast iron "bull-ring" R to obviate the possibility of a steel piston working on a cast iron cylinder and thus obviate "scoring" or cutting of the cylinder.

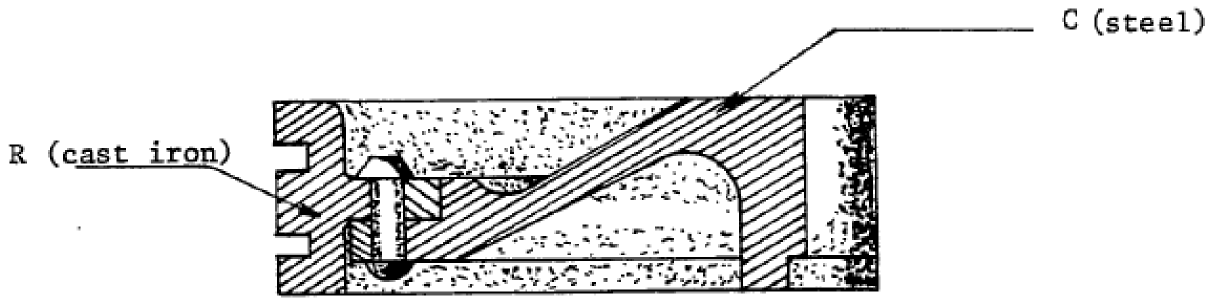


Figure 5.4

5.3 Piston rings

Piston rings are usually made of cast iron. As will be seen from **Figure 5.5** the two piston rings P are placed one on each side of a restraining ring R, and this latter limits the outwards movement of the split piston rings.

When these piston rings become slightly worn they fit the cylinder closely and are prevented from expanding further by means of the shoulders S on the restraining rings.

On top there is the junk ring J, made usually of steel or cast iron and this is provided so that the piston rings may be replaced when necessary without taking the piston off the piston rod and out of the cylinder.

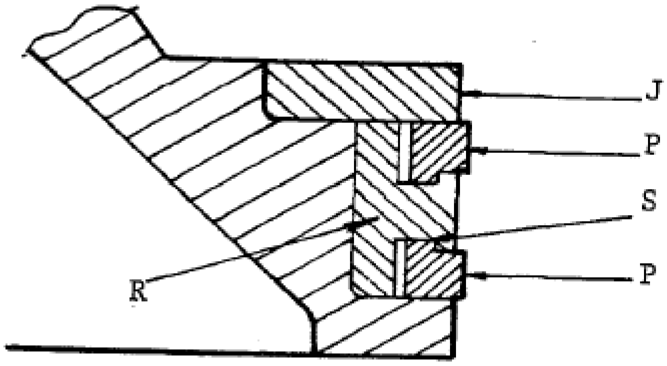


Figure 5.5

5.3.1 Proportions for piston rings

Thickness of piston ring measured radially $= \frac{piston\ diameter}{40}$

Width of piston ring measured axially $= \frac{piston\ diameter}{25}$

5.4 Piston rods

The piston rod of a steam engine is in compression and tension on alternate strokes as shown in **Figure 5.6**.

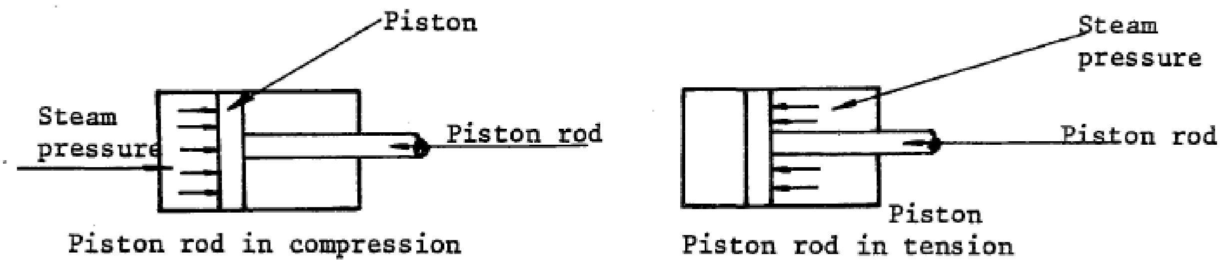


Figure 5.6

On the stroke when the rod is in compression it tends to buckle and it will do so at a load much less than that which would break it in tension.

It must therefore be designed with a very low working stress of (that is a high factor of safety) 17 MPa to 28 MPa for mild steel. It is immaterial whether this stress is considered as a tensile or compressive stress as in either case we have:

$$Load\ on\ piston\ rod = Area\ of\ piston\ rod \times stress\ in\ piston\ rod$$

$$P = \frac{\pi d^2}{4} \times \sigma$$

where $d = diameter\ of\ piston\ rod$ also

$$Load\ on\ piston\ rod = Area\ of\ piston \times cylinder\ pressure$$
$$P = \frac{\pi d^2}{4} \times p$$

where $D = cylinder\ diameter$

This applies only to the body or centre part of the rod where buckling would occur. The ends of the rod cannot buckle so they are designed with a normal working stress (ie medium factor of safety).

5.4.1 Fixing Piston rod to Piston

The end of the piston rod to which the piston is attached is almost always tapered and threaded as is shown in **Figure 5.7**.

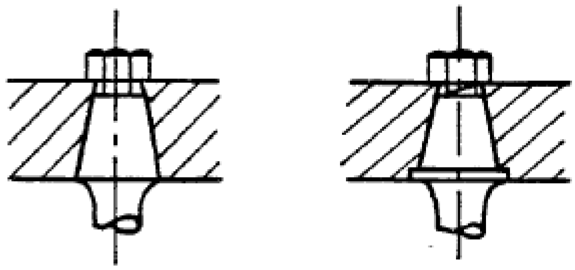


Figure 5.7

The nut must be securely locked. Because the nut has been tightened, the screwed part is subject to tension only; therefore the permissible stress in the screwed end may be higher than that for the main body of the rod.

A permissible stress at the screwed end may be taken to be from 48 MPa to 55 MPa. The design of this end consists of finding the diameter of the threaded part, using the core area as it will carry the tensile load.

Core diameter of screwed end $(d_r) = \sqrt{\frac{4 \times Load}{\pi \times \sigma_t}}$

Screw threads may be one of three series, namely:
1) Coarse pitch series.
2) Fine pitch series.
3) Constant pitch series.

Nominal sizes for standard threads in the above three series are.

Coarse pitch series from 1,6 mm to 68 mm.
Fine pitch series from 8 mm to 68 mm.

Constant pitch series:
a 0,35 mm constant pitch from 1,5 mm to 3,5 mm diameter.

a 0,5 mm constant pitch from 4 mm to 5,5 mm diameter.
a 0,75 mm constant pitch from 6 mm to 11 mm diameter.
a 1 mm constant pitch from 8 mm to 30 mm diameter.
a 1,25 mm constant pitch from 10 mm to 14 mm diameter.
a 1,5 mm constant pitch from 12 mm to 80 mm diameter.
a 2 mm constant pitch from 18 mm to 150 mm diameter.
a 3 mm constant pitch from 30 mm to 250 mm diameter.
a 4 mm constant pitch from 42 mm to 300 mm diameter.
a 6 mm constant pitch from 70 mm to 300 mm diameter.

To find the nominal diameter of a threaded portion it can be looked up, from a table of screw threads, as done before or as follows:

$$Nominal\ diameter\ of\ bolt = Core\ diameter + 1{,}227 \times pitch$$
$$d = d_r + 1{,}227\,p$$

The length of the taper is given as approximately one -quarter of the cylinder diameter $L = \frac{D}{4}$. The taper varies from 1 in 4 to 1 in 24 on the diameter. The smaller the taper the firmer can the rod be secured to the piston, but the larger the taper the easier it will be for the piston to be removed from the rod.

5.4.2 Fixing piston rod to cross-head

The cross-head is either forged solid with the rod as in **Figure 5.8** or attached by means of a cotter as in **Figure 5.9** and this joint is designed as per Module 4.

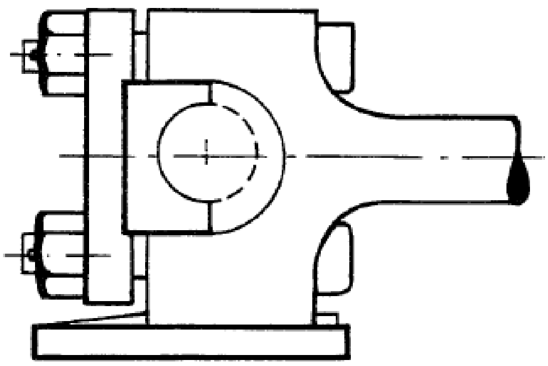


Figure 5.8

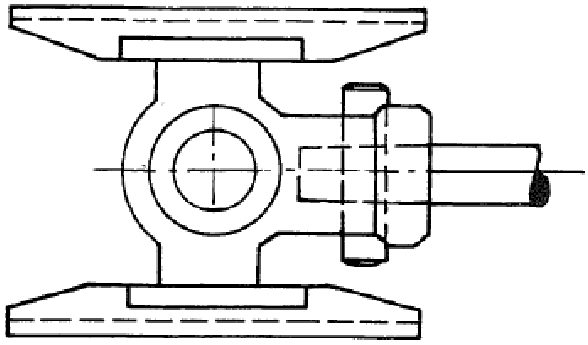


Figure 5.9

Worked Example 5.1

Calculate the diameter of a piston rod for a safe stress of 17 MPa. The greatest force on the rod is 260 kN.

<u>Solution:</u>

$$load = Area \times Stress$$
$$P = \frac{\pi d^2}{4} \times \sigma$$

$$d = \sqrt{\frac{P \times 4}{\pi \times \sigma}}$$
$$= \sqrt{\frac{260 \times 10^3 N \times 4}{\pi \times 17 \times 10^6 N/m^2}}$$
$$d = 0{,}1395 \text{ Say } 140\ mm\ diameter$$

Worked Example 5.2

In a horizontal steam engine the cylinder diameter is 915 millimetres and the greatest difference in steam pressure on the two sides of the piston is 380 kPa.

If the width of the piston is approximately one-quarter of the cylinder diameter, design the piston rod. Show the tapered part on which the piston fits, the screwed end, nut, etc by means of neat sketches. Use safe stresses of 17 MPa for the rod and 41 MPa for the screwed part.

Solution:

$$Maximum\ load\ on\ rod = Piston\ area \times steam\ pressure$$
$$P = \frac{\pi D^2}{4} \times p$$
$$= \frac{\pi (0{,}915\ m)^2}{4} \times 380 \times 10^3\ N/m^2$$
$$= 249{,}87 \times 10^3\ N$$
$$P = 249{,}87\ kN$$

$$Load\ on\ rod = Area\ of\ rod \times stress\ in\ rod$$
$$P = \frac{\pi d^2}{4} \times \sigma$$
$$d = \sqrt{\frac{P \times 4}{\pi \times \sigma}}$$
$$= \sqrt{\frac{249{,}87 \times 87 \times 10^3 N \times 4}{\pi \times 17 \times 10^6 N/m^2}}$$
$$= 0{,}1368\ metre$$

Say 137 $mm\ diameter$

$$\text{Length of taper portion} = \frac{D}{4}$$
$$= \frac{915\ mm}{4}$$
$$= 228{,}75$$

Say 230 mm

Diameter at end of taper
Assume a taper of 1 in 6 on the diameter.

A taper of 1 in 6 on the diameter means that the diameter decreases 1 mm for each 6 mm in length. Therefore in a 230 mm length the diameter decreases by $1 \times \frac{230}{6} = 38{,}33\ mm$

Hence diameter at end of taper.

$$= 137\ mm - 38{,}33\ mm$$
$$= 98{,}67\ mm$$

Say $99\ mm\ diameter$

$$Load\ in\ threaded\ portion = Load\ in\ piston\ rod$$
$$Load\ in\ threaded\ portion = Core\ area\ of\ threaded\ portion \times stress\ in\ threaded\ portion$$
$$P = \frac{\pi {d_r}^2}{4} \times \sigma_t$$
$$d_r = \sqrt{\frac{P \times 4}{\pi \times \sigma_t}}$$
$$= \sqrt{\frac{249{,}87 \times 10^3 N \times 4}{\pi \times 41 \times 10^6 N/m^2}}$$
$$= 0{,}0881\ metre$$
$$d_r = 88\ millimetre$$

A constant pitch series thread will be used with a constant pitch of 3 mm, 4 mm or 6 mm.

Let us take a 6 millimetre pitch.

$$Nominal\ diameter\ of\ threaded\ portion = d_r + 1{,}227\ p$$
$$d = 88 + 1{,}227\ \times 6$$
$$d = 95{,}362$$

Say $96\ mm\ diameter$

$$Constant = 00{,}12\ D\sqrt{p}$$
$$= 0{,}12 \times 0{,}915\sqrt{380} \times 10^3$$
$$= 67{,}69$$

$$Width = 2{,}5\ to\ 4\ multiply\ with\ the\ constant$$
$$= 3{,}5 \times 67{,}69$$
$$= 237\ millimetres$$

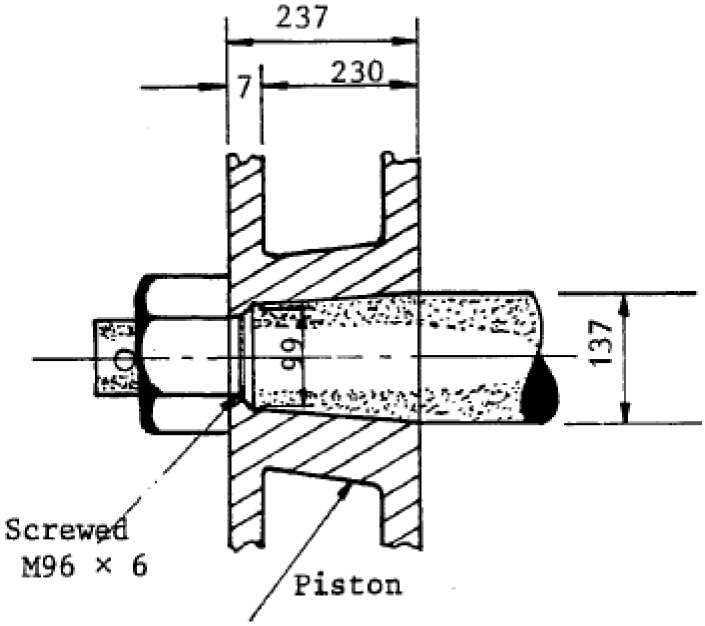


Figure 5.10

Worked Example 5.3

A steam engine cylinder has a bore of 762 millimetre and the maximum steam pressure is 1,03 MPa. Find the diameter of the piston rod for a direct stress of 27,5 MPa.

At the piston end the rod is tapered 1 in 4 on the diameter for a length of 152millimetres and then runs parallel for 38 millimetres beyond which it is threaded for the piston nut. The nominal diameter of the threaded part is 102 millimetres and the total thickness of the piston boss is 202 millimetres.

Sketch the rod end and piston boss.

Calculate the maximum tensile stress in the threaded portion.

Solution:

$$Maximum\ load\ on\ piston\ rod = Area\ of\ piston \times Steam\ pressure$$

$$P = \frac{\pi D^2}{4} \times p$$

$$= \pi \times \frac{(0{,}762\ m)^2}{4} \times 1{,}03 \times 10^6\ N/m^2$$

$$= 469{,}72\ kN$$

$$For\ the\ rod\ Load = Area\ of\ rod \times stress\ in\ rod$$

$$P = \frac{\pi d^2}{4} \times \sigma$$

$$d = \sqrt{\frac{P \times 4}{\pi \times \sigma}}$$

$$= \sqrt{\frac{469{,}72 \times 10^3 N \times 4}{\pi \times 27{,}5 \times 10^6}}$$

$$d = 0{,}1476\, metre$$

Say $d = 148\, mm$

A taper of 1 in 4 on diameter means that the diameter decreases 1 mm for each 4 mm in length. Therefore in a 152 mm length the diameter decreases by $1 \times \frac{152}{4} = 38\, mm$. Hence diameter at end of taper

$$= 152 - 38\, mm$$

$$= 114\, mm$$

Threaded portion

$$Nominal\ diameter\ of\ bolt = Core\ diameter + 1{,}227 \times pitch$$

$$Core\ diameter = Nominal\ diameter - 1{,}227 \times pitch$$

For a nominal diameter of 102 mm a pitch of 2mm, 3 mm, 4 mm or 6 mm can be used.

Assume a pitch of 6 mm

$$d_r = 102 - 1{,}227 \times 6\, mm$$

$$= 94{,}638\, mm$$

Say 95 mm

$$Tensile\ stress\ in\ threaded\ portion = \frac{Load}{Area}$$

$$\sigma_t = \frac{P}{\frac{\pi {d_r}^2}{4}}$$

$$\sigma_t = \frac{469{,}72 \times 10^3 N \times 4}{\pi \times (0{,}095\, m)^6}$$

$$= 66{,}27\, MPa$$

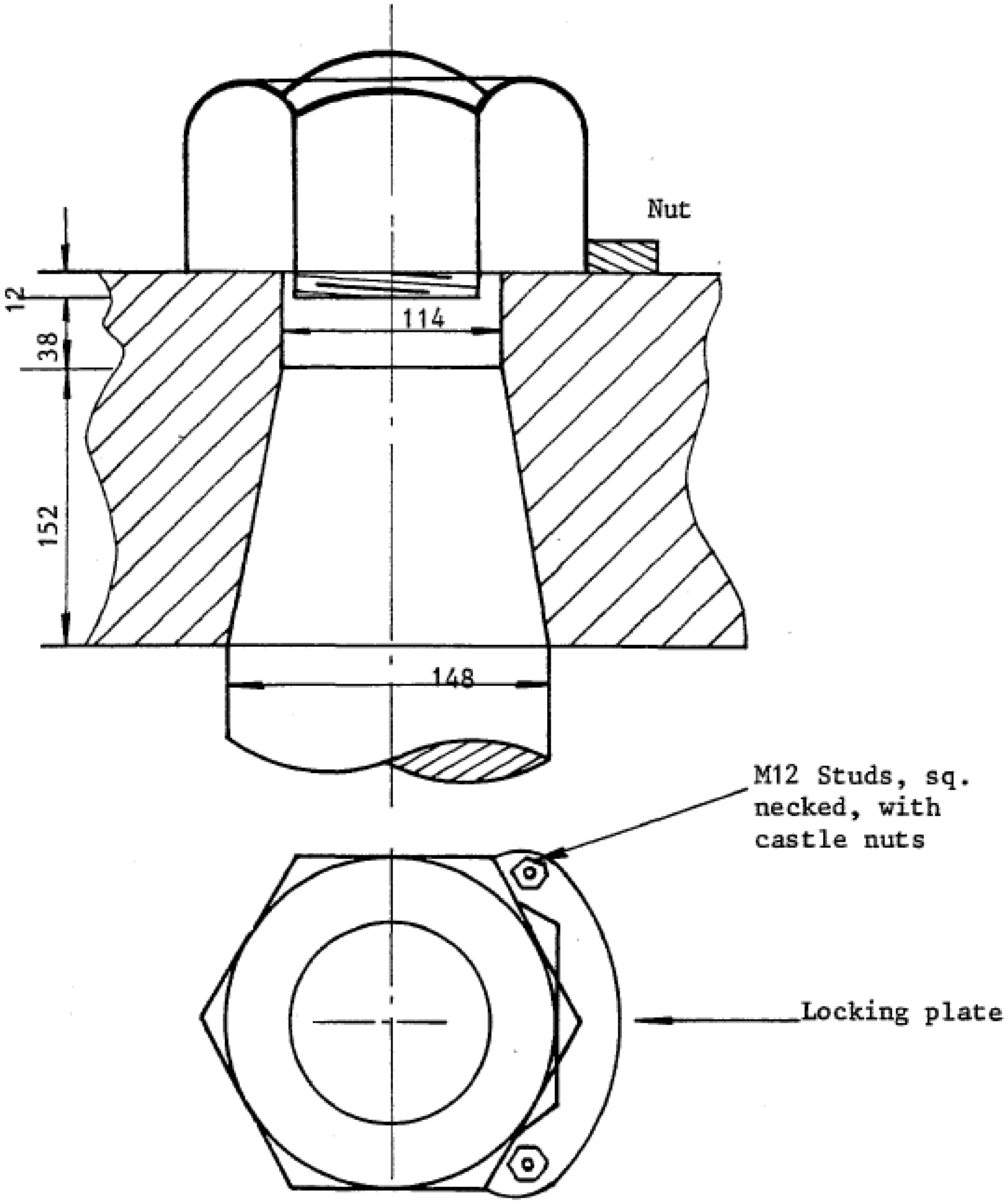


Figure 5.11

Worked Example 5.4

The maximum pull in the piston rod of a steam engine is 71 kN and the diameter of the rod is 64 mm. At the crosshead end the rod diameter is increased to 75 mm diameter and is then tapered to 70 mm diameter over a length of 140 mm where it fits the socket which has an outside diameter of 114 mm. A cotter 64 mm by 19 mm is fitted at the centre of the joint.

Calculate:

a) tensile stress in the body of the rod.

b) shear stress in the cotter.
c) tensile stress in the socket.
d) crushing stress on the cotter in the rod and the socket.

Solution:

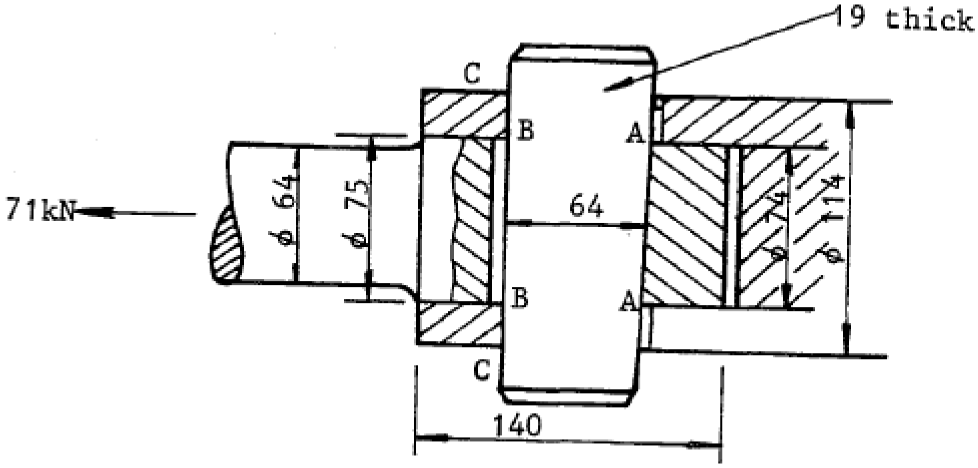


Figure 5.12

a) Tensile stress in the body of the rod

$$Tensile\ stress = \frac{Tensile\ Load}{Cross-sectional\ area}$$

$$\sigma_t = \frac{P}{\frac{\pi d^2}{4}}$$

$$= \frac{71 \times 10^3 N \times 4}{\pi \times (0{,}064\ m)^2}$$

$$= 22{,}07\ MPa$$

b) Shear stress in the cotter

$$Shear\ stress = \frac{Shear\ load}{Cross-sectional\ area}$$

$$\tau = \frac{P}{2 \times b \times t}$$

$$= \frac{71 \times 10^3 N}{2 \times 0{,}064\ m \times 0{,}019\ m}$$

$$= 29{,}2\ MPa$$

c) Tensile stress in the socket

Maximum stress will occur where cross-section of socket is smallest for example at CC.

Diameter BB can be obtained from direct measurement from a scale drawing or from calculations.

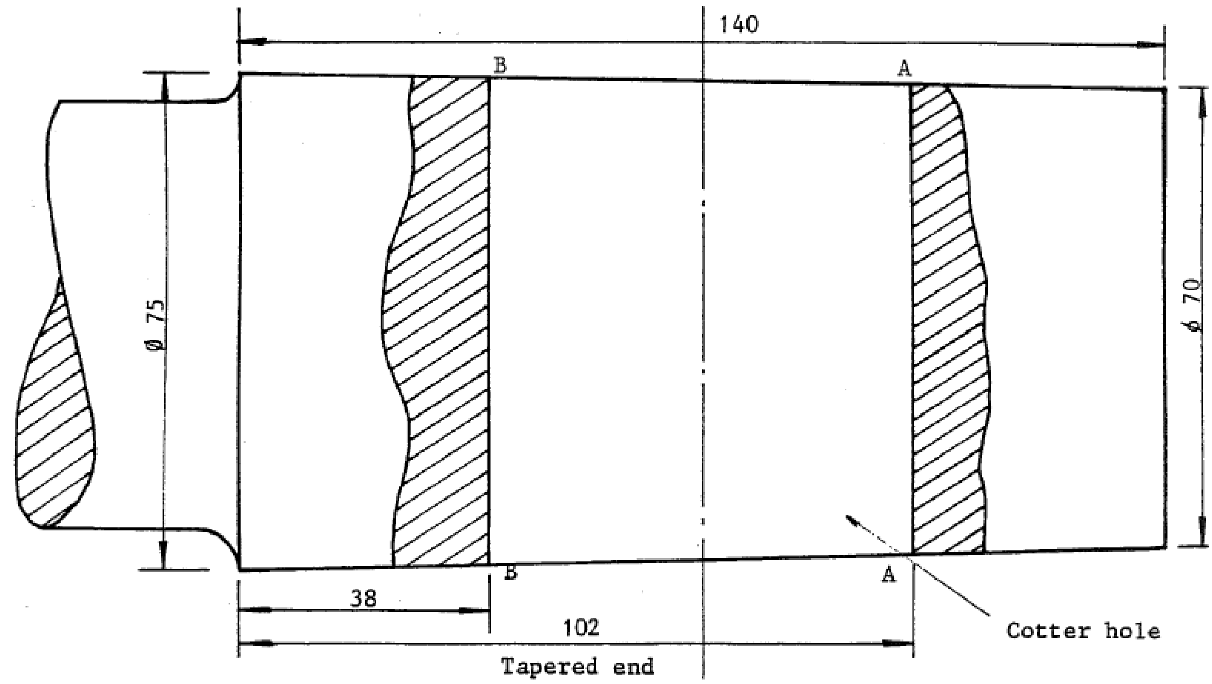


Figure 5.13

$$Taper = \frac{Difference\ between\ end\ diameters}{Length}$$
$$= \frac{75\ mm - 70\ mm}{140\ mm}$$
$$= \frac{5}{140}$$
$$Taper = \frac{1}{28} = 1\ in\ 28\ on\ diameter$$

To find diameter at BB.

$$Taper = \frac{Difference\ between\ the\ two\ diameters}{Length}$$
$$\frac{1}{28} = \frac{75\ mm - Diameter\ at\ BB}{38\ mm}$$
$$\frac{38}{28} mm = 75\ mm - Diameter\ at\ BB$$
$$Diameter\ at\ BB = 75\ mm - \frac{38}{28} mm$$
$$= 75\ mm - 1{,}357\ m$$
$$= 73{,}643\ mm$$

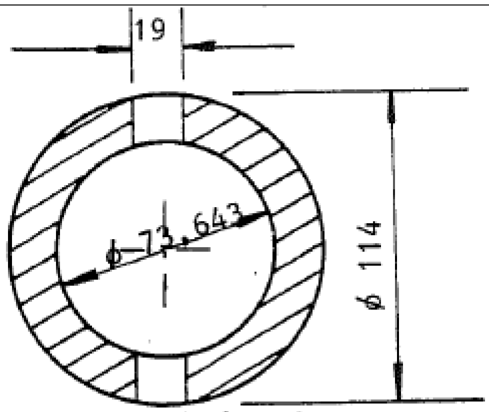


Figure 5.14

$$Tensile\ stress = \frac{Tensile\ Load}{Cross-sectional\ area}$$

$$\sigma_t = \left\{\frac{P}{\frac{\pi}{4}(D^2-d^2)-t(D-d)}\right\}$$

$$= \frac{71\times10^3}{\left\{\frac{\pi}{4}(0{,}114)^2-0{,}073643^2-0{,}019\ (0{,}114-0{,}073643)\right\}}$$

$$= \frac{71\times10^3 N}{5{,}181\times10^{-3} m^2}$$

$$= 13{,}704\ MPa$$

d) Crushing stress on the cotter in the rod

Diameter at AA

$$\text{Taper} = \frac{\text{Difference between the two diameters}}{\text{Length}}$$

$$\frac{1}{28} = \frac{75\text{ mm}-\text{Diameter at AA}}{102\text{ mm}}$$

$$Diameter\ at\ AA = 75\text{ mm} - \frac{102\ mm}{28}$$

$$= 71{,}357\text{ mm}$$

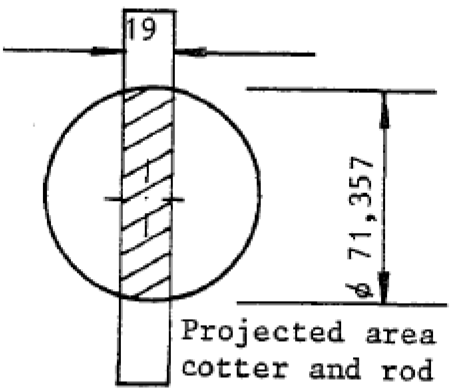


Figure 5.15

$$Crushing\ stress = \frac{Crushing\ load}{Projected\ area}$$

$$\sigma_c = \frac{P}{d \times t}$$
$$= \frac{71 \times 10^3 N}{0{,}071357\, m \times 0{,}019\, m}$$
$$= 52{,}37\ MPa$$

Crushing stress on the cotter in the socket

$$Crushing\ stress = \frac{Crushing\ load}{Projected\ area}$$
$$\sigma_c = \frac{P}{(D-d) \times t}$$
$$= \frac{71 \times 10^3 N}{(0{,}114\, m - 0{,}073643\, m) \times 0{,}019\, m}$$
$$= 92{,}59\ MPa$$

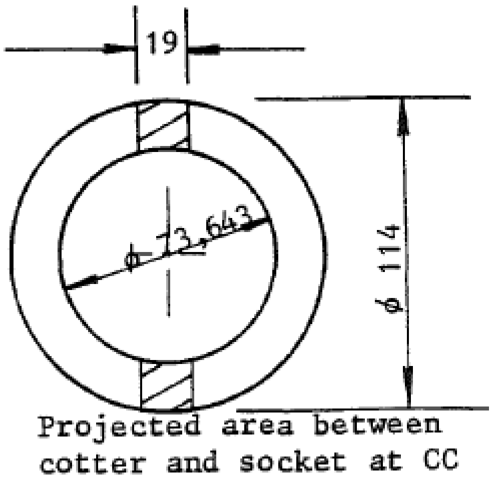


Figure 5.16

Worked Example 5.5

The piston rod of a steam engine is connected to the crosshead by means of a cottered joint, having a tapered solid rod entering the corresponding tapered socket of the cross-head. Rod, socket and cotter are of mild steel, with the following strengths.

Tensile stress = 46 MPa
Shearing stress = 42 MPa
Crushing stress = 84 MPa

<u>Design the following:</u>

1) The diameter of the tapered rod (d_1)
2) The thickness of the cotter.
3) The width of the cotter.
4) The diameter of the socket at the cotter hole.
5) The diameter of the socket at the large end of the socket.

Make a neat dimensioned sketch of the assembly, if the piston rod is 38 mm in diameter. Other dimensions may be obtained using standard proportions.

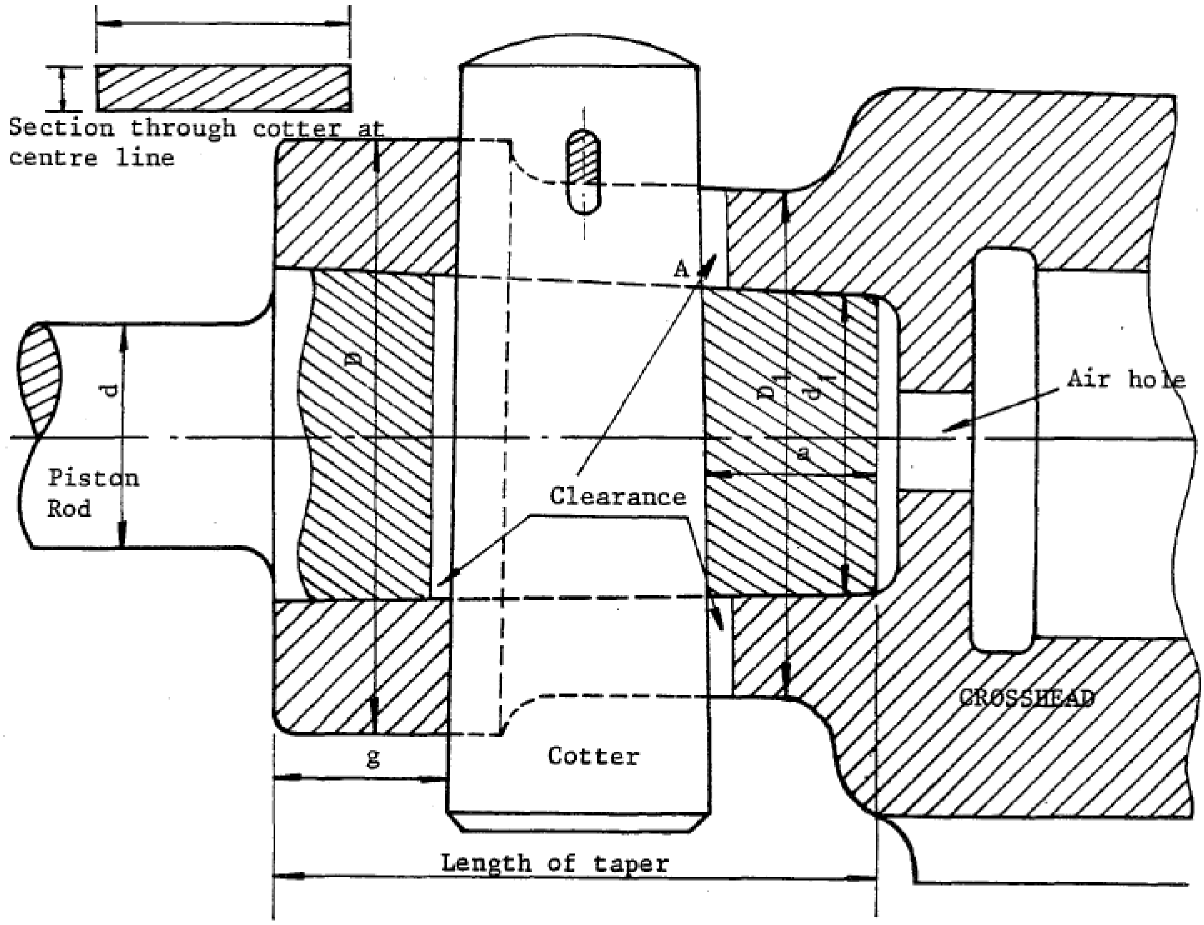


Figure 5.17 Cottered joint

Load in the rod

$$Tensile\ load = Tensile\ stress \times Cross - sectional\ area$$

$$P = \sigma_t \times \frac{\pi d^2}{4}$$

$$= 46 \times 10^6\ N/m^2\ \frac{\pi\times(0{,}038m)^2}{4}$$

$$= 52{,}17\ kN$$

1. *Diameter (d_1) at AA pierced rod*

$$d_1 = \sqrt{\frac{4\times P(\sigma_c+\sigma_t)}{\pi(\sigma_c+\sigma_t)}}$$

$$= \sqrt{\frac{4 \times 52{,}17 \times 10^3\ (84 \times 10^6+46\times10^6)}{\pi\times84 \times 10^6\times46\times10^6}}$$

$$= \sqrt{2{,}235}$$

$$= 0{,}0473\ m$$

Say 48 mm diameter

2. *Thickness of the cotter*

$$Tensile\ load\ in\ pierced\ rod = Crushing\ load\ in\ cotter\ against\ the\ rod$$
$$\sigma_t\left(\frac{\pi d_1^2}{4} - d_1 \times t\right) = \sigma_c \times d_1 \times t$$
$$t = \frac{\sigma_t \times \pi \times d_1}{4(\sigma_c + \sigma_t)}$$
$$= \frac{46 \times 10^6 \times \pi \times 0{,}048\ m}{4(84 \times 10^6 + 46 \times 10^6)}$$
$$= 0{,}0133$$
Say $t = 14\ mm$

3. *The width of the cotter*

$$Shear\ load\ in\ cotter = Crushing\ load\ in\ cotter\ against\ rod$$
$$\tau \times 2 \times t \times b = \sigma_c \times d_1 \times t$$
$$b = \frac{\sigma_c \times d_1}{\tau \times 2}$$
$$= \frac{84 \times 10^6 \times 0{,}048}{42 \times 10^6 \times 2}$$
$$= 0{,}048\ m$$
Say $b = 48\ mm$

4. *Diameter (D_1) of the socket at the cotter hole*

$$Tensile\ load\ in\ pierced\ socket = Tensile\ load\ in\ pierced\ rod$$
$$\sigma_t \times \left\{\frac{\pi}{4}(D_1^2 - d_1^2) - t(D_1 - d_1)\right\} = \sigma_t\left(\frac{\pi d_1^2}{4} - d_1 \times t\right)$$
$$0{,}785D_1^2 - 1{,}57d_1^2 - D_1 t + 2d_1 t = 0$$
$$0{,}785D_1^2 - 1{,}57 \times 0{,}048^2 - D_1 \times 0{,}014 + 2 \times 0{,}048 \times 0{,}014 = 0$$
$$0{,}785D_1^2 - 0{,}00362 - 0{,}014D_1 + 0{,}001344 = 0$$
$$D_1^2 - 0{,}0178D_1 - 0{,}0029 = 0$$

$$D_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{+0{,}0178 \pm \sqrt{0{,}000317 + 0{,}0116}}{2}$$
$$= \frac{+0{,}0178 \pm 0{,}122}{2}$$
$$= 0{,}0697$$
Say $= 70\ mm$

5. *The diameter of the socket at the large end of the socket (D)*

$$Crushing\ load\ in\ cotter\ against\ rod = Crushing\ load\ in\ cotter\ against\ socket$$
$$\sigma_c \times d_1 \times t = \sigma_c \times (D - d_1)t$$
$$D = 2d_1$$
$$= 2 \times 0{,}048\ m$$
$$D = 96\ mm$$

Length a

$$\begin{aligned} Shear\ load\ in\ end\ of\ pierced\ rod &= Shear\ load\ in\ cotter \\ \tau \times 2 \times d_1 \times a &= \tau \times 2 \times t \times b \\ a &= \frac{t \times b}{d_1} \\ &= \frac{0{,}014\ m \times 0{,}048\ m}{0{,}048} \\ &= 0{,}014\ M \\ &= 14\ mm \end{aligned}$$

Length g

$$\begin{aligned} Shear\ load\ in\ end\ of\ socket &= Shear\ load\ in\ cotter \\ \tau \times 2(D - d_1)g &= \tau \times 2 \times t \times b \\ g &= \frac{t \times b}{D_1 - d_1} \\ &= \frac{0{,}014\ m \times 0{,}048\ m}{0{,}096\ m \times 0{,}048\ m} \\ &= 14\ mm \end{aligned}$$

Assume a taper of 1 in 16 on the diameter.
Small diameter of taper

$$\begin{aligned} Taper &= \frac{Diameter\ at\ AA - Small\ diameter\ of\ taper}{Length\ (a)} \\ \frac{1}{16} &= \frac{0{,}048\ m - Small\ diameter\ of\ taper}{0{,}014\ m} \\ small\ diameter\ of\ taper &= 0{,}048\ m - \frac{1}{16} \times 0{,}014\ m \\ &= 0{,}047125\ m \\ &= 47{,}125\ mm \end{aligned}$$

Large diameter of taper

$$Taper = \frac{Large\ diameter - Small\ diameter}{Length\ of\ taper}$$

Note: Length of taper $= a + g + clearance$
assume a clearance of 5 mm

$$\begin{aligned} \therefore Length\ of\ taper &= 14\ mm + 14\ mm + 5\ mm \\ &= 23\ mm \\ Large\ diameter &= \frac{1}{16} \times 23\ mm + 47{,}125\ mm \\ &= 48{,}563\ mm \end{aligned}$$

Other dimensions obtained from standard proportions.

Worked Example 5.6

In a high pressure conical type of cast steel piston, the diameter D is 1 524 mm and the difference of pressure between the two sides of the piston is 655 kPa. The base angle of the conical part is 37,5 degrees.

Find the respective thicknesses of the conical part near the boss ($x = 178\ mm$) and at a point where the conical part meets the rim(or crown) of the piston, ($y = 666\ mm$). See **Figure 5.2**.

$$T = \frac{p}{2 \times \sigma \times \sin\theta} \times \frac{R^2 - x^2}{x}$$

$$= \frac{655 \times 10^3\ N/m^2}{2 \times 62 \times 10^6\ N/m^2 \times \sin 37{,}5°} \quad \frac{x(0{,}762\ m) - (0{,}178\ m)^2}{0{,}178\ m}$$

$$T = 0{,}0268\ m$$

Say T = $27\ mm$

$$t = \frac{p \times y}{\sigma \times x \times \sin\theta}$$

$$= \frac{655 \times 10^3\ N/m^2 \times 0{,}666\ m}{62 \times 10^6\ N/m^2 \times \sin 37{,}5°}$$

$$t = 0{,}0116\ m$$

Say t = $12\ mm$

Note: For compound or triple-expansion steam engines, it will be seen that for a low-pressure piston of conical type (as compared with a high-pressure one) the base angle of the cone is much less. When designing a low-pressure piston certain practical contingencies must be guarded against.

One is, getting a very much higher initial steam pressure than the normal working pressure in the cylinder, especially when starting the engine. To meet such contingencies TWICE the normal difference of steam pressure between the two sides of the piston is used when designing t for a low-pressure piston.

(This doubling of pressure does not apply to the design of any piston other than a LOW-PRESSURE one).

Worked Example 5.7

The normal difference in pressure between the two sides of a conical cast steel low pressure piston is 165 kPa. The diameter of the piston is 2,74 m and the base angle of the cone is 19 degrees.

Calculate the thicknesses, T and t, taking a stress value of 62 MPa.

Take x and y (as per **Figure 5.2**), to be 178 mm and 280 mm respectively.

<u>Solution:</u>

$$Pressure\ to\ be\ taken = 2 \times 165 \times 10^3\ N/m^2$$

$$= 330\ KPa$$

$$T = \frac{p}{2 \times \sigma \times \sin\theta} \times \frac{R^2 - x^2}{x}$$

	$= \frac{330\times10^3\ N/m^2}{2\times62\times10^6\ N/m^2\times\sin 19°} \times \frac{(1{,}372\ m)-(0{,}178\ m)^2}{0{,}178\ m}$
	$= 8{,}174 \times 10^{-3} \times 10{,}397\ m$
	$= 0{,}0849\ m$
Say T	$= 85\ mm$
t	$= \frac{p\times y}{\sigma\times x\times\sin\theta}$
	$= \frac{330\times10^3\ N/m^2\times1{,}280\ m}{62\times10^6\ N/m^2\ \times\sin 19°}$
	$= 0{,}0209\ mm$
Say t	$= 21\ mm$

5.5 Shafting

Shafts are generally subjected to a combination of bending and torsion, but in some cases either of these may be so small as to be negligible.

An example of pure torsion occurs when an electric motor is directly coupled to a centrifugal pump. The shafting between the motor and the pump is in pure torsion; there is no bending action on it.

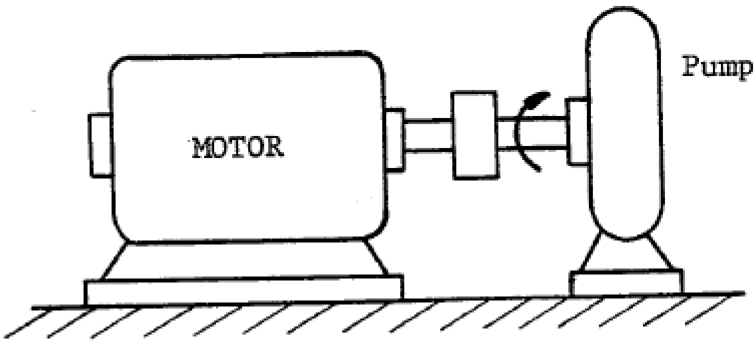


Figure 5.18

A railway carriage axle, for instance, may be considered to be subjected to bending only.

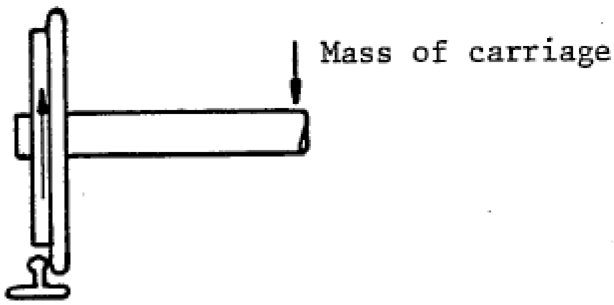


Figure 5.19

In the case of an electric motor fitted with a pulley for a belt drive the shaft is in torsion and also bending, as a result of the pull in the belts.

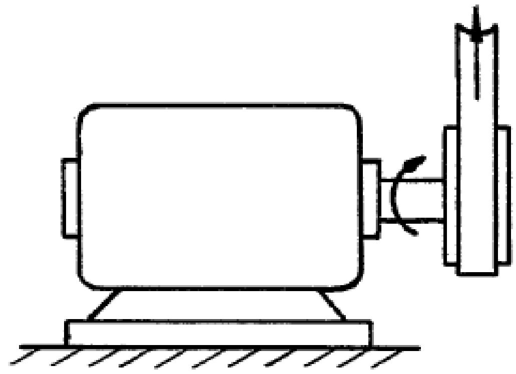


Figure 5.20

We will deal with solid and hollow shafts subjected only to pure torsion, ie twisting only with no bending.

5.6 The torsion equation for shafts

$$\frac{T_{max}}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

Where T_{max} = Maximum torque or twisting moment (Nm)
J = Polar moment of inertia (m^4)
d_s = Diameter of solid shaft
D = External diameter of hollow shaft (m)
d = Internal diameter of hollow shaft (m)
τ = Allowable shear stress in shaft material (Pa)
r = $\frac{d_s}{2}$ for solid shafts and $\frac{D}{2}$ for hollow shafts (m)
G = Modulus of rigidity of shaft material (Pa)
θ = Angle of twist in radians
One radian = $\frac{180}{\pi}$ degrees
One radian = $57{,}295\ degrees, say\ 57{,}3\ degrees$
l = Length of shaft (m)

5.6.1 Torque

Torque or turning moment is the product of force and perpendicular distance from the fulcrum or turning point.

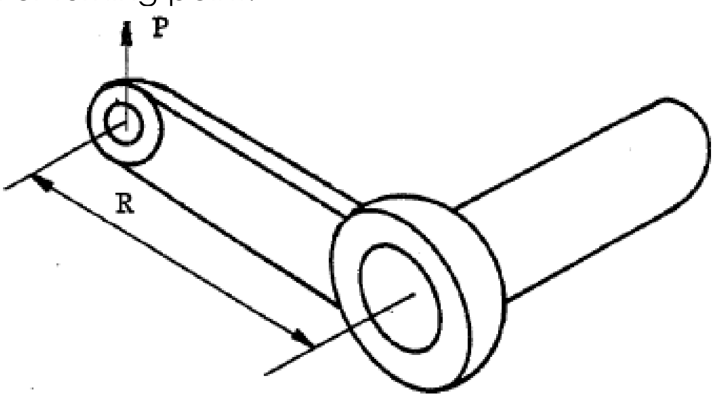


Figure 5.21

$$Torque = Force \times Perpendicular\ distance$$
$$T = p \times R \qquad P \text{ in newtons and } R \text{ in meter}$$
$$T = PR\ Nm$$

5.6.2 Maximum and Mean Torque

An electric motor or a turbine provides an almost uniform torque, because the driving force is applied steadily and at the same radius throughout a revolution.

A single-cylinder reciprocating engine, on the other hand, does not supply a uniform torque, as the pressure, on the piston varies and so does the effectiveness of the crank. (In the dead centre positions the crank is completely non-effective, for no amount of pressure on the piston will turn it).

This variation in torque is smoothed out to some extent by the flywheel or by using more cylinders, but is never entirely eliminated. Thus there is a maximum torque at some point in the cycle, and it is at this point that the greatest stress will occur in the shaft, and it is therefore for this torque that the shaft must be designed.

Note:
It is usual to assume the maximum torque to be approximately 1,3 times the mean torque.

5.6.3 Polar Moment of Inertia

It is the moment of inertia, about an axis through the centre of gravity of a body, but perpendicular to the plane of the body.

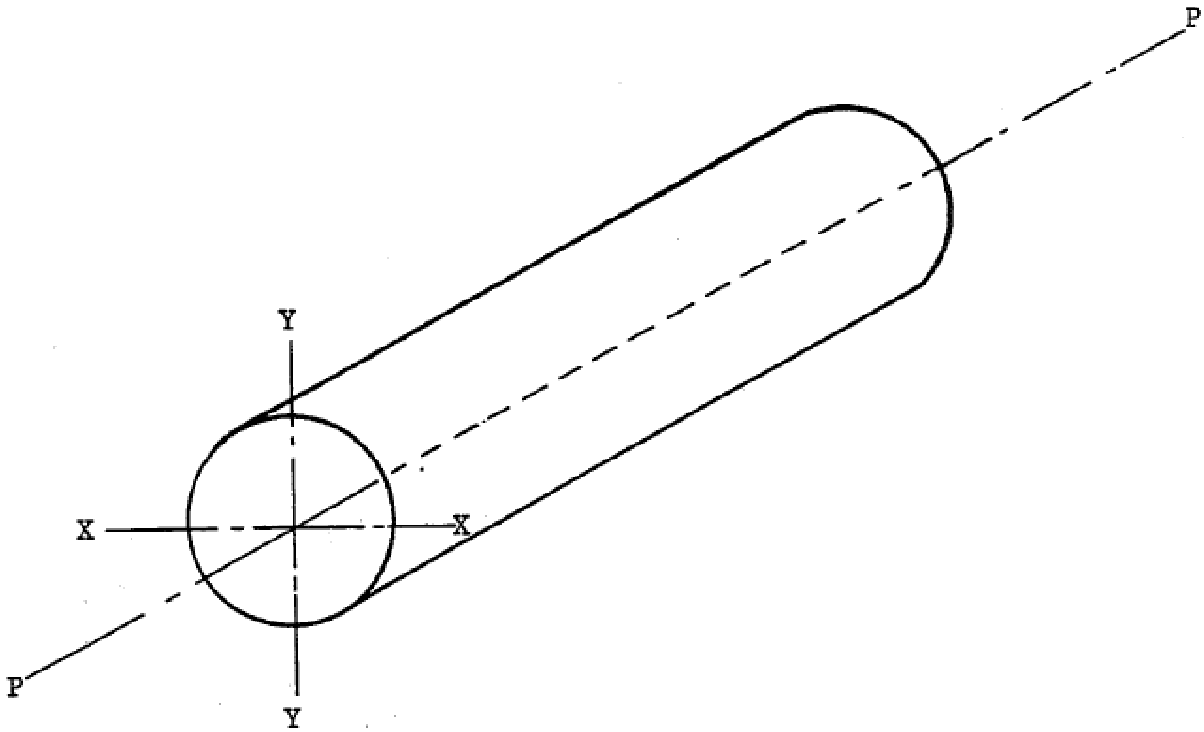


Figure 5.22

$$
\begin{aligned}
J_p &= I_{xx} + I_{yy} \\
&= \frac{\pi d_s^4}{64} + \frac{\pi d_s^4}{64} \\
&= \frac{\pi d_s^4}{32} \text{ for solid shafts}
\end{aligned}
$$

Similarly, $J_p = \frac{\pi}{32}\{D^4 - d^4\}$ for hollow shafts

5.6.4 Shear stress

When a torque is applied to a shaft each section tends to slide round on the next one and hence the material is in shear. The shear will not be uniform across the section, but will be greatest at the outside, where there is most tendency to sliding between the layers.

Secondly, the area of each ring of metal carrying these stresses is greater at the outside than at the centre, and therefore the forces at the outside are greater. Thirdly, the forces at the outside are acting at a greater radius than at the inside, and so can resist a higher torque.

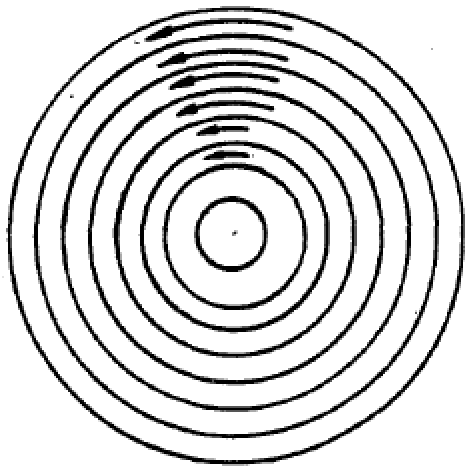


Figure 5.23

We see, therefore, that the outside layers of the shaft are the important ones, as they carry the highest stress and resist most of the torque. The formula connecting torque and stress cannot therefore be a simple one like we have for direct tension, shear or compression.

5.6.4.1 Solid Shafts

$$\frac{T_{max}}{J} = \frac{\pi}{r} \quad \text{Note } r = \frac{d_s}{2} \text{ and } \frac{\pi d_s^4}{32}$$

$$
\begin{aligned}
T_{max} &= \frac{\tau}{r} \times J \\
&= \tau \times \frac{\pi d_s^4}{32} \times \frac{2}{d_s} \\
T_{max} &= \frac{\pi d_s^3}{16} \tau
\end{aligned}
$$

5.6.4.2 Hollow Shafts

$$\frac{T_{max}}{J} = \frac{\pi}{r} \qquad r = \frac{D}{2} \text{ and } J = \frac{\pi}{32}[D^4 - d^4]$$

$$T_{max} = \frac{\tau}{r} \times J$$

$$= \tau \times \frac{\pi}{32}[D^4 - d^4] \times \frac{2}{D}$$

$$T_{max} = \frac{\pi}{16}\left[\frac{D^4 - d^4}{D}\right]\tau$$

5.6.5 Modulus of rigidity

This is the ratio of shear stress to shear strain for a linear elastic material. It is denoted by G. It is also known as the shear modulus.

$$G = \frac{Shear\ stress}{Shear\ strain}$$

5.6.6 Angle of twist

The rotary motion of prime movers such as steam engines and electric motors is transmitted by means of circular shafts.

The driver exerts a torque or turning moment on the shaft at one place along its length, and the torque is transmitted to other places along the shaft to overcome the resisting torques of the driven mechanisms.

In transmitting a torque, the shaft is twisted. The driven end lags more or less behind the driving end and, for any particular shaft, the angle of twist is proportional to the torque transmitted.

⚠	**Note:** An enquiry into the action of the torque on the material of the shaft shows that the material is in shear, and therefore the twist is a shear strain.

In **Figure 5.24** a circular shaft of length t and diameter d is represented as fixed at one end.

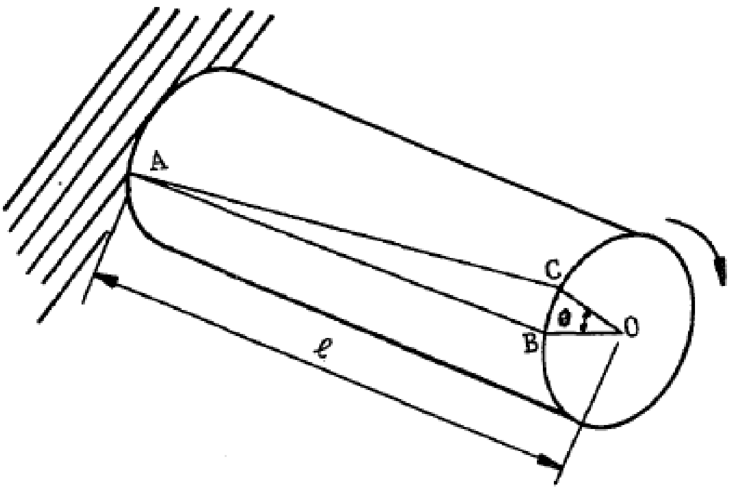


Figure 5.24

A line AB is scribed along the length of the shaft, and a corresponding radial line OB on the circular end.

If now the free end of the shaft is subjected to a twisting moment or torque T, the shaft will be twisted. The twist will vary uniformly from zero at the fixed end to a maximum at the other end.

The line AB will take up the position AC, which is really a portion of a helix of very long pitch, but may be considered as a straight line when the angle of twist is small. The radius OB will swing through an angle to the position OC. The angle BOC is the angle of twist in the length l.

Note:
Provided that the elasticity of the shaft is not impaired, the radius OB remains straight in its new position OC.

If angle BOC is e in radian measure, then:

$$\theta = \frac{Arc}{Radius} = \frac{BC}{OB} = \frac{BC}{\frac{1}{2}d}$$

$$\therefore BC = \frac{d \times \theta}{2}$$

Now, the shear strain on the outside of the shaft

$$= \frac{BC}{AB} = \frac{BC}{l} = \frac{d \times \theta}{2l}$$

But $shear\ stress = shear\ strain \times modulus\ of\ rigidity$

ie $\tau = \frac{d \times \theta}{2l} \times G$

or Angle of twist θ for solid shaft $= \frac{2\tau l}{dG}$

Angle of twist θ for hollow shaft $= \frac{2\tau l}{GD}$

Note:
e is in radians and that the units used must be consistent, ie t and d in meters and T and G both in pascals.

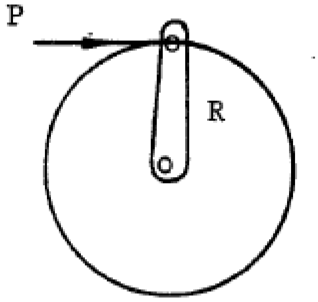
This can also be seen when the torsion equation is used.

$$\frac{\tau}{r} = \frac{G\theta}{l}$$

$$\theta = \frac{\tau l}{rG} \quad \text{but } r = \frac{d}{2}$$

$$\theta = \frac{2\tau l}{dG}$$

5.7 Power developed by a torque



P in newtons and
R in meters

Figure 5.25

Let a force of P newtons acting at a radius of R meters, move the arm around at N r/min.

$$\begin{aligned} Torque_{\ mean} &= Force \times Radius \\ &= p \times R\ Nm \\ Work\ done\ by\ force &= Force \times Distance\ moved \\ Work\ done\ in\ 1\ revolution &= Force \times Circumference\ of\ circle \\ &= P \times 2\,\pi\,R\ Nm \end{aligned}$$

$$\begin{aligned} Work\ done\ in\ 1\ minute &= Work\ done\ per\ one\ revolution \times number\ of\ revolutions \\ &= 2\,\pi\,PR \times N \\ &= 2\,\pi\,TN \end{aligned}$$

Note: $P \times R = T_{mean}$

$$\begin{aligned} Power\ Developed &= Work\ done\ per\ second \\ &= \frac{2\pi T_{\ meanN}}{60} \\ T_{\ mean} &= mean_{\ torque}\ (Nm) \\ N &= Revolutions\ per\ minute\ (r/min) \\ Power\ transmitted &= Power\ Developed\ (WATTS\ (W)) \end{aligned}$$

5.8 Design of shafts

Shafts may be designed for either (a) strength or (b) stiffness, and the expressions involved in calculating their dimensions are:

5.8.1 Designing for strength

5.8.1.1 Solid Shafts

$$\begin{aligned} T_{max} &= \frac{\pi d_s^3}{16}\tau \\ d_s &= \sqrt[3]{\frac{T_{max} \times 16}{\pi \times \tau}} \end{aligned}$$

5.8.1.2 Hollow Shafts

$$T_{max} = \frac{\pi}{16}\left[\frac{D^4-d^4}{D}\right]\tau$$

$$\therefore \frac{D^4-d^4}{D} = \frac{T_{max}\times 16}{\pi\times\tau}$$

5.8.2 Designing for stiffness

5.8.2.1 Solid Shafts

$$\frac{T_{max}}{J} = \frac{G\theta}{l}$$

$$\frac{T_{max}}{4} = \frac{G\theta}{l}$$

$$\frac{\pi d_s^4}{32}$$

$$\therefore d_s = \sqrt[4]{\frac{32\,T_{max}l}{\pi G\theta}}$$

5.8.2.2 Hollow Shafts

$$\frac{T_{max}}{J} = \frac{G\theta}{l}$$

$$\frac{T_{max}}{\frac{\pi}{32}(D^4-d^4)} = \frac{G\theta}{l}$$

$$(D^4-d^4) = \frac{32\,T_{max}l}{\pi G\theta}$$

5.9 Saving in mass

By using a hollow shaft instead of a solid shaft, a saving in mass is obtained. To obtain this saving in mass, we compare the volume of unit length of the two shafts being considered.

The lengths will both be the same; the mass per cubic metre will be the same, so that it is only a matter of comparing the cross-sectional areas of the two shafts.

$$Percentage\ saving = \left\{\frac{A_s-A_h}{A_s}\right\}\times 100$$

$$A_s = area\ of\ solid\ shaft$$

$$A_h = area\ of\ hollow\ shaft$$

5.10 Standard sizes of bright steel bars

5.10.1 Solid

Nominal diameter in millimeters						
3	11	19	32	50	80	120
4	12	20	35	55	85	125
5	13	22	36	60	90	130
6	14	24	38	*64	95	140

7	15	25	40	65	100	150
8	16	26	42	70	105	160
9	17	*28	45	75	110	180
10	18	30	48	*76	115	200

*These sizes are non-preferred

Table 5.1

5.10.2 Hollow

Outside diameter (mm)	Wall thickness (mm)	Mass per unit length kg/m	Area of section (cm²)	Second moment of area I(cm⁴)	Radius of gyration (cm)	Modulus of section Z (cm³)	Approx. number of metres per ton
27	3,2	1,89	2,43	1,75	0,846	1,28	530
34	2,6	2,01	2,61	3,25	1,12	1,90	498
	3,2	2,42	3,15	3,79	1,10	2,23	414
	4,0	2,95	3,84	4,41	1,07	2,59	339
42	2,6	2,57	3,34	6,78	1,42	3,16	390
	3,2	3,11	4,05	7,99	1,40	3,72	322
	4,0	3,81	4,95	9,41	1,38	4,39	263
48	3,2	3,59	4,61	11,8	1,60	4,88	279
	4,0	4,41	5,66	14,0	1,57	5,80	227
	4,9	5,24	6,65	16,0	1,55	6,60	191
60	3,2	4,54	5,83	23,8	2,02	7,90	221
	4,0	5,59	7,16	28,6	1,99	9,47	179
	4,9	6,67	8,52	32,9	1,97	10,9	150
76	3,2	5,80	7,42	49,5	2,59	13,0	173
	4,5	7,92	10,1	64,9	2,54	17,0	127
	5,4	9,41	12,0	75,3	2,51	19,8	107
89	3,2	6,81	8,77	80,3	3,02	18,0	147
	4,0	8,43	10,8	97,8	3,00	22,0	119
	5,4	11,1	14,1	124	2,95	27,9	90,1
114	3,6	9,90	12,7	195	3,91	34,1	102
	4,5	12,1	15,4	233	3,89	40,8	82,7
	5,4	14,5	18,5	274	3,86	47,9	69,0
	6,3	16,8	21,5	315	3,81	55,1	59,6

Table 5.2

Worked Example 5.8

What power can be transmitted by a 65 mm diameter shaft at 280 r/min if the torsional stress in the shaft is not to exceed 62 MPa?

Solution:

As no mention is made of maximum and mean torques, it is assumed that the torque is constant.

$$T = \frac{\pi d_s^3}{16}\tau$$
$$= \frac{\pi}{16} \times (0{,}065\ m)^3 \times 62 \times 10^6\ N/m^2$$
$$= 3343{,}2\ Nm$$

$$Power = \frac{2\ \pi\ T\ N}{60}$$
$$= \frac{2\times\ \pi\ \times 3343{,}2\ Nm \times 280}{60}$$
$$= 98028\ watts$$
$$= 98{,}028\ kW$$

Worked Example 5.9

Calculate the diameter of a solid shaft to transmit 261 kW at 180 r/min. The maximum torque exceeds the mean torque by 15 %.

Shearing strength of forged steel = 432 MPa.
Factor of safety = 8.

Solution:

$$Working\ stress = \frac{Ultimate\ stress}{Factor\ of\ safety}$$
$$= \frac{432\times10^6\ N/m^2}{8}$$
$$= 54\ MPa$$

$$Power = \frac{2\pi T_{mean} N}{60}$$
$$T_{mean} = \frac{Power \times 60}{2\ \pi\ N}$$
$$= \frac{261 \times 10^3 \times 60}{2 \times \pi \times 180}$$

$$T_{mean} = 13846{,}5\ Nm$$
$$T_{max} = T_{mean} \times \frac{115}{100}$$
$$= 13846{,}5\ \times \frac{115}{100}$$
$$= 15923{,}5\ Nm$$

$$d = \sqrt[3]{\frac{T_{max}\times16}{\pi\times\tau}}$$
$$= \sqrt[3]{\frac{15923{,}5\times16}{\pi\times54\times10^6}}$$
$$= \sqrt[3]{1{,}502\ \times\ 10^{-3}}$$
$$d = 0{,}1145$$

Use 115 mm standard-size bar

5.11 Geared shafts

It is important to understand that when two shafts are connected by a pair of gears only the SPEED and the TORQUE are changed. The power remains the same. Actually-there is a small loss of power owing to friction, but this is usually neglected in designing the shaft.

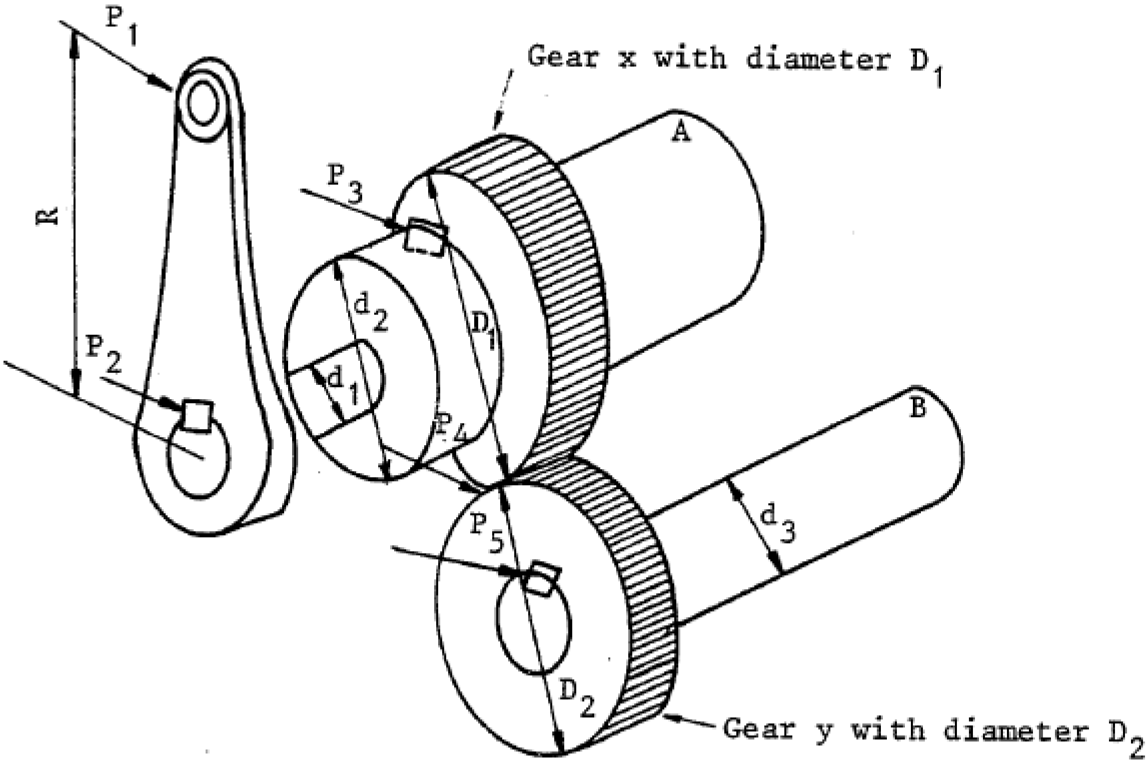


Figure 5.26

$$Torque\ on\ gear\ x = P_1 \times R = P_2 \times \frac{d_1}{2} = P_3 \times \frac{d_2}{2} = P_4 \times \frac{D_1}{2}$$

$$Torque\ on\ shaft\ B = P_4 \times \frac{d_2}{2} = P_5 \times \frac{d_3}{2}$$

$$\frac{Torque\ on\ gear\ x}{Torque\ on\ gear\ y} = \frac{P_4 \times \frac{d_1}{2}}{P_4 \times \frac{d_2}{2}} = \frac{D_1}{D_2}$$

$$\frac{Number\ of\ teeth\ in\ gear\ x}{Number\ of\ teeth\ in\ gear\ y} = \frac{r/min\ in\ gear\ y}{r/min\ in\ gear\ x}$$

$$Power\ output\ by\ shaft\ B = Power\ input\ to\ A\ x\ Efficiency$$

Worked Example 5.10

A 40 MW, 1 740 r/min electric motor is fitted with a 24 tooth pinion which drives a 60 tooth gear on a pump shaft. Find the torque on the motor shaft and on the pump shaft.

Solution:

Motor shaft

$$Power = \frac{2\,\pi\,T\,N}{60}$$

$$T = \frac{Power \times 60}{2\pi N}$$

$$= \frac{40 \times 10^6 \times 60}{2 \times \pi \times 1740}$$

$$= 219524\ Nm$$

Pump shaft

$$Speed\ of\ pump\ shaft = 1740 \times \frac{24}{60}$$

$$= 696\ r/min$$

$$Power = \frac{2\,\pi\,T\,N}{60}$$

$$T = \frac{Power \times 60}{2 \times \pi \times N}$$

$$= \frac{40 \times 10^6 \times 60}{2 \times \pi \times 696}$$

$$= 548810\ Nm$$

$$\frac{Torque\ on\ motor\ shaft}{Torque\ on\ pump\ shaft} = \frac{Teeth\ on\ motor\ pinion}{Teeth\ on\ pump\ gear}$$

$$Torque\ on\ pump\ shaft = \frac{Torque\ on\ motor\ shaft \times Teeth\ on\ pump\ gear}{Teeth\ on\ motor\ pinion}$$

$$= \frac{219524 \times 60}{24}$$

$$= 548810\ Nm$$

Worked Example 5.11

If the angle of twist in a 150 mm mild-steel shaft is to be limited to 1° in twenty diameters, determine the maximum shear stress in the material. $G = 90\ GPa$

Solution:

$$\pi\ radians = 180°$$

$$\therefore 1° = \frac{1 \times \pi}{180} = 0{,}0175\ radians$$

$$Angle\ of\ twist\ \theta = \frac{2\tau l}{dG}$$

But $l = 20d$

$$\therefore \tau = \frac{\theta \times d \times G}{2 \times 20 \times d}$$

$$= \frac{0{,}0175 \times 90 \times 10^9\ N/m^2}{2 \times 20}$$

$$= 39{,}38\ MPa$$

Worked Example 5.12

A shaft transmits 75 kW at 120 r/min. The maximum torque is forty per cent greater than the mean. Taking maximum stress allowable as 65 MPa, determine the necessary diameter for:

(i) a solid shaft
(ii) a hollow shaft whose internal diameter is half its external diameter.

Solution:

$$Power = \frac{2\,\pi\,T_{mean}\,N}{60}$$
$$T_{mean} = \frac{Power \times 60}{2 \times \pi \times N}$$
$$= \frac{75 \times 10^3 \times 60}{2 \times \pi \times 120}$$
$$T_{mean} = 5968\ Nm$$
$$T_{max} = T_{mean} \times \frac{140}{100}$$
$$= 5968 \times \frac{140}{100}$$
$$T_{max} = 8355\ Nm$$

Solid shaft

$$d = \sqrt[3]{\frac{T_{max} \times 16}{\pi \times \tau}}$$
$$= \sqrt[3]{\frac{8355 \times 16}{\pi \times 65 \times 10^6}}$$
$$= \sqrt[3]{6{,}546 \times 10^{-4}}$$
$$= 0{,}0868$$

Say 90 mm diameter standard size

Hollow Shaft

$$\frac{D^4 - d^4}{D} = \frac{T_{max} \times 16}{\pi \times \tau} \qquad \text{but } d = \frac{D}{2}$$
$$\frac{D^4 - \left(\frac{D}{2}\right)^4}{D} = \frac{T_{max} \times 16}{\pi \times \tau}$$
$$\frac{D^4 - \frac{D^4}{16}}{D} = \frac{8355 \times 16}{\pi \times 65 \times 10^6}$$
$$\frac{D^4 - 0{,}0625\,D^4}{D} = 6{,}546 \times 10^{-4}$$
$$D = \sqrt[3]{\frac{6{,}546 \times 10^{-4}}{0{,}9375}}$$
$$= \sqrt[3]{6{,}9824 \times 10^{-4}}$$
$$= 0{,}887$$

Say 89 mm outside diameter standard size

Inside diameter $= \frac{89}{2} = 44{,}5\ mm$

Worked Example 5.13

A line shaft running at 150 r/min is 15m long and is to transmit 45 kW. The shaft is of mild steel having a working shear stress of 58 MPa. The permissible torsional deflection is to be 12° and the modules of rigidity is to be 83 GPa.

Calculate a suitable diameter for this shaft.

Solution:

$$Power = \frac{2\,\pi\,T_{mean}\,N}{60}$$

$$T_{mean} = \frac{Power \times 60}{2\,\pi\,N}$$

$$= \frac{45 \times 10^3 \times 60}{2 \times \pi \times 150}$$

$$= 2865\ Nm$$

Assume $T_{mean} = T_{max}$

$$\therefore T_{max} = 2865\ Nm$$

Suitable shaft diameter
Design for strength

$$d_s = \sqrt[3]{\frac{T_{max} \times 16}{\pi \times \tau}}$$

$$= \sqrt[3]{\frac{2865 \times 16}{\pi \times 58 \times 10^6}}$$

$$= \sqrt[3]{2{,}516 \times 10^{-4}}$$

$$= 0{,}06312$$

Say $= 64\ mm$

Design for stiffness

$$d_s = \sqrt[4]{\frac{32 T_{max} l}{\pi\ G\ \theta}} \qquad \theta = 12°$$

$$= \sqrt[4]{\frac{32 \times 2865 \times 15}{\pi \times 83 \times 10^9 \times 0{,}2094}} \qquad = \frac{12° \times \pi}{180°}$$

$$= \sqrt[4]{2{,}5186 \times 10^{-5}} \qquad = 0{,}2094\ radians$$

$$= 0{,}0708$$

Say $= 70\ mm$

Therefore a 70 mm-diameter standard-size shaft will be used.

Worked Example 5.14

A hollow shaft with a ratio of inside diameter to outside diameter of 1 to 2, is required to transmit 600 kW at 130 r/min. The maximum torque exceeds the mean torque by 15%. The shear stress is not to exceed 60 MPa and the twist in a length of 4 metres is not to exceed 1°.

$G = 80\,GPa$

Calculate suitable outside and inside diameters.

Solution:

$$Power = \frac{2\,\pi\,T_{mean}\,N}{60}$$
$$T_{mean} = \frac{Power \times 60}{2\times\pi\times N}$$
$$= \frac{600\times10^3\times60}{2\times\pi\times130}$$
$$= 44074\,Nm$$
$$T_{max} = 44074\times\frac{115}{100}$$
$$= 50685\,Nm$$

Suitable shaft diameters
Design for strength

$$\frac{D^4-d^4}{D} = \frac{T_{max}\times16}{\pi\times\tau} \qquad \frac{d}{D}=\frac{1}{2}$$
$$\frac{(2d)^4-d^4}{2d} = \frac{50685\times16}{\pi\times\tau} \qquad \therefore D = 2d$$
$$\frac{16d^4-d^4}{2d} = 4{,}3023\times10^{-3}$$
$$7{,}5d^3 = 4{,}3023\times10^{-3}$$
$$d = \sqrt[3]{\frac{4{,}3023\times10^{-3}}{7,}}$$
$$d = 0{,}083\,m$$
$$D = 2\times0{,}083$$
$$= 0{,}166\,m$$

Design for stiffness

$$(D^4-d^4) = \frac{32\,T_{max}l}{\pi G\theta} \qquad \theta = 1°$$
$$(2d)^4-d^4 = \frac{32\times50685\times4}{\pi\times80\times10^9\times0{,}0175} \qquad = \frac{1°\times\pi}{180°}$$
$$15d^4 = 1{,}4751\times10^{-3} \qquad = 0{,}0175\,radians$$
$$d = \sqrt[4]{\frac{1{,}4751\times10^{-3}}{15}}$$
$$d = 0{,}0996\,m \quad \text{(Say 100 mm)}$$
$$D = 2\times100 = 200\,mm$$

Therefore use a hollow shaft with inside diameter = 100 mm and outside diameter 200 mm.

Worked Example 5.15

A hollow propeller shaft for an automobile having an engine developing a maximum power of 80 kW at 1 500 r/min is driven through a gear-box with a low gear ratio of 3:1. The driving efficiency is 90%. The shear stress in the shaft material must not exceed 50 MPa.

Calculate:
i. the torque transmitted by the engine
ii. the power at the propeller shaft
iii. the speed of the propeller shaft
iv. the torque transmitted by the propeller shaft
v. the inside and outside diameters of the propeller shaft, assuming that the internal diameter is 0,9 times the external diameter.

Solution:

i) Engine torque

$$Power = \frac{2\,\pi\,T_{mean}\,N}{60}$$

$$T_{mean} = \frac{Power \times 60}{2 \times \pi \times N}$$

$$= \frac{80 \times 10^3 \times 60}{2 \times \pi \times 1500}$$

$$= 509{,}3\ Nm$$

ii) Power output by propeller shaft

$$Power\ output\ by\ propeller\ shaft = Power\ output\ by\ engine\ x\ Efficiency$$

$$= 80 \times 10^3 \times \frac{90}{100}$$

$$= 72\ kW$$

iii) Propeller shaft speed

$$Speed\ of\ propeller\ shaft = Engine\ speed \times \frac{1}{3}$$

$$= \frac{1500}{3}$$

$$= 500\ r/min$$

iv) Torque transmitted by propeller shaft

$$T_{mean} = \frac{Power \times 60}{2 \times \pi \times N}$$

$$= \frac{72 \times 10^3 \times 60}{2 \times \pi \times 500}$$

$$= 1375\ Nm$$

v) Propeller shaft diameters

Assume $T_{mean} = T_{max} = 1375\ Nm$

Design for strength

$$\frac{D^4 - d^4}{D} = \frac{T_{max} \times 16}{\pi \times \tau} \qquad d = 0{,}9\ D$$

$$\frac{D^4 - (0{,}9d)^4}{D} = \frac{1375 \times 16}{\pi \times 50 \times 10^6} \qquad \therefore D = 2d$$

$$\frac{D^4 - 0{,}656 D^4}{D} = 1{,}401 \times 10^{-4}$$

$$0{,}344 D^3 = 1{,}401 \times 10^{-4}$$

$$D = \sqrt[3]{\frac{1{,}401\times10^{-4}}{0{,}344}}$$

$$D = 0{,}074\ m$$ (Say 76 mm standard size)

$$d = 0{,}9\times 76\ mm$$

$$= 68{,}4\ m$$

Worked Example 5.16

Find the maximum value of torsional shear stress in a 70 mm-diameter shaft transmitting 38 k W at 80 r/min, if the maximum twisting moment exceeds the mean by 40 per cent. What is the greatest twis in degrees per metre length if the modulus of rigidity is 83 MPa?

Solution:

$$T_{mean} = \frac{Power\times60}{2\ \pi\ N}$$

$$= \frac{38\times10^3\ \times60}{2\times\pi\times80}$$

$$= 4536\ Nm$$

$$= 4{,}536\ kNm$$

$$T_{(max)} = T_{(mean)}\times\frac{140}{100}$$

$$T_{(max)} = T_{(mean)}\times 1{,}4$$

$$= 4536\times1{,}4$$

$$= 6{,}35\ kNm$$

$$\tau = \frac{16\times T_{max}}{\pi D^3}$$

$$= \frac{16\times6350\ Nm}{\pi\times(0{,}07)^3 m^3}$$

$$= 94{,}3\ MPa$$

$$Angle\ of\ twist\ \theta = \frac{2\tau l}{dG}$$

$$= 20d$$

$$= \frac{2\times94{,}3\times10^6\ \times1}{0{,}7\times83\times10^9}\ radians$$

$$= 0{,}03246\ radians$$

$$\theta = \frac{0{,}03246\times180^\circ}{\pi}\ degrees$$

$$= 1{,}86^\circ$$

Worked Example 5.17

A hollow steel shaft is required to transmit a torque of 7 960 Nm with a maximum shear stress not exceeding 41 MPa. If the outer diameter is to be twice the inner diameter, calculate the two diameters. If the shaft is 6,1 m long and the modulus of rigidity is 83 GPa, calculate the angle of twist.

Solution:

$$T = \frac{\pi\tau}{16}\left[\frac{D^4-d^4}{D}\right]$$

$$7960\ Nm = \frac{\pi\tau}{16}\left[\frac{(2d)^4-d^4}{2d}\right]$$

$$7960\ Nm = 0{,}196 \times 41 \times 10^6\ N/m^2 \times 7{,}5\ d^3$$

$$d^3 = \frac{7960\ Nm}{0{,}196 \times 41 \times 10^6\ N/m^2 \times 7{,}5}$$

$$= 0{,}0509\ m$$

$$\therefore d = 50\ mm$$

$$D = 2d$$

$$= 2 \times 50$$

$$D = 100\ mm$$

Angle of twist

$$\frac{T_{max}}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

$$\theta = \frac{\tau l}{rG} \qquad r = \frac{D}{2}$$

$$= \frac{2\tau l}{GD}$$

$$= \frac{2 \times 41 \times 10^6 \times 6{,}1}{83 \times 10^9 \times 0{,}1}$$

$$= 0{,}0603\ radians$$

$$\theta = \frac{0{,}0603 \times 180}{\pi}\ degrees$$

$$= 3{,}46°$$

Worked Example 5.18

Determine the percentage that a hollow shaft of outside diameter 250 mm and inside diameter 125 mm is stronger in torsion than a solid shaft having the same mass per metre run.

Solution:

(Cross-sectional areas must be equal)

$$\frac{\pi d_s^2}{4} = \frac{\pi}{4}\left(D^2 - {d_1}^2\right)$$

$$\therefore d_s^2 = D^2 - {d_1}^2$$

$$= 250^2 - 125^2$$

$$= 46875$$

$$\therefore d_s = \sqrt{46875} = 216{,}5\ mm\ diam$$

Torque on solid shaft

$$T = \frac{\pi {d_s}^3}{16}\tau$$

$$= \frac{\pi \times 0{,}2165^3}{16} \times \tau$$

$$= 0{,}001993\ \tau$$

Torque on hollow shaft

$$T = \frac{\pi\tau}{16}\left[\frac{D^4-d^4}{D}\right]$$

$$= \frac{\pi\tau}{16}\left[\frac{0{,}25^4-{,}125^4}{0{,}25}\right]\tau$$

$$= 0{,}00288\tau$$

The hollow shaft is $= \left\{\frac{0{,}00288\tau-0{,}001993\tau}{0{,}001993\tau}\right\} \times 100$

$$= 44{,}51\%$$

Worked Example 5.19

1 500 kW at 210 r/min is to be transmitted by a hollow nickel steel shaft whose outside diameter is 1,8 times the inner diameter. If the shaft is subjected to torsion only, determine:

(a) the size of the hollow shaft required, assuming an allowable stress of 200 MPa

(b) the size of the solid shaft for the above conditions, using an allowable stress of 170 MPa

(c) the percentage saving in mass of the hollow shaft over the solid one.

Solution:

a)

$$T = \frac{Power\times 60}{2\times\pi\times N}$$

$$\therefore T = \frac{1500\times 60\times 10^3}{2\times\pi\times 210}$$

$$= 68\ kNm\ and\ for\ hollow\ shaft$$

$$T = \left(\frac{\pi}{16}\frac{D^4-d^4}{D}\right)\tau$$

$$\therefore 68\times 10^3 = \frac{\pi\times 200\times 10^6}{16}\left(\frac{1{,}8d^4-d^4}{1{,}8d}\right)$$

$$\frac{9{,}5\ d^3}{1{,}8} = 5{,}27\ d^3 = \frac{68\times 10^3\times 16}{\pi\times 200\times 10^6}$$

$$\therefore d^3 = \frac{68\times 10^3\times 16}{5{,}27\times\pi\times 200\times 10^6}$$

And $d = 0{,}069\ m = 70\ mm$

$\therefore internal\ diameter = 70\ mm$

And $eternal\ diameter = 1{,}8\times 70$

$= 126\ mm$

b) For solid shaft

$$T = \frac{\pi d^3}{16} f_s$$

$$\therefore d^3 = \frac{T\times 16}{\pi\times f_s}$$

$$\therefore d^3 = \frac{68\times 10^3\times 16}{\pi\times 170\times 10^6}$$

And $d = 0{,}127\ m$

$= 0{,}127\ mm$

(c) For equal lengths

$$Cross-sectional\ area\ of\ hollow\ shaft = \frac{\pi}{4}(126-70)(126+70)$$
$$= 8600\ mm^2$$
$$Cross-sectional\ area\ of\ solid\ shaft = \frac{\pi}{4} \times 127^2$$
$$= 12650\ mm^2$$

$$Percentage\ saving = \left\{\frac{A_s - A_h}{A_s}\right\} \times 100$$
$$= \left\{\frac{12650-8600}{12650}\right\} \times 100$$
$$= 32\%$$

5.12 Forces acting on parts of driving mechanism of steam and similar engines

The driving mechanism of a steam engine is shown in **Figure 5.27**.

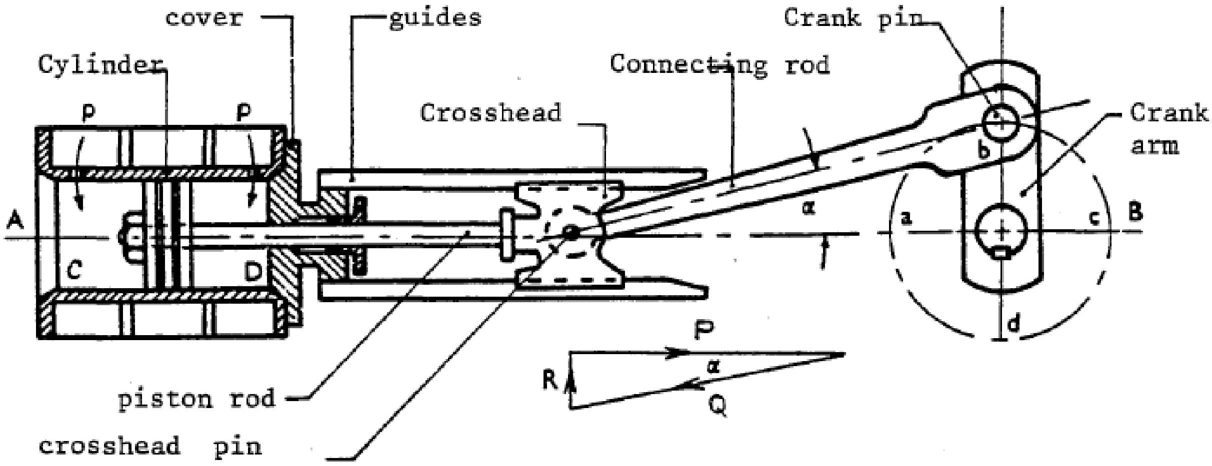


Figure 5.27

The force P on the piston brings the piston and piston rod into motion, and they in turn, bring the crosshead on the piston rod line AB into motion. The motion direction of the crosshead is laid down by the guides.

The connecting rod hinges at the crosshead pin and drives the crank pin in a circular path a b c d.

Steam is admitted in turn at the front end and back end of the cylinder - into the front to move the piston from C to D and into the back to move the piston from D to C. During the stroke from c to D, a compressive stress is set up in the piston rod, while on the return stroke from D to C, a tensile stress is set up in the piston rod.

Note:
With each stroke, a crushing stress and a shear stress are set up in the crosshead pin and the crank pin.

$$Force\ on\ piston = Steam\ pressure \times Cross - sectional\ area\ of\ piston$$

$$P = p_i \times \frac{\pi D^2}{4}$$

$$Force\ on\ piston\ at\ piston\ rode\ side\ (back\ end) = Steam\ pressure \times (Cross - sectional\ area\ of\ piston\ rod)$$

$$P = p_i \times \left(\frac{\pi D^2}{4} - \frac{\pi d^2}{4}\right)$$

5.13 Crossheads

Two types of crossheads are shown in **Figure 5.28**. Their function is to transmit the side thrust due to the obliquity of the connecting rod on to the guide and so prevent the piston rod from being bent

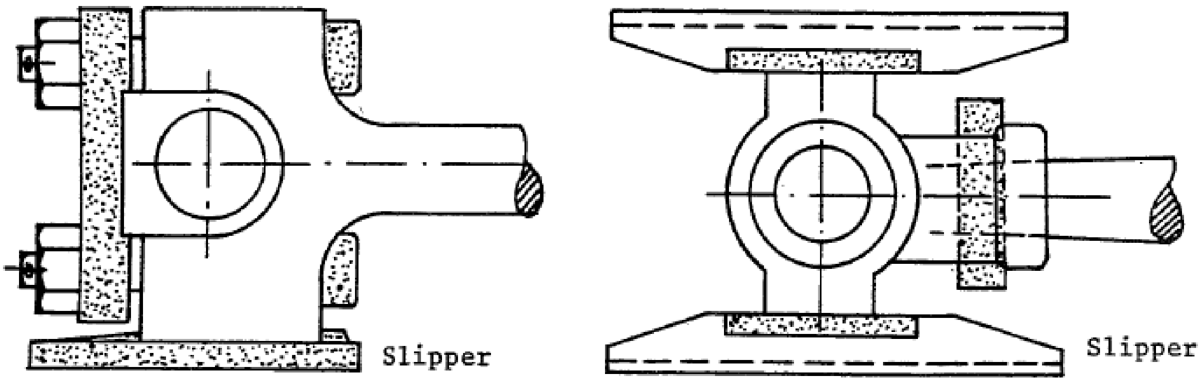


Figure 5.28

As the crosshead forms a swivel for the connecting rod, the joint must therefore be in the form of a knuckle joint.

Either the crosshead is the eye and the connecting rod the fork (**Figure 5.29**) or the crosshead is the fork and the connecting rod the eye (**Figure 5.30**).

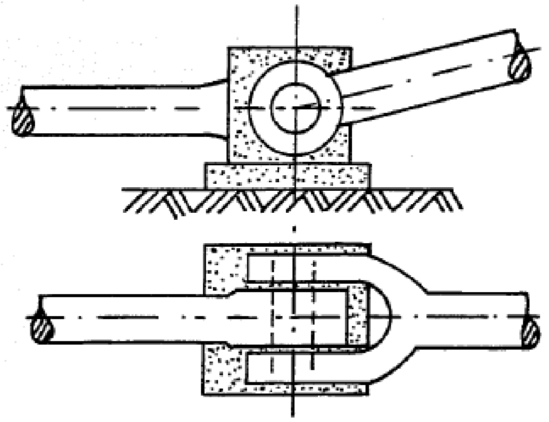


Figure 5.29

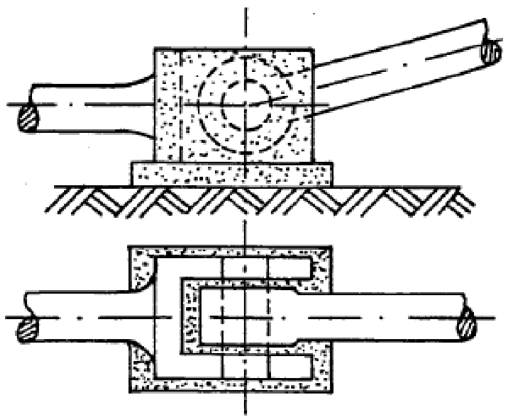


Figure 5.30

There are three forces acting on the crosshead pin, as will be seen from **Figure 5.31** which shows diagrammatically the piston rod, crosshead, connecting rod, etc of a steam engine.

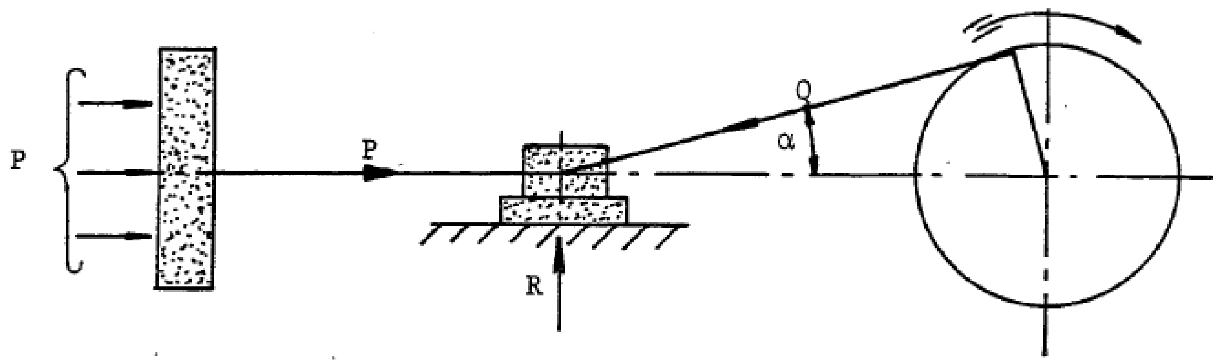


Figure 5.31

There are

- the horizontal force P along the piston rod
- a vertical reaction R at the guides
- the thrust Q which is acting along the connecting rod. From the initial steam pressure and area of piston, the force P may be calculated, and since the lines of action of all the forces are known, the triangle of forces (**Figure 5.32**) may be drawn.

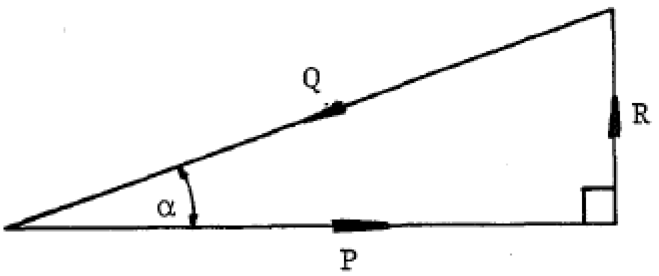


Figure 5.32

⚠	**Note:** Note that whatever angle α the connecting rod makes with the horizontal, (**Figure 5.31**) a corresponding angle α is made in **Figure 5.32**.

The force Q and reaction R may now be calculated, or measured off to scale.

The direction of the arrows for Q and R may not be clear. Remember, we are considering the forces acting on the crosshead. The crosshead presses downwards on to the guide, but the reaction of the guide on the crosshead is upwards.

Similarly, the crosshead pin pushes along the connecting rod, but the resistance is back along the rod on to the pin.

An examination of the following diagrams shows that, if P remains constant, then Q and R both increase to a maximum.

Actually P remains constant only up to the position of cut-off, which usually occurs before this maximum position of the crank is reached.

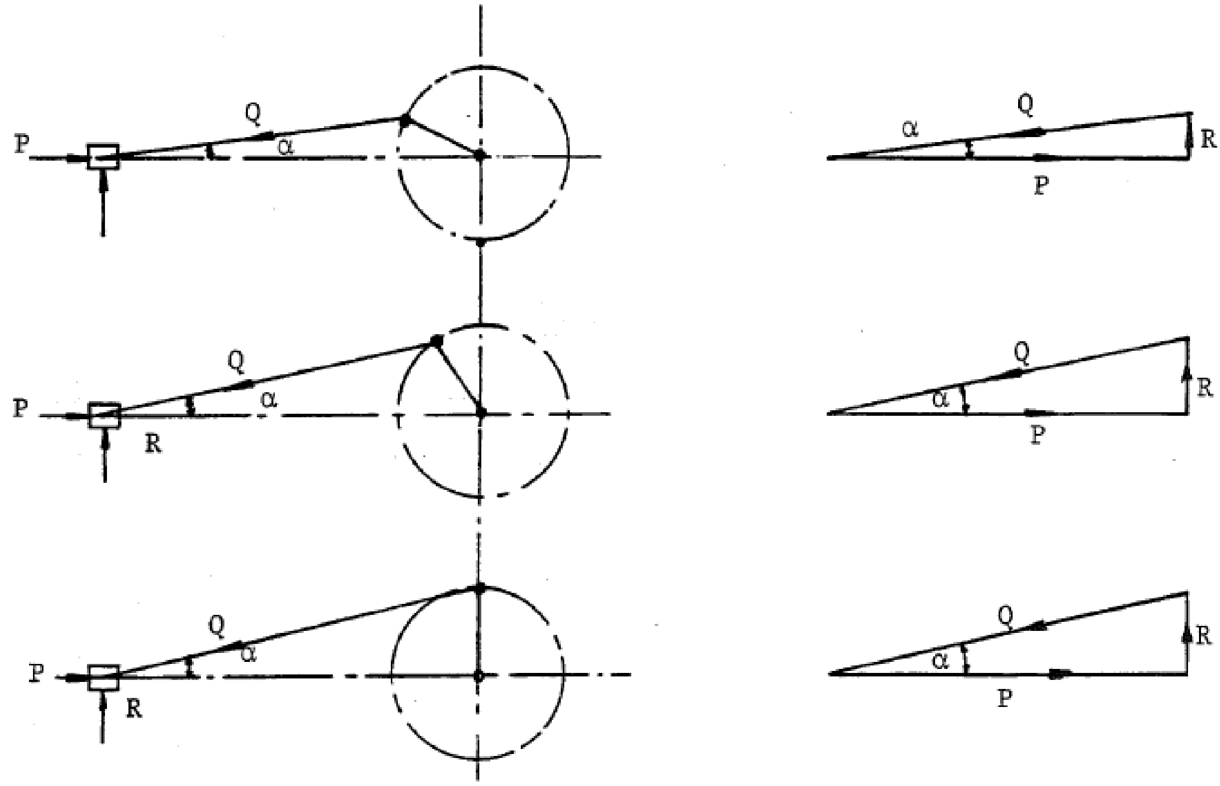


Figure 5.33

5.13.1 Maximum Pressure on Guides
5.13.1.1 Steam Engine

It is usual to speak of the length of any connecting rod in terms of the radius of the crank.

Let:- n = ratio of length of connecting rod to that of the crank.
L = length of connecting rod
r = radius of crank.

Then:- $n = \frac{L}{r}$ or $\frac{1}{n} = \frac{r}{L}$

Maximum pressure on the guides occurs when the connecting rod and crank are at 90° to each other, provided that steam is cut off from behind the piston after $\frac{5}{8}$ of the stroke.

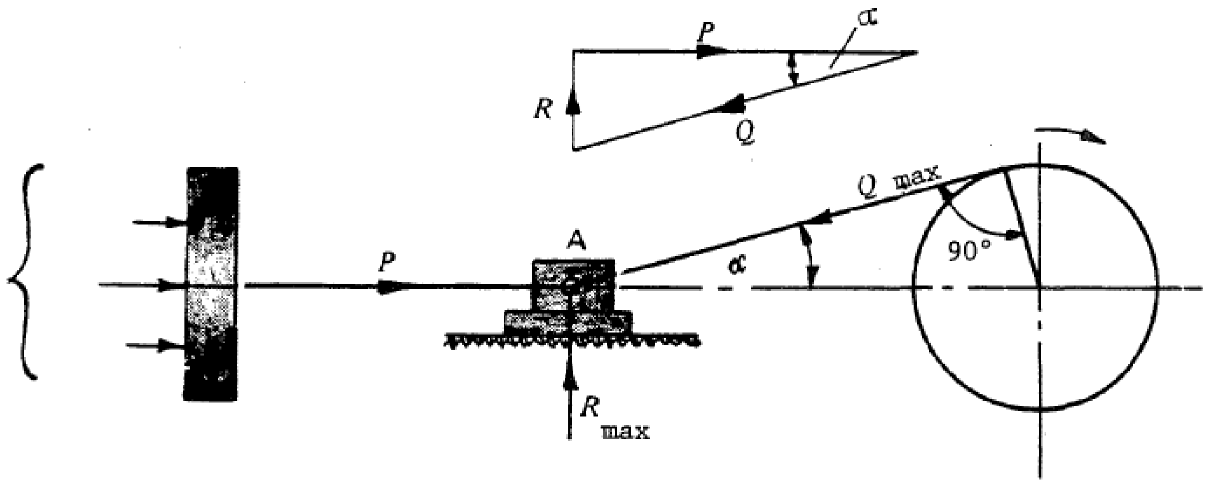


Figure 5.34

Maximum pressure on guides (R_{max}) = $\frac{P}{\sqrt{n^2-1}}$

This formula indicates that if n ($\frac{L}{r}$) is decreased, then the value of R_{max} increases. It is, therefore, necessary to make the ratio $\frac{L}{r}$ as large as possible by employing a long connecting rod. Connecting rods for steam engines are usually made not less than 4 to 5 cranks in length.

The maximum force Q_{max} in the connecting rod is given by $\frac{P \times n}{\sqrt{n^2-1}}$.

The only design calculation that can be made for a crosshead is the area of the slipper (guide), the size of the pin and the cotter joint for the piston rod, if any.

The latter has already been considered.

5.13.2 Area of slippers

The bearing area of the top or bottom slipper (in square metres) is thus:-

$$Area\ of\ slipper = \frac{R_{max}}{Safe\ bearing\ pressure\ (p_b)}$$

The intensity of the safe bearing pressure between the sliding surfaces of the crosshead slippers and guides depends principally upon the metals of which these surfaces are made and also upon the sliding speeds.

⚠	**Note:** Pressure on slippers can be taken between 200 kPa and 700 kPa. Note that these values assume that the surfaces will be properly lubricated.

The ratio of the length (t) to breadth (b) of the surface of the slipper is left to the discretion of the designer, but a fairly good average value might be:-

$$l = 2{,}5 \times b$$

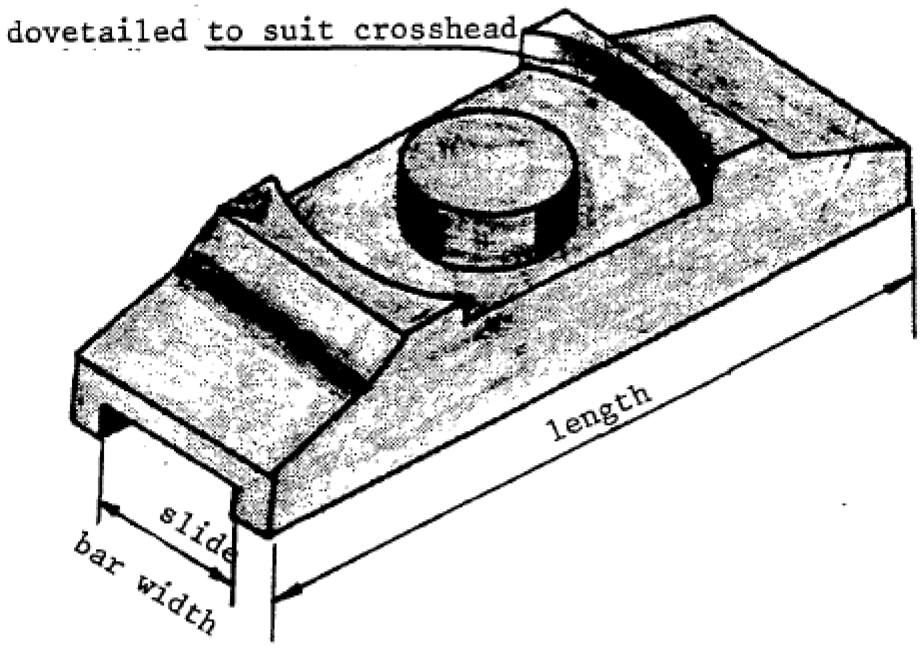


Figure 5.35

In all cases, the centre of the crosshead shoe should pass through the centre of the gudgeon pin.

5.13.3 Size of the gudgeon pin

The pin is designed for wear and not strength - that is, on bearing pressure and not on shear or bending, but this may be checked.

The bearing surface (also called a journal) for a crosshead pin may be supported as at **Figure 5.36** or as at **Figure 5.37**.

M = $Maximum\ bending\ moment$

= $\frac{Q_{max} \times l}{8}$

Also

M = $\sigma_b \times Z,$

Where σ_b = $safe\ bending\ stress$ and

Z = $section\ modulus$

Solid pin

$$Z = \frac{\pi d_e{}^3}{32}$$

Hollow pin

$$Z = \frac{\pi}{32}\left\{\frac{d_e^{\,4} - d_i^{\,4}}{d_e}\right\}$$

The length (l) of the gudgeon pin is made equal to its external diameter or possibly up to 1,3 times its external diameter.

5.14 Design of the crankshaft

The crankshaft is made of forged steel, and is located in the crankcase directly below the cylinders. The bearing surfaces on the shaft are accurately ground to size, and the entire shaft is delicately and accurately balanced.

Note:
The crankshaft is supported in the crankcase by bearings known as main bearings. Each main bearing fits on a main bearing journal.

The purpose of the crankshaft is to change the reciprocating motion of the piston in the cylinder to a rotary motion.

5.14.1 Types of cranks

5.14.1.1 Overhang crank (case 1)

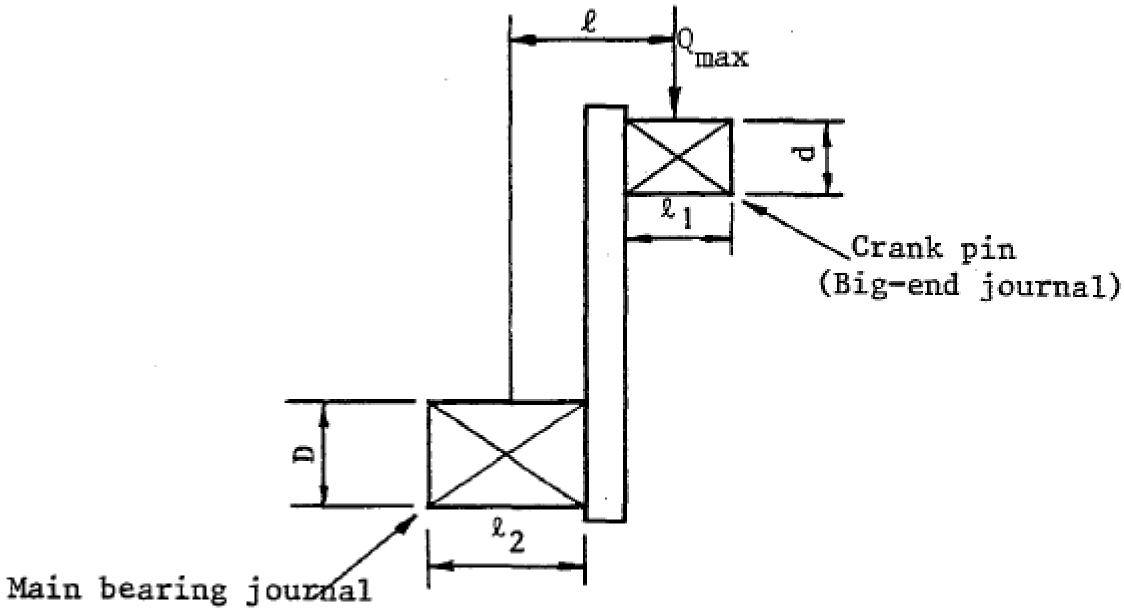


Figure 5.36

5.14.1.2 Centre crank (case 2)

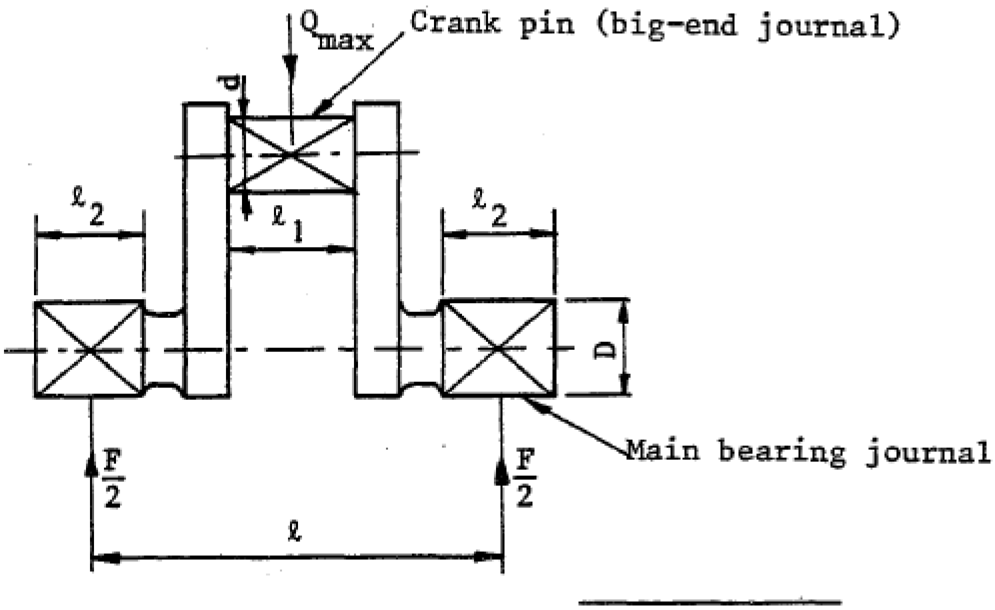


Figure 5.37

5.141.3 Off-centre crank (case 3)

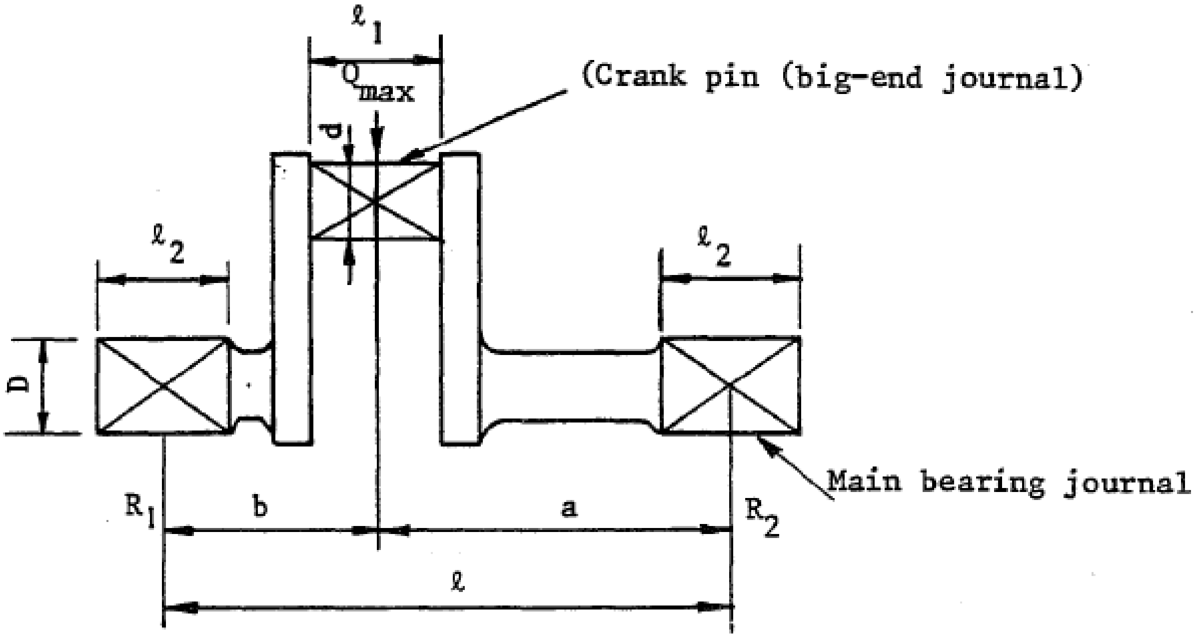


Figure 5.38

5.14.2 Design of the crank pin (Big-end journal)

The size of the crank pin is usually estimated from the safe bearing pressure, which varies between 4,1 MPa to 13,8 MPa.

The length (l_1) of the crank pin can be taken between 1d to 1,25d.

$$Force = Bearing\ pressure \times projected\ area$$
$$Q_{max} = P_b \times l_1 \times d$$

5.14.3 Design of the main bearing journal

The diameter of the journal may be designed for:

- twisting moment
- bending moment
- bearing pressure

- Twisting moment

  We have already seen that the effective force on the piston is transmitted through the connecting rod to the crank pin, thus causing a torque or twisting moment at the crankshaft.

> **Note:**
> There exist a number of twisting-moment factors whose usefulness is questionable.

The torque produced is the product of the force acting along the connecting rod and the crank arm when it is perpendicular to the connecting rod.

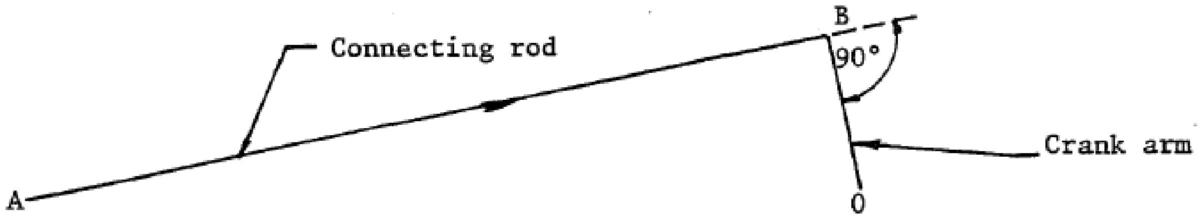


Figure 5.39

$$Maximum\ Torque = Maximum\ force\ in\ connecting\ rod \times\ crank\ radius$$
$$T_{max} = Q_{max} \times r$$

Torque can also be found from the brake power of the engine.

$$Brake\ power = \frac{2\pi T_{mean} N}{60}$$
$$T_{mean} = \frac{Brake\ power \times 60}{2\pi N}$$
$$T_{max} = T_{mean} + \%\ over\ load$$

Now

$$T_{max} = \frac{\pi}{16} D^3 \times \tau$$
$$D = \sqrt[3]{\frac{T_{max} \times 16}{\pi \times \tau}}$$

- Bending moment in main bearing journals

$$M = Maximum\ bending\ moment$$
$$M\ for\ overhung\ crank\ (case\ 1) = Q_{max} \times l$$

$$M\ for\ centre\ crank\ (case\ 2) = \frac{Q_{max} \times l}{4}$$

$$M\ for\ off-centre\ crank\ (case\ 3) = \frac{Q_{max} \times a \times b}{l}$$

$$Z = section\ modulus$$

$$Z\ for\ solid\ shaft = \frac{\pi D^3}{32}$$

$$Z\ for\ hollow\ shaft = \frac{\pi}{32}\left(\frac{D^4 - {d_i}^4}{D}\right)$$

Also

$$M = \sigma_b \times z,$$

Where $\sigma_b = safe\ bending\ stress$

- Bearing pressure

$$Force = bearing\ pressure \times\ projected\ area$$

$$Force = p_b \times D \times l_2$$

Where:

$$force\ for\ overhung\ crank = Q_{max}$$

$$force\ for\ centre\ crank = \frac{Q_{max}}{2}$$

$$force\ for\ off-centre\ crank = largest\ of\ two\ reactions$$

Safe bearing pressure can be taken between 1,7 MPa to 5,5 MPa.

The length (l_2) of the main bearing can be taken between 1,25 D to 2D.

5.15 Internal combustion engine

In an internal combustion engine there is no crosshead, the side thrust being taken by the piston pressing on the cylinder wall. The thrust in the connecting rod and the reaction on the cylinder wall are found in the same way as for a steam engine.

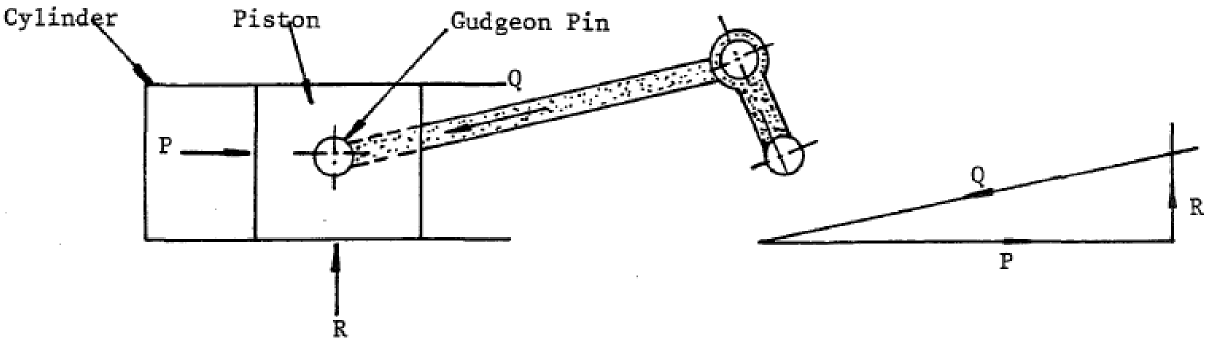


Figure 5.40

Note:
Because the cylinder pressure in an internal combustion engine drops considerably during the early portion of the expansion stroke,

	the maximum force (Q_{max}) in the gudgeon pin, connecting rod and crankshaft may be taken as P where P is the maximum explosion force on the piston.

Worked Example 5.20

A single-acting steam engine has a cylinder bore of 380mm and a stroke of 450mm. The connecting rod is 680mm long. The steam pressure is 690 kPa, assumed constant until late in the stroke.

Determine the thrust in the correcting rod and the reaction at the crosshead guide when:
a) the crank is 30° from dead centre
b) when the crank is at right angles to the line of stroke.

Solution:

$$Force\ on\ piston = Steam\ pressure \times Cross-sectional\ area\ of\ piston$$
$$P = p_i \times \frac{\pi D^2}{4}$$
$$= 690 \times 10^3 N/m^2 \times \frac{\pi(0{,}38\ m)^2}{4}$$
$$= 78254\ N$$
$$P = 78{,}254\ kN$$

Choosing a suitable scale, we draw the crank and connecting rod in the two positions A and B.

Note that the length of the crank arm is half the stroke

$$Length\ of\ crank\ arm = \frac{Stroke}{2}$$
$$= \frac{450\ mm}{2}$$
$$= 225\ mm$$

A) Scale 1mm = 10mm

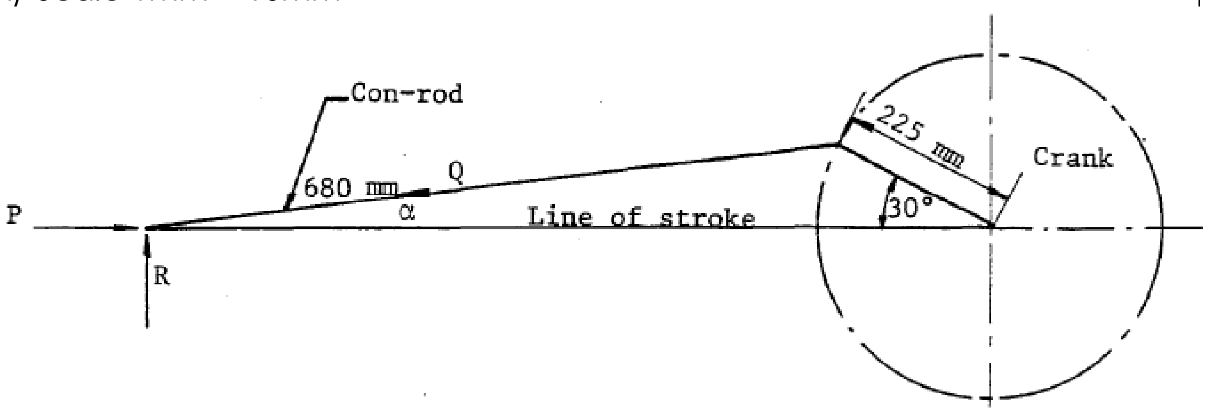


Figure 5.41

Choosing a suitable scale, we now draw the triangle of forces, R being perpendicular and Q being parallel to the connecting rod. Rand Q are then found by measurement or calculation.

Scale 1mm = 1 kN

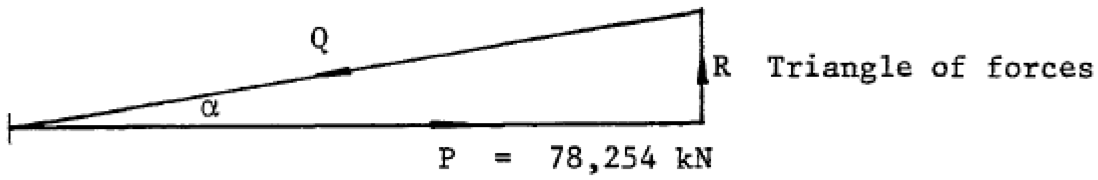


Figure 5.42

By measurement $Q = 79\,mm = 79 \times 1\,kN = 79\,kN$
$R = 14\,mm = 14 \times 1\,kN = 14\,kN$
$\alpha = 10°$

B) Scale 1mm = 10 mm

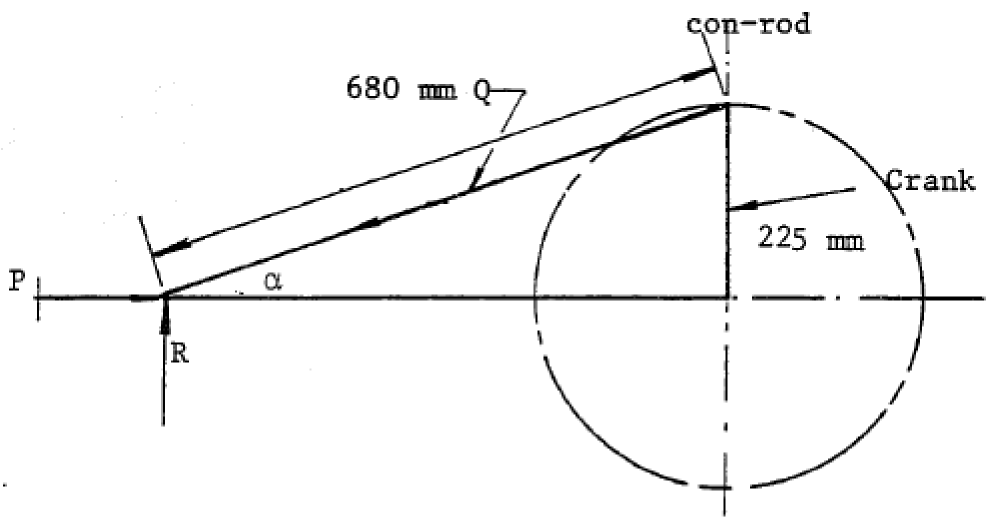


Figure 5.43

Scale 1 mm = 1 kN

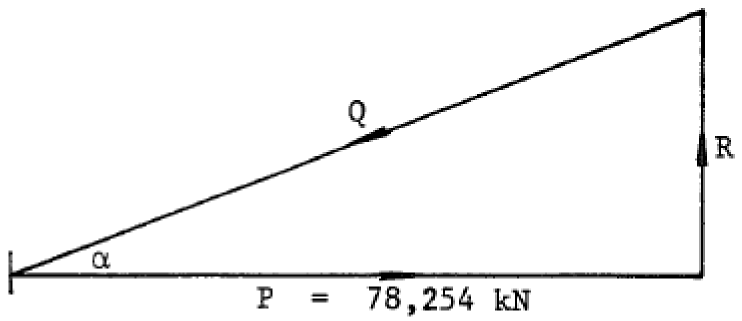


Figure 5.44

By measurement $Q = 85\,mm = 85 \times 1\,kN = 85\,kN$
$R = 31\,mm = 31 \times 1\,kN = 31\,kN$

$\alpha = 20°$

Note that the scales used in the diagrams are far too small for accuracy. You should use a much larger scale. More accurate results may be obtained using trigonometry.

Worked Example 5.21

The total force behind the piston of a steam engine is 280 kN.

Calculate:
1. the reaction R provided by the guide.
2. the magnitude of the compressive force along the connecting rod.

The connecting rod makes an angle (α) of 20 degrees with the axis of the cylinder.

Solution:

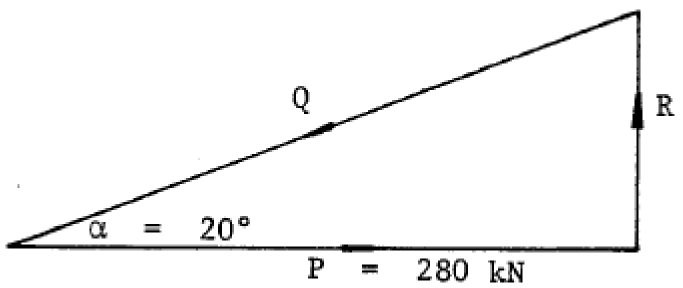


Figure 5.45

1. Reaction (R)

$$\tan\alpha = \frac{R}{P}$$
$$\therefore R = P\tan\alpha$$
$$= 280 \times 10^3 N \times \tan 20°$$
$$= 101{,}9\ kN$$

2. Force in connecting rod (Q)

$$\cos\alpha = \frac{P}{Q}$$
$$Q = \frac{P}{\cos\alpha}$$
$$= \frac{280 \times 10^3 N}{\cos 20°}$$
$$Q = 298\ kN$$

Worked Example 5.22

Particulars for a vertical petrol engine are:

- Piston diameter 90 mm
- stroke 114 mm
- length of connecting rod 200 mm
- gudgeon pin 20 mm in diameter
- length of small end bearing = 25 mm
- crank pin (big end) = 45 mm in diameter and 38 mm long.
- Gas pressure on piston = 1,1 MPa at the position when the crank is at right angles to the line of stroke

Calculate:

1. the force in the connecting rod for the given position
2. the reaction between the piston and the cylinder wall for the given position
3. the pressure on the top and bottom end bearings of the connecting rod

Solution:

$$Force\ on\ piston = Gas\ pressure\ on\ piston \times\ Cross - sectional\ area\ of\ piston$$

$$P = p_i \times \frac{\pi D^2}{4}$$

$$= 1{,}1 \times 10^6 N/m^2 \times \frac{\pi(0{,}09\ m)^2}{4}$$

$$= 6998\ N$$

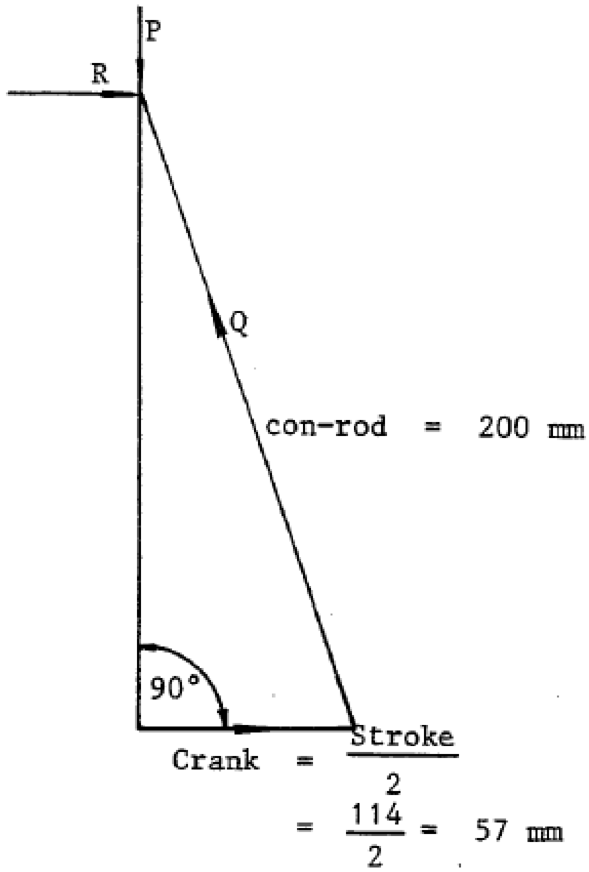


Figure 5.46

To find α

$$sin\ \alpha = 57\ mm$$

$\alpha = 16{,}56°$

1. Force in connecting rod

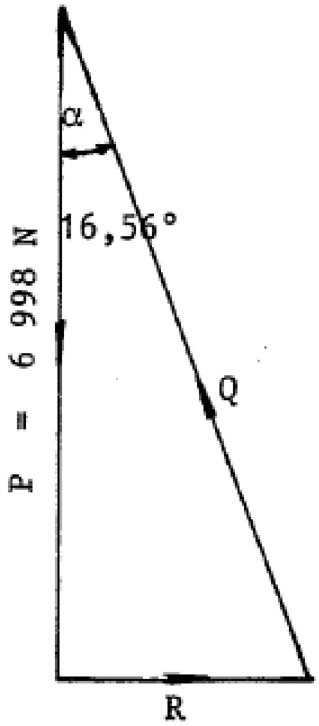


Figure 5.47

$$\cos\alpha = \frac{P}{Q}$$
$$Q = \frac{P}{\cos\alpha}$$
$$= \frac{6998N}{\cos 16{,}56°}$$
$$Q = 7301\ N$$

2. The reaction between the piston and cylinder wall

$$\tan\alpha = \frac{R}{P}$$
$$\therefore R = P\tan\alpha$$
$$= 6998 \times \tan 16{,}56°$$
$$= 2081\ N$$

3. Pressure on con-rod bearings
   3.1 Top end (small end on gudgeon pin)

$$Bearing\ pressure = \frac{Force}{Projected\ Area\ of\ gudgeon\ pin}$$
$$P_b = \frac{Q}{l \times d_e}$$
$$= \frac{7301\ N}{0{,}025\ m \times 0{,}020\ m}$$
$$= 14{,}6\ MPa$$

   3.2 Bottom end (crank pin bearing)

$$Bearing\ pressure = \frac{Force}{Projected\ Area}$$

$$P_b = \frac{Q}{l_1 \times d}$$
$$= \frac{7301\ N}{0{,}038\ m \times 0{,}045\ m}$$
$$= 4{,}27\ MPa$$

Worked Example 5.23

In the same engine as in **Worked Example 5.22**, determine the thrust in the connecting rod and the side force on the piston when the connecting rod is at right angles to the crank. The gas pressure for this position is 1,4 MPa.

Solution:

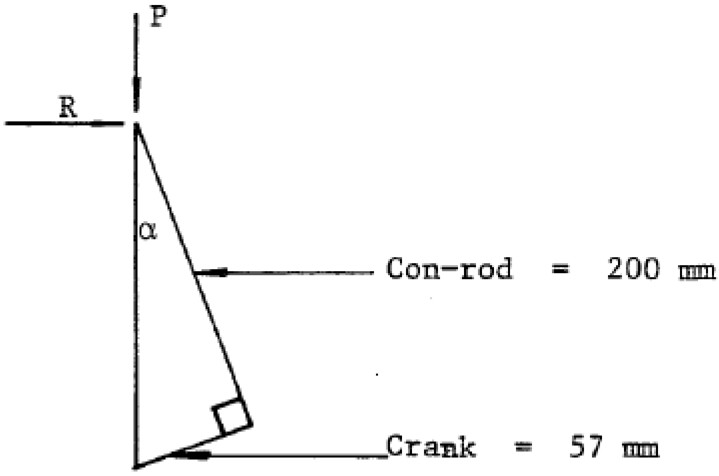


Figure 5.48

$$\tan\alpha = \frac{57}{200}$$
$$\alpha = 15{,}91°$$

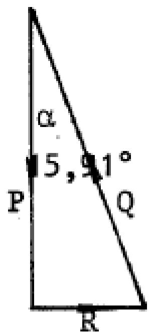


Figure 5.49

$$Force\ on\ piston = Gas\ pressure\ on\ piston \times\ Cross - sectional\ area\ of\ piston$$
$$P = p_i \times \frac{\pi D^2}{4}$$
$$= 1{,}4 \times 10^6 N/m^2 \times \frac{\pi(0{,}09\ m)^2}{4}$$
$$= 8906\ N$$

Force in the connecting rod

$$\cos\alpha = \frac{P}{Q}$$

$$Q = \frac{P}{\cos\alpha}$$

$$= \frac{8906\ N}{\cos 15{,}91°}$$

$$Q = 9261\ N$$

Force reaction between the piston and cylinder wall

$$\tan\alpha = \frac{R}{P}$$

$$\therefore R = P\tan\alpha$$

$$= 8906 \times \tan 15{,}91°$$

$$= 2539\ N$$

Worked Example 5.24

The maximum load behind the piston of a steam engine is 280 000 N.

Determine the diameter and the length of bearing surface for the solid steel gudgeon pin for the crosshead of this engine. The following assumptions will be made:

length of gudgeon pin	= 1,3 x diameter of gudgeon pin
Ratio n	= 5
permissible bearing pressure	= 8 300 kN/m²
safe bending stress for steel	= 48 000 kN/m²

Solution:

Design gudgeon pin for wear

$$Q_{max} = \frac{P \times n}{\sqrt{n^2-1}}$$

$$= \frac{280 \times 10^3 \times 5}{\sqrt{25-1}}$$

$$= 285{,}77\ kN$$

$$Q_{max} = P_b \times l \times d_e$$

$$285{,}77 \times 10^3 = 8{,}3 \times 10^6 \times 1{,}3d_e \times d_e$$

$$d_e = \sqrt{\frac{285{,}77 \times 10^3}{1{,}3 \times 8{,}3 \times 10^6}}$$

$$= 0{,}1627$$

Say $163\ mm$ in diameter, and length equals $1{,}3 \times 163\ mm = 212\ mm$

Check gudgeon pin for bending strength

$$M = \sigma_b \times Z$$

$$\frac{Q_{max} \times l}{8} = \sigma_b \times \frac{\pi d^3}{32}$$

$$\sigma_b = \frac{Q_{max} \times l}{8} \times \frac{32}{\pi d^3}$$
$$= \frac{285,77 \times 10^3\ N \times 0,212\ m}{8} \times \frac{32}{\pi \times (0,163\ m)^3}$$
$$= 17,81\ MPa$$

This is safe, since the permissible stress is 48 MPa.

Worked Example 5.25

The total load P behind the piston of a steam engine is 280 000 N. Find the length l and breadth b of the guide shoe.

Assume that $l = 2,5 \times b$, and take the safe bearing pressure between the sliding surface to be 520 kN/m² The connecting rod is 5 times the radius of the crank.

Solution:

$$R_{max} = \frac{P}{\sqrt{n^2 - 1}}$$
$$= \frac{280 \times 10^3 N}{\sqrt{25 - 1}}$$
$$= 57,5\ kN$$

$$Area\ of\ slipper = \frac{R_{max}}{Safe\ bearing\ pressure\ P_b}$$
$$= \frac{57,5 \times 10^3 N}{520 \times 10^3 N/m^2}$$
$$l \times b = 0,110\ m^2$$
$$2,5b \times b = 0,110\ m^2$$
$$b = \sqrt{\frac{0,110\ m^2}{2,5}}$$
$$= 0,2098\ m$$

Say $b = 210\ mm$

and $l = 2,5\ b$
$$= 2,5 \times 210\ mm$$
$$= 525\ mm$$

Worked Example 5.26

a) An internal combustion engine has a bore and stroke of 65 mm and 90-mm respectively, with a connecting rod 180 mm long. When the crank is 60° past top dead centre on the power stroke, the effective pressure on the piston is 1,05 MPa.

Calculate:

i) The effective force on the piston.

ii) The side thrust on the piston.
iii) The thrust along the connecting rod.

b) If the above condition is assumed to give maximum side thrust on the piston, calculate the length of piston if the allowable pressure is not to exceed 105 kPa.

Solution:

a) i) Effective force on piston

$$\begin{aligned} Force &= pressure \times bore\ area \\ P &= 1{,}05 \times 10^6 N/m^2\ \frac{\pi}{4} \times (0{,}065)^2 \\ &= 3484\ N \end{aligned}$$

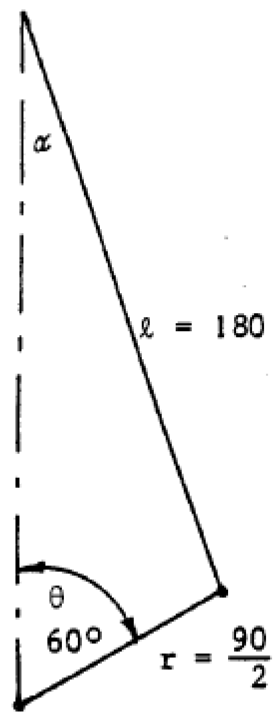


Figure 5.50

$$\begin{aligned} \sin\alpha &= \sin\theta \times \frac{r}{l} \\ &= \sin 60° \times \frac{0{,}045}{0{,}18} \\ &= 0{,}866 \times \frac{0{,}045}{0{,}18} \\ &= 0{,}2165 \\ \therefore \alpha &= 12{,}5° \end{aligned}$$

ii) Side thrust on piston

$$\begin{aligned} R &= P\tan\alpha \\ &= 3484\ N \times \tan 12{,}5° \\ &= 3484\ N \times 0{,}2217 \\ &= 772{,}38\ N \end{aligned}$$

iii) Thrust along the connecting rod

$$P = Q\cos\alpha$$
$$3484 = Q\cos 12{,}5°$$
$$Q = \frac{3484\ N}{\cos 12{,}5°}$$
$$= \frac{3484\ N}{0{,}976}$$
$$Q = 3569{,}6\ N$$

b) Length of piston

$$R = P_b \times D \times l$$
$$772{,}38\ N = 105 \times 10^3\ N/m^2 \times 0{,}065 \times l$$
$$l = \frac{772{,}38\ N}{105 \times 10^3\ N/m^2 \times 0{,}065}$$
$$= 0{,}113\ m$$
$$l = 0{,}113\ m$$

Worked Example 5.27

The maximum force on a connecting rod was found to be 7 kN. Calculate the dimensions for a suitable gudgeon pin if the bearing pressure is imited to 12 MPa.

Solution:

Diameter of gudgeon pin

$$Q_{max} = P_b \times l \times d_e \qquad \text{Assume } l = 1{,}5d_e$$
$$7\ kN = 12 \times 10^3\ N/m^2 \times 1{,}5d_e \times d_e$$
$${d_e}^2 = \frac{7\ kN}{12 \times 10^3\ N/m^2 \times 1{,}5}$$
$$d_e = \sqrt{0{,}000388}$$
$$d_e = 0{,}0197\ m$$
$$d_e = \text{say } 20\ m$$

and

$$l = 1{,}5d_e$$
$$= 1{,}5 \times 20\ mm$$
$$l = 30\ mm$$

Worked Example 5.28

The following data are given for a single cylinder four-stroke compression ignition engine.

a) Maximum explosion force 70 kN.

b) Allowable shear stress for the hollow gudgeon pin 35 MPa.

c) Ratio of external diameter to internal diameter is 2:1 and the bearing length = 2 x external diameter of pin.

Calculate:

i. The dimensions of the hollow gudgeon pin.
ii. The stresses due to bending in the gudgeon pin, stating clearly the assumptions made.
iii. Check for bearing pressure and state whether the design of the pin is safe.

Solution:

Dimensions of the hollow gudgeon pin

$$P = Q_{max} = \tau \times area\ of\ resistance \times 2$$
$$= \tau \times \frac{\pi}{4}\left[{d_e}^2 - {d_i}^2\right] \times 2 \text{ (use 2 for double shear)}$$
$$70 \times 10^3 N = 35 \times 10^6\ N/m^2 \times \frac{\pi}{4}\left[(2d_i)^2 {d_i}^2\right] \times 2 \text{ (given } d_e = 2d_i)$$
$$\frac{70\times10^3 N\times4}{\pi\times35\times10^6 N/m^2\times2} = \left[4{d_i}^2 - {d_i}^2\right]$$
$$0{,}001273 = 3{d_i}^2$$
$$d_i = \sqrt{\frac{0{,}001273}{3}}$$
$$= 0{,}0206\ m$$
$$d_i = \text{say } 20\ mm$$

and
$$d_e = 2 \times d_i \text{ (given)}$$
$$= 2 \times 20$$
$$d_e = 40\ mm$$

Bending stress in gudgeon pin

a)
$$l = 2 \times d_e$$
$$= 2 \times 40\ mm$$
$$= 80\ mm$$

b)
$$M = \frac{Q_{max}\times l}{8} \text{ (simply supported with uniformly distributed loading)}$$
$$M = \sigma_b \times Z$$
$$\frac{Q_{max}\times l}{8} = \sigma_b \times \frac{\pi}{32}\left[\frac{{d_e}^4 - {d_i}^4}{d_e}\right]$$
$$\frac{70\times10^3 N\times0{,}080\ m}{8} = \sigma_b \times \frac{\pi}{32}\left[\frac{(0{,}04)^4-(0{,}02)^4}{0{,}04}\right] m^3$$
$$700\ Nm = \frac{\pi}{32}\sigma_b\left[\frac{2{,}56\times10^{-6}-0{,}16\times10^{-6}}{0{,}04}\right] m^3$$
$$700\ Nm = \frac{\pi}{32}\sigma_b\left[\frac{2{,}4\times10^{-6}}{0{,}04}\right] m^3$$
$$700\ Nm = \frac{5{,}89}{10^6} m^3 \times \sigma_b$$
$$\frac{700\ Nm\times10^6}{5{,}89 m^3} = \sigma_b$$
$$118{,}8 \times 10^6\ Pa = \sigma_b$$

$$\sigma_b = 118{,}8\ MPa$$

Checking bearing pressure
The bearing pressure should not exceed 13,8 MPa

$$Q_{max} = P_b \times l \times d_e$$
$$70 \times 10^3 N = P_b \times 0{,}08\ m \times 0{,}04\ m$$
$$P_b = 21{,}9\ MPa \quad \text{(Design not safe)}$$

To decrease the bearing pressure from 21,9 MPa to 13,8 MPa, the diameter of the gudgeon pin should be increased. From this we can see that the bearing pressure is the criterion of gudgeon pin design.

Worked Example 5.29

Design a suitable crank pin for a crankshaft used in a petrol engine. The explosion force is 20 kN, and a safe bearing pressure of 7 MPa is to be used. Assume $length\ of\ crank\ pin = 1{,}3 \times diameter\ of\ crank\ pin.$

Solution:
Diameter and length of crank pin

$$P = Q_{max} = P_d \times d \times l_1$$
$$\therefore = P_b \times d \times 1{,}3\ d$$
$$20 \times 10^3 = 7 \times 10^6\ N/m^2 \times 1{,}3d^2$$
$$d = \sqrt{\frac{20 \times 10^3 N}{7 \times 10^6 N/m^2 \times 1{,}3}}$$
$$= 0{,}047\ m$$
$$\text{say } d = 48\ mm$$
$$\text{and } l_1 = 48 \times 1{,}3$$
$$= 62{,}4\ mm$$
$$\text{say } l_1 = 63\ mm$$

The lining for the big-end may be a suitable white metal, such as Hoyt's metal, in a brass or gunmetal liner or in the rod itself, and for the small-end a chilled phosphor bronze bush.

Worked Example 5.30

A diesel engine develops 260 kW at 180 r/min. Calculate the diameter for the main bearing journal if the maximum torque exceeds the mean torque by 15% and the shear stress in the material is 54 MPa.

Solution:

Mean torque

$$
\begin{aligned}
T_{mean} &= \frac{Power \times 60}{2 \times \pi \times N} \\
&= \frac{260 \times 10^3 W \times 60}{2 \times \pi \times 180\ r/min} \\
&= 13{,}79 \times 10^3\ Nm
\end{aligned}
$$

Maximum torque

$$
\begin{aligned}
T_{maximum} &= T_{mean} + \%\ overload \\
&= 13{,}79 \times 10^3 + \frac{15}{100} \times 13{,}79 \times 10^3 \\
&= 13{,}79 \times 10^3 \times 2{,}069 \times 10^3 \\
&= 15{,}859 \times 10^3\ Nm
\end{aligned}
$$

Diameter of solid shaft

$$
\begin{aligned}
T_{maximum} &= \frac{\pi}{16} \tau D^3 \\
15{,}859 \times 10^3\ Nm &= \frac{\pi}{16} \times 54 \times 10^6\ N/m^2 \times D^3 \\
D &= \sqrt[3]{\frac{15{,}859 \times 10^3\ Nm \times 16}{\pi \times 54 \times 10^6\ N/m^2}} \\
&= 0{,}114\ m \\
D &= 114\ mm
\end{aligned}
$$

Worked Example 5.31

An engine has a bore and stroke of 65 mm and 90 mm respectively with a connecting rod 180 mm long. When the crank is 60° past the top dead centre on the power stroke, the effective pressure on the piston is 1,05 MPa.

Calculate

i. the twisting moment on the crankshaft. (Assume that the calculated torque in (i) is the maximum).
ii. the diameter of the main journal if the shear stress in the shaft is 42 MPa.

Solution:

(From **Worked Example 5.26** $\alpha = 12{,}5°$)

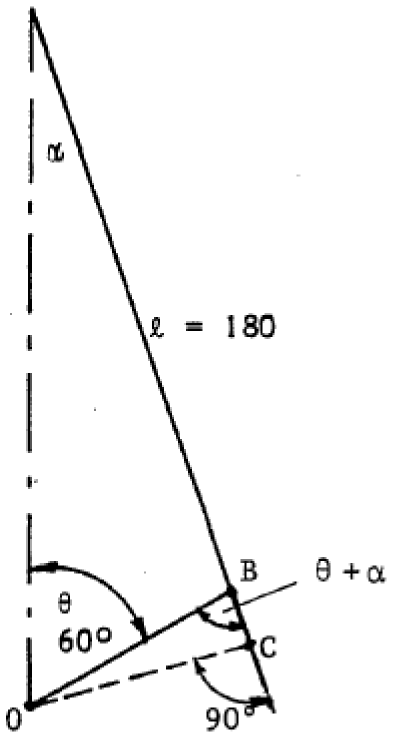


Figure 5.51

Length OC

$$OC = \sin(\theta + \alpha) \times OB$$
$$= \sin(60° + 12{,}5°) \times \frac{0{,}09}{2}$$
$$= \sin 72{,}5 \times 0{,}045$$
$$= 0{,}9537 \times 0{,}045$$
$$OC = 0{,}0429\ m$$

Force along the connecting rod
(from **Worked Example 5.26** $Force = 3\,569{,}6\ N$).

Twisting moment

$$T = Q_{max} \times OC$$
$$= 35696\ N \times 0{,}0429\ m$$
$$= 153{,}14\ Nm$$

Diameter of crankshaft

$$T = \frac{\pi}{16}\tau D^3$$
$$153{,}14\ Nm = \frac{\pi}{16} \times 42 \times 10^6\ N/m^2 \times D^3$$
$$D = \sqrt[3]{\frac{153{,}14\ Nm \times 16}{\pi \times 42 \times 10^6\ N/m^2}}$$
$$= 0{,}026\ 26\ m$$

say $D = 28\ mm$

Worked Example 5.32

The following data are given for a single-cylinder compression ignition engine:

a) Maximum explosion force 70 kN.
b) The centre of the big-end bearing is at mid-position between the main bearings, which are 450 mm apart.
c) The bending stress ·;n the main bearing journals is not to exceed 70 MPa.

State clearly the assumptions made, and calculate:

i. The diameter of the main bearing journals.
ii. The bearing pressure in the main bearing journals.

<u>Solution:</u>

Bending moment for centre crank

$M = \frac{Q \times l}{4}$ (Treat as simply supported beam with a central 4 concentrated load)

$= \frac{70 \times 10^3\ N \times 0{,}45\ m}{4}$

$M = 7{,}875 \times 10^3\ N$

Diameter of main bearing journals

$$M = \sigma_b \times Z$$
$$M = \sigma_b \times \frac{\pi}{32} D^3$$
$$7{,}875 \times 10^3\ Nm = 70 \times 10^6\ N/m^2 \times \frac{\pi}{32} D^3$$
$$D = \sqrt[3]{\frac{7{,}875 \times 10^3\ Nm \times 32}{\pi \times 70 \times 10^6\ N/m^2}}$$
$$= 0{,}1046\ m$$
$$\text{say } D = 105\ mm$$

Bearing pressure in main bearing journals

Assume (a) $l_2 = 1{,}25\ D$ (b) $Force = \frac{F}{2}$

$$Force = p_b \times D \times l_2$$
$$\frac{F}{2} = p_b \times D \times 1{,}25\ D$$
$$\frac{70 \times 10^3\ N}{2} = p_b \times 1{,}25 \times (0{,}105)^2 m^2$$
$$p_b = \frac{70 \times 10^3\ N}{2 \times 1{,}25 \times (0{,}105)^2 m^2}$$
$$= 2{,}54 \times 10^6 Pa$$
$$p_b = 2{,}54\ MPa$$

Activity 5.1

1. The maximum pressure in a cylinder of 200 mm diameter is 1 MPa. Calculate the diameter of a piston rod for a safe stress of 20 MPa.
2. In a high pressure conical type of cast-steel piston, the diameter D is 1 000 mm and the difference of pressure between two sides of the piston is 500 kN/m². The base angle of the conical part is 37,5 degrees.

Find the respective thicknesses of the conical part near the boss where $x = 120\ mm$ and at a point where the conical part meets the rim (or crown) of the piston where $y = 520\ mm$, x and y referring to **Figure 5.2**.

3. The piston rod of a horizontal steam engine has a diameter of 100 mm and takes a pull of 222 kN. At the piston end it is tapered to 82 mm diameter and then screwed 75 mm diameter for the piston nut. The screw thread is of the constant type pitch having a pitch of 4 mm. The thickness of the piston boss is 140 mm.
   At the crosshead end it is tapered to 90 mm diameter over a length of 190 mm to fit the socket. A cotter, 94 mm wide by 22 mm thick, fixes the rod to the socket at the centre of the tapered length.
   Calculate:
   i) tensile stress in threaded portion.
   ii) shear stress in cotter.
   iii) tensile stress in piston rod at cotter hole.
4. A piston rod for a steam engine of cylinder diameter 610 mm and steam pressure 690 kPa is tapered and screwed at the piston end and enlarged and tapered with a cotter fixing at the crosshead end.
   Calculate, using a factor of safety of 15 for direct tension, the diameter of the body of the rod.
   Calculate all the other sizes at the rod ends using a factor of safety of 8. The taper is 1 in 16 on the diameters at both ends and the lengths of the tapers may be assumed as 160 mm at the piston end and 200 mm at the crosshead end.
   The cotter may be assumed as 30 mm thick. The socket need not be designed.
   Tensile strength of steel = 463 MPa.
   Shearing strength of steel = 386 MPa.
   Crushing strength of steel = 618 MPa.
   Draw, free-hand, dimensioned views of both ends of the piston rod.

Activity 5.2

1. Two steel shafts, a driver transmitting 64 kW at 220 r/min and a driven, run parallel side by side with their centres 600 mm apart, and with a speed reduction of 5 to 1, obtained through gear wheels keyed to the shafts Calculate the diameters of the two shafts for a working shear stress of 65 MPa.
2. Compare the strength of a 150 mm diameter solid shaft and a hollow shaft of the same weight per unit of length. The hollow shaft has an external diameter of 200 mm.
3. A motor-car shaft consists of a steel tube with an internal diameter of 31,75 mm and is 3,175 mm thick. The engine develops 8,95 kW. at 2000 r/min. What will the maximum stress in the tube be when the power is transmitted through a 4:1 gearing?

4. A shaft is 20 mm in diameter and 380 mm long. When running st 2 000 r/min under a certain load, the torsion meter showed a deflection of 3,06°. The modulus of rigidity of the shaft material is 82 x 109 Pa. What power is being transmitted and what will the shearing stress in the shaft be?

Activity 5.3

1. The particulars of a steam engine are: Cylinder diameter 500 mm; stroke 600 mm; length of connecting rod 1,5 m.
   At the moment when the crank is at right angles to the connecting rod, the effective steam pressure on the piston is 550 kPa.
   For this position, determine the thrust on the crosshead guide and the force in the connecting rod.
   From this data obtain the bearing pressure on the crosshead pin which is 90 mm diameter, by 114 mm long, and the bearing pressure on the slipper which is 150 mm wide by 250 mm long.
2. The following information is given for a four-cylinder four-stroke petrol engine:
   a) Maximum pressure in cylinder = 2 400 kPa.
   b) Diameter of piston = 95 mm.
   c) Length of small-end bearing = 48 mm.
   d) Length of big-end bearing = 1,4 x diameter of big-end.
   e) Allowable bearing pressure for the small-end bearing = 14 MPa.
   f) Allowable bearing pressure for big-end bearing = 7,5 MPa.
   Calculate
   i) the maximum force on piston.
   ii) the diameter of small-end bearing.
   iii) the length and diameter of big-end bearing.
3. The total force P on the piston of a steam engine is 300 kN. Find the pressure on the guides
   a) when the connecting red makes an angle of 15 degrees with the line of stroke;
   b) when it is a maximum. (Ratio $\frac{l}{r} = 3$).

Self-Check

I am able to:	Yes	No
• Describe pistons		
o Rings		
o Rods		
• Describe shafting		
o Torsion equation for shafts		
o Power developed by a torque		
• Describe the design of shafts		

• Describe the saving of mass		
• Describe the standard sizes of bright steel bars		
• Describe geared shafts		
• Describe the forces which act on the parts of the driving mechanism of steam and similar engines		
o Crossheads		
o Design of the crankshaft		
o Internal combustion engine		
If you have answered 'no' to any of the outcomes listed above, then speak to your facilitator for guidance and further development.		

Module 6

Friction

Learning Outcomes
On the completion of this module the student must be able to:

- Describe friction
- Describe lubrication
    - Lubricants
    - Lubricating methods
- Describe bearings
- Describe velocities and bearing pressures
- Describe bearing and losses
- Describe he working pressure for bearings and crossheads
- Describe the size of solid and hollow shafts to transmit a given power (pure torsion)

6.1 Introduction

Friction is the resistance between two surfaces which prevents the movement of one surface against the other.

One advantage of friction is the usefulness of friction in the clutch of a car; a disadvantage would be friction in a car's gearbox.

The earth's gravity plays a role (10 m/s^2) in friction as it exerts a force making it difficult to move, eg a block of wood or other material over a surface.

6.2 Friction

When two surfaces are in contact and slide relative to each other, a tangential force acting so as to resist the motion is set up along the plane of contact; this force is referred to as the friction force.

The law for dry friction states that the friction force is:
1. independent of the area of contact
2. directly proportional to the normal force holding the surfaces together;
3. independent of the velocity of sliding.

The frictional resistance is greater at the moment when the surfaces are about to commence sliding than during the process of sliding.

These two forms of resistance are referred to as static and kinetic friction, respectively.

$F = \mu Q$ where F = tangential friction force resisting sliding,
Q = normal force holding the surfaces together
and μ = coefficient of friction which represents the static or kinetic coefficient of friction, depending upon whether F is equal to the force required to cause or to maintain sliding.

Type of bearing	Coefficient of friction
Ball bearing, radial	0,002 to 0,003 (rolling)
Thrust	0,001 (rolling)
Deep groove	0,001 3 (rolling)
Self-aligning	0,001 0 (rolling)
Roller bearing	0,005 to 0,008 (rolling)
Self-aligning	0,002 7 (rolling)
Collar bearing, thrust	0,03 (sliding)
Step bearing	0,015 (sliding)
Ring-oiling	0,010 to 0,015 (sliding)
Plain	0,009 (sliding)

Table 6.1 Coefficient of friction of bearings with good lubricant

6.3 Lubrication

Lubrication implies the introduction of grease or oil between the bearing surfaces of machine components with the object of reducing friction (and hence the power required to overcome frictional resistance) and of minimising surface wear.

The different types of lubrication follow.

6.3.1 Hydrostatic lubrication

Oil is pumped to the bearing under pressure sufficiently great to overcome the average pressure at the bearing surfaces. The parts are thereby "floated" on a layer of a fluid which is continuously maintained.

Hydrostatic lubrication is also used in machine tools which have a reciprocating motion, such as shapers, planers, and slotting machines, the oil being pumped to the slides carrying the reciprocating parts.

6.3.2 Hydrodynamic lubrication
The bearing surfaces involved in the transmission of motion and power are usually so shaped that they do not lie parallel, but are inclined or curved relative to each other.

Examples are:
(a) the anti-friction bearings in which ball or roller surfaces engage with races in a rolling motion associated with a small amount of slip;
(b) gear teeth which combine rolling and slipping motions;
(c) thrust bearings of the tilted pad type involving sliding motion and
(d) journal bearings, in which the surfaces slide over each other but which, because of the eccentricity of the journal and its bearings, are not parallel.

Hydrodynamic lubrication occurs when engaging surfaces have relative sliding motion and converge to produce a wedge shaped film of lubricant.

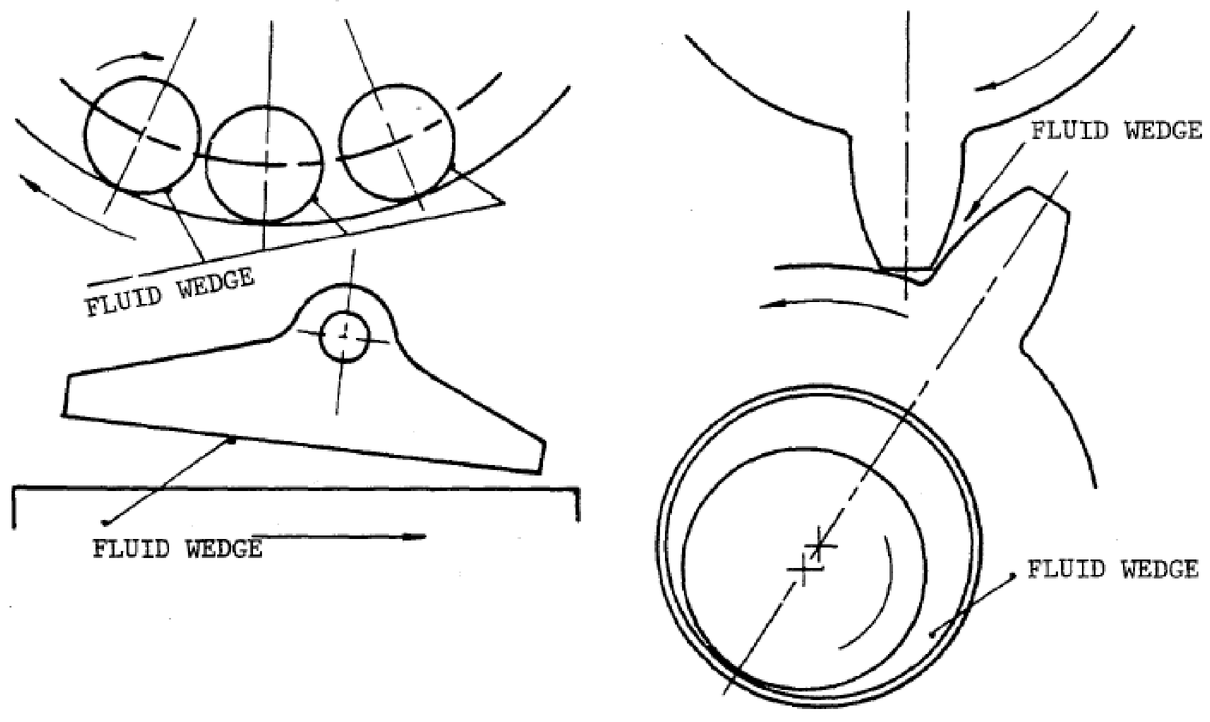


Figure 6.1

6.4 Lubricants
6.4.1 Grease
Grease is a mineral oil which has been thickened by the addition of graphite, soaps, or other ingredients.

Grease can be used where oil is not a practical proposition, for example, in the wheel bearings of a motor-car, where oil would be washed out by water thrown up from the road.

Greases are used where slow or medium speeds are required and the working temperature is below 120°C.

When choosing a grease the following points should be considered:

- It must be of the right viscosity to ensure complete distribution and to form the required protecting film.
- It must resist the formation of deposits.
- It must not thicken during working conditions.
- It must have good heat-carrying properties.
- If possible it should contain graphite, as graphite flakes break down and are caught in the 'rough spots', whilst the remainder form a highly polished surface.

When comparing grease with oil, it is found that grease has an important place in the field of lubrication.

Greasing methods can be placed in three categories:

1. compression cups
2. a one-shot system, where the grease is piped to various points by intermittent pressure from a reservoir
3. as in 2., but with a constant feed.

6.4.2 Oil

Oil lubrication is used when working speeds and temperatures are high. Many modern machines use a centralised lubricating system in order to simplify the lubrication tasks, and oil is conveyed under pressure from a central reservoir along pipe runs to the various bearings. Oil can be used very successfully in lubricators, or rings, and it is important to note that the oil can be filtered and used again so long as it retains its lubricating properties.

6.4.3 Viscosity

The viscosity of a lubricant is best thought of as its resistance to motion; hence for low viscosities the resistance to motion is low and movement takes place easily, while a lubricant with a high viscosity does not flow easily.

Note:
As temperature increases, the viscosity decreases. A lubricant with a low viscosity may flow so easily that it is squeezed out from the parts it is lubricating.

The 'oiliness' of a lubricant can be described as its ability to maintain a film between surfaces even when subjected to pressure. It is obvious therefore that a lubricant for a particular machine or engine must be carefully chosen by taking all these points into consideration.

Several factors should be considered when choosing a lubricant:

- the velocity of the shaft
- the coefficient of friction between shaft and bush
- the temperature conditions

- the atmospheric conditions
- the pressure of lubrication required

6.4.4 Precautions to be observed in using lubricants

It is essential that:

- the correct lubrication should be used for a particular job
- the best method should be used to apply the lubricant
- for high speeds and low pressures a light oil should be used
- for low speeds and high pressures a lubricant with enough 'body' to prevent wear should be used.

6.5 Lubricating methods

6.5.1 Pressure lubrication

This method is largely used in machine tools, internal combustion engines, etc.

The principle is that the oil is forced under pressure between the rubbing surfaces, ie into the gap between the bush and the shaft.

Note:
As the pressure increases, the shaft is forced to rise on an oil film. This condition is known as 'fully floating'.

It has the following disadvantages:

- As rotation stops, the oil ceases to flow and tends to drain away from the bearing.
- It takes time for the oil pressure to build up when starting, with a consequent increased rate of wear during this period.
- If the oil is cold, the viscosity is greater, so extra power is required to overcome the extra friction.

6.5.2 Lubricators

6.5.2.1 Ring-lubricated bearing

On long high-speed bearings as well as on long countershaft bearings, ring lubrication is often resorted to **Figure 6.2** shows that the body proper of the casting is provided with a trough large enough to hold a considerable quantity of oil.

The upper brass is so cut that one or more jointed rings may be hung over the shaft, their lower portion dipping into the oil chamber. When the shaft rotates, the ring also rotates, and in doing so it deposits a quantity of oil on the shaft.

After working along the shaft the oil drains back into the trough below.

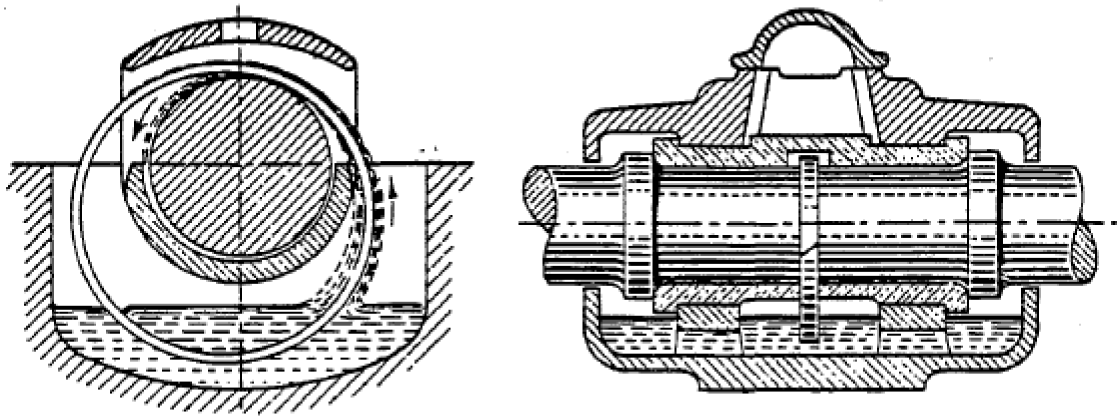


Figure 6.2

6.5.2.2 The needle lubricator

The needle lubricator is often used on plain bearing caps, where the shafts carry moderate. loads at average speeds.

The bottle is filled with oil, the double-tapered cork or wooden stopper carrying the needle is inserted, and the lower part of the stopper is fitted into the bearing cap.

The rotation of the shaft shakes the needle so that air bubbles rise in the oil chamber and displaced drops of oil flow down the needle onto the shaft.

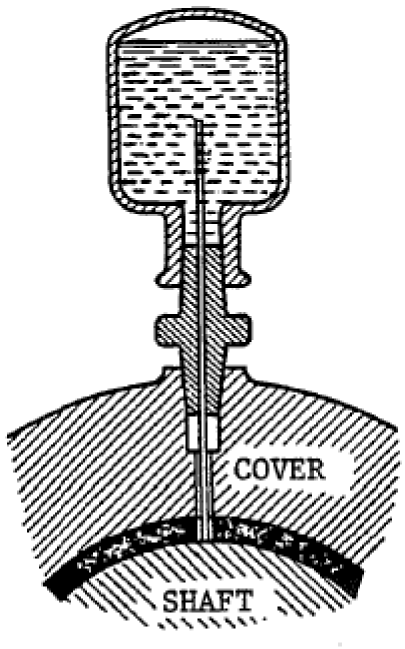


Figure 6.3

6.5.2.3 The wick lubricator

The wick lubricator acts on the syphon principle. If a wick is run from an oil reservoir in the cap of a bearing to the surface requiring lubrication, it will syphon oil over. When the machine is stationary, the wick can be withdrawn by means of a wire and left in the oil reservoir until required again.

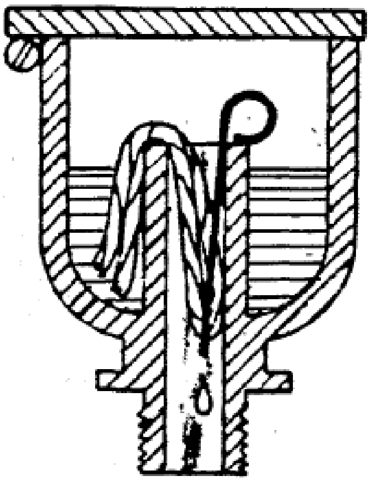


Figure 6.4

6.5.2.4 The hand-pressure grease lubricator

The hand-pressure grease lubricator may be of brass or iron. A turn of the knurled top forces lubricant upon the shaft.

An alternative method is to force the grease downward by means of a light piston, the pressure on which is maintained by a spiral spring.

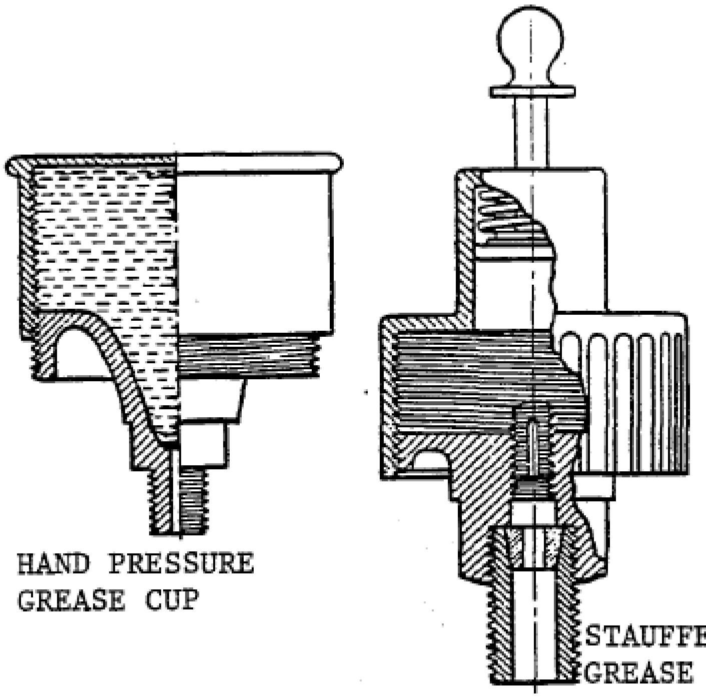


Figure 6.5

6.6 Bearings

The function of a bearing is to support a part which has motion relative to the portion containing the bearing with as small a frictional resistance as possible.

> **Note:**
> The relative motion is most frequently rotation about a common axis.

If a shaft is machined in one or more places in order that the shaft may be supported by some form of bearing, the suitably machined part of the shaft is called a journal. The combination of shaft and support is called a journal bearing.

> **Think about it!**
> When designing simple journal bearings, wear, which is inevitable, must be considered.

If a material softer than the journal is used at the surface of the bearing, wear will occur mainly at that surface, and adjustment and replacement are facilitated by providing a surface in the form of a bush or steps. The latter may be lined wholly or partly with a suitable material.

The choice of this material will depend on its thermal conductivity and on the bearing pressure and speed, since the possibility of overheating must not be overlooked. Babbitt metal (89 percent tin, 3,5 percent copper, 75, percent antimony) and similar alloys are frequently used.

Engine bearings are usually the plain or bushed type, and they work under most exacting conditions of speed and load. The most heavily loaded engine bearings are the big and small-end bearings of the connecting rod and main bearings of the crankshaft.

The big-end bearing consists of "half" (split) steel or bronze shells lined with white metal. Most modern engines are fitted with the thin-shell pre-finished type of bearing which consists of a steel shell about 1,55 mm thick, lined with metal whose thickness is only about 0,255 - 0,385 mm.

Thin-shell bearings have a long life, and they are readily assembled or replaced without boring, scraping or hand fitting of any kind. In fact, it is damaging to carry out any of this work because of the fine tolerances and the excellence of the surface finish obtained by means of diamond boring or broaching.

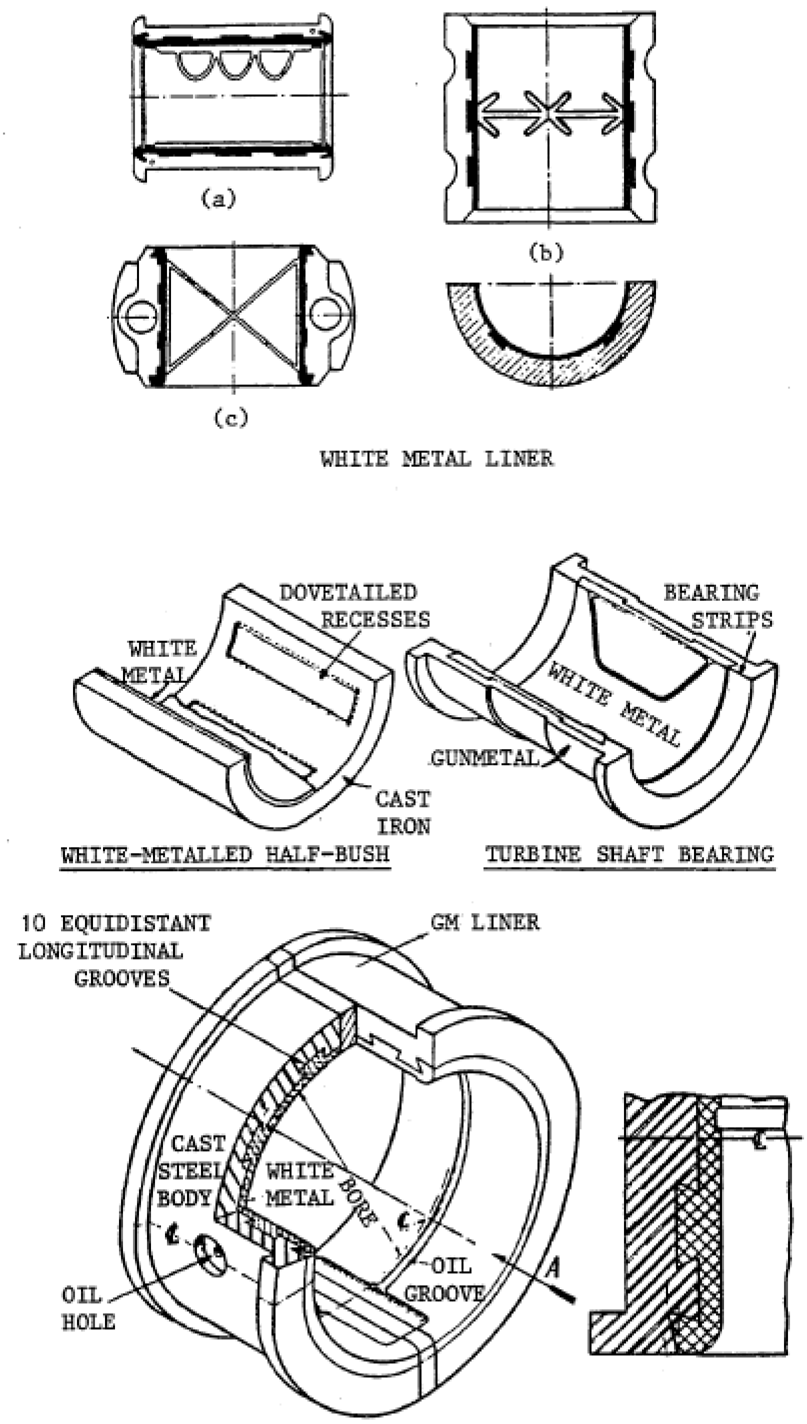


Figure 6.6

6.6.1 Classes of bearings

Bearings may be classified into three main groups, namely:

- Journal bearings
- Thrust bearings
- Footstep bearings

6.6.1.1 Journal bearings

In this type of bearing the supporting pressure of the bearing is at right angles to the shaft axis.

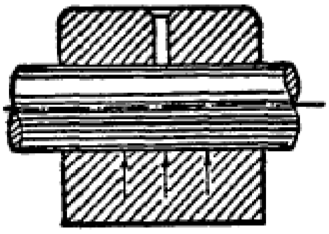


Figure 6.7

6.6.1.2 Thrust bearings

In this type of bearing the pressure is largely parallel to the axis of a shaft (having "end thrust" and passing through and beyond the bearing) as in propeller drives etc. (Also called "collar bearings").

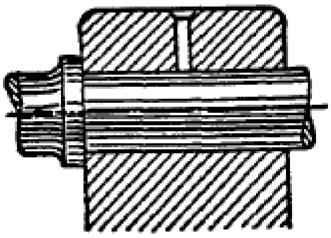


Figure 6.8

6.6.1.3 Footstep bearings

In this type of bearing the supporting pressure is vertically upwards, ie parallel to the shaft axis (the end of which rests within the bearing).

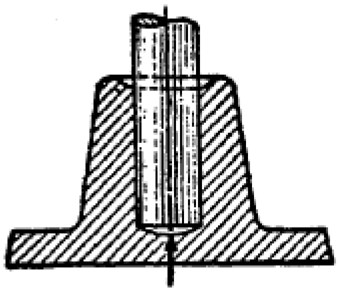


Figure 6.9

6.6.2 Solid bearings

The simplest form of bearings, the solid bearing, is shown in **Figure 6.7**, **Figure 6.8** and **Figure 6.9**. These bearings are used for shafts which rotate slowly or at infrequent intervals. The hole in the bearing is bored so that the shaft is a running fit.

The shaft must be passed into the bearing axially, ie endwise. No provision is made for wear or for adjustment on account of wear, and the complete bearing has to be replaced, should the wear become excessive. This type of bearing has a very limited field of application.

6.6.3 Bushed bearings

The next stage in bearing development is represented by **Figure 6.10**, where the simple solid bearing is bushed. Bushes of this type are of brass, gun metal or phosphor bronze, and are renewable when worn.

The bushes are kept in position either by using set-screws or pins, or the outside of the bush is a driving or interference fit in the hole of the casting, and they are pressed in position by means of hydraulic pressure.

The inside of the bush is bored as a running fit for the shaft.

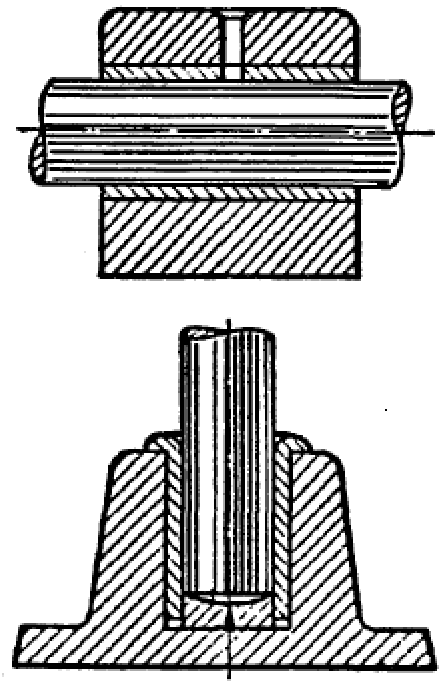


Figure 6.10

6.6.4 Split type of bearing

Another method of taking up wear is by dividing the bearing with a plane containing the axis of the shaft and normal to the direction of the load. In **Figure 6.11** the upper half or cap is secured to the machine frame by means of studs and nuts.

Alignment between the parts is maintained by a narrow offset A on the line of division to ensure that no longitudinal relative movement occurs, the offset portion in the cap need not extend the whole length and may then fit into a recess arranged in the lower half. Wear is taken up by removing some of the metal at B.

Thin metal strips called shims are frequently provided initially at B; wear may then be taken up by removing one or more of the strips. The bottom of the offset at A is usually clear of the pedestal. The bore is then turned to size and the bearing fits snugly around the shaft again.

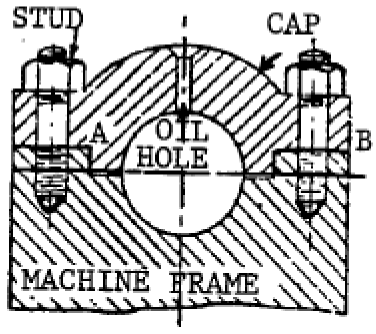


Figure 6.11

If a thrust bearing of the type shown in **Figure 6.12** is used, the bearing must consist of two parts (split bearing), otherwise it is not possible to place the shaft in position.

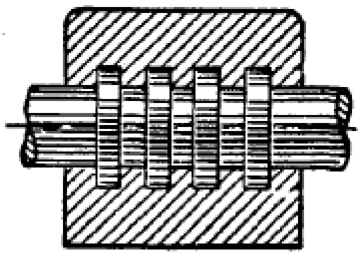


Figure 6.12

6.6.5 Pedestal bearing

As mentioned previously, bearings are split into halves to facilitate installation and also to allow for adjustment for wear.

For higher speeds and in the larger sizes shafts run in brass, phosphor-bronze or gun-metal "brasses" made in two halves, supported in a rigid cast-iron block, and held down by a separate bolted cap.

The brasses must be provided with some means of preventing their rotation and their axial movement along the shaft. Means of lubrication must also be provided.

Figure 6.13 shows a pedestal bearing with split bearing brasses C and B. The bearing housing consists of two parts, namely, the bearing cap D and the base A, which are bolted together and house the split brasses.

The bearing cap does not rest on the base K. The clamping action is therefore maintained when the split brasses are machined at E and F to take up the wear of the bearing.

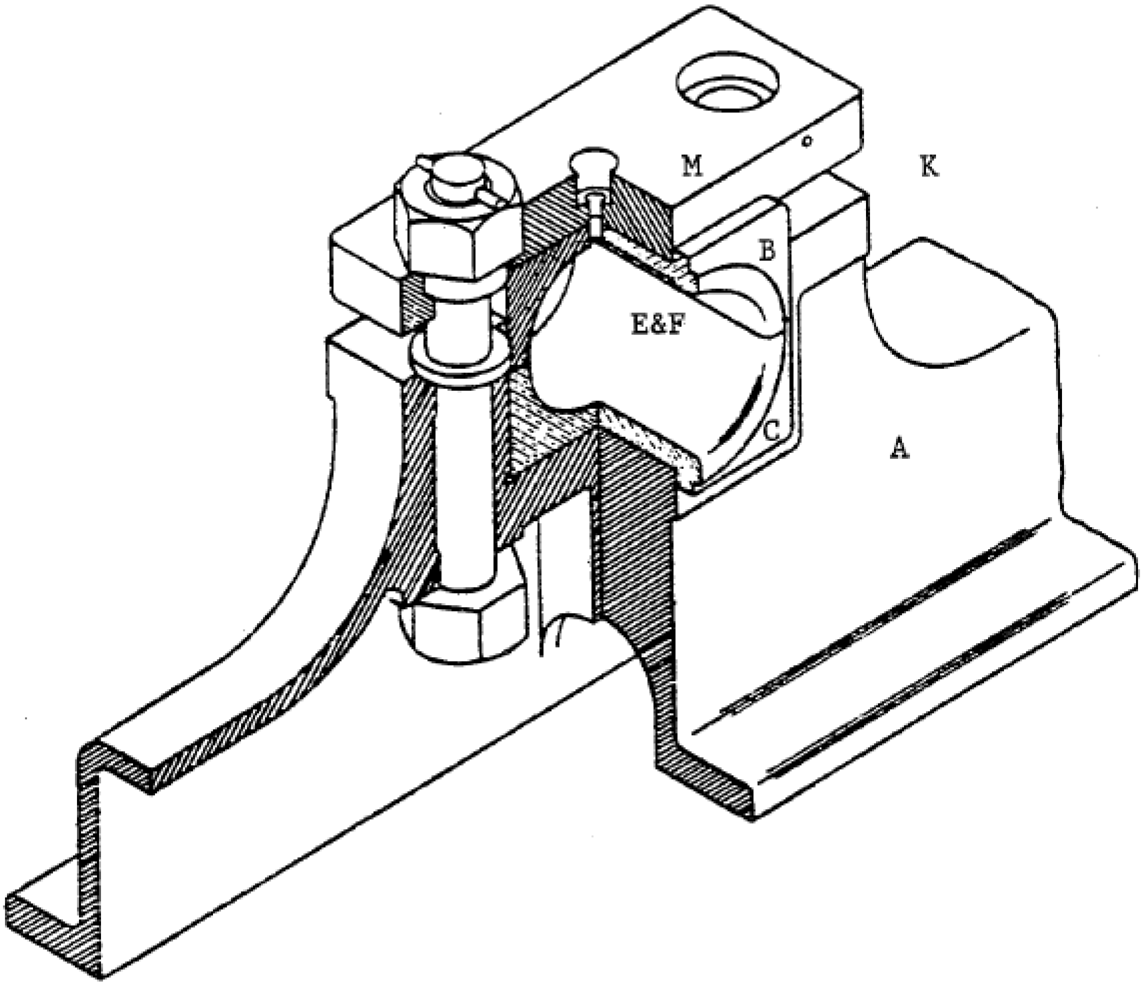


Figure 6.13

6.6.6 Prevention of rotation of brasses

When a shaft rotates in split bearing brasses, it tends to rotate the bearings with it. To counteract this rotation of the halves, various methods are employed, as shown below:

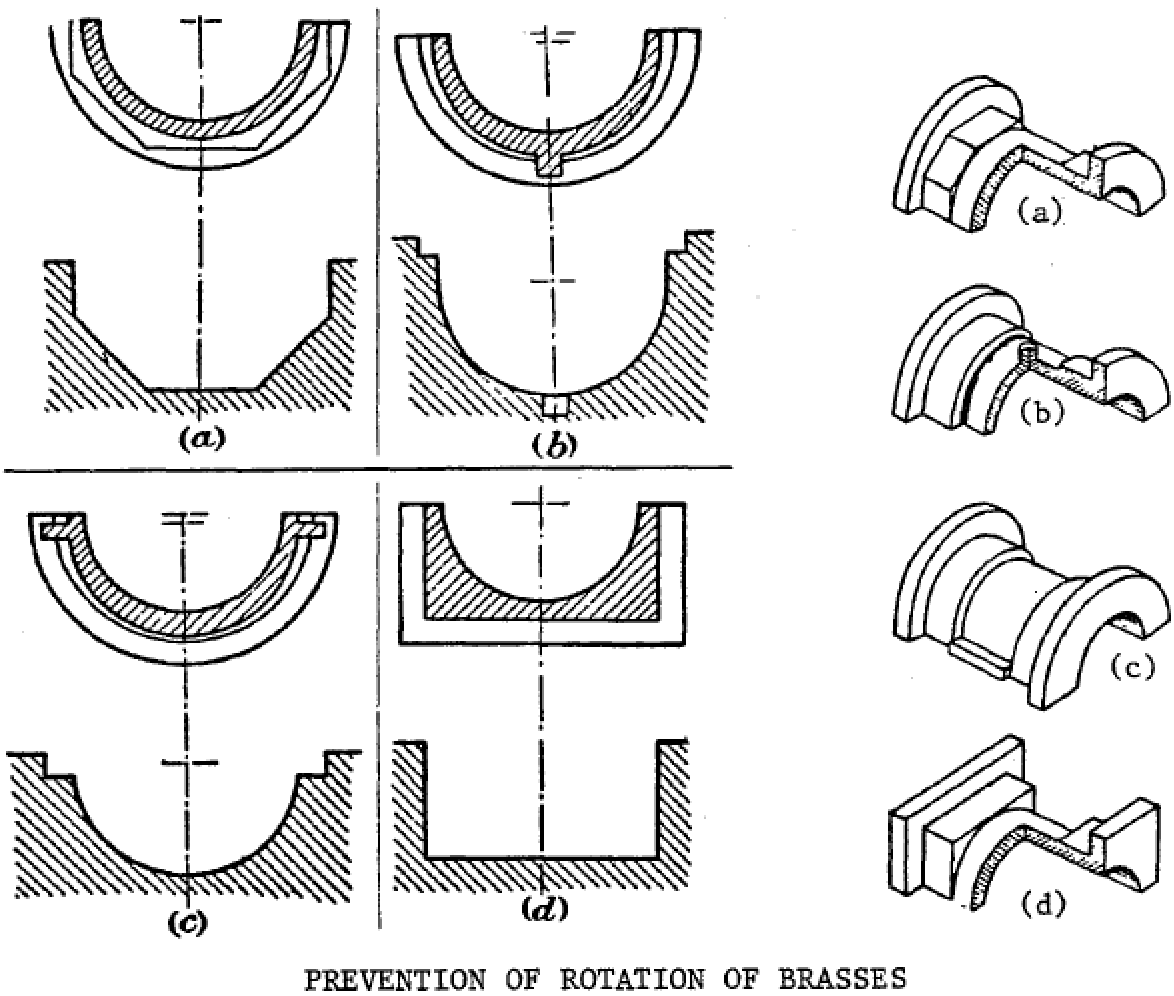


Figure 6.14

At (a) the casting is octagonal. The outside of the brasses are also octagonal, except for the middle portion which is recessed. This recessing facilitates fitting and prevents rock.

At (b) a pip, snug, or register is cast on the lower brass. This fits a corresponding hole in the casting.

At (c) lugs are left at the sides of the brass and corresponding recesses left in the casting.

At (d) the lower brass is squared and fits a casting to correspond.

6.6.7 Ball and roller bearings

Ball and roller bearings have the following advantages over plain bearings. The coefficient of friction is lower and is practically the same at starting as in motion; heavier loads and higher speeds are permissible; less speed is required; less lubricant is used; and wear is practically nil.

On the other hand, the first cost is greater, the bearing cannot be used in halves, any defects may produce serious consequences, and the bearings are frequently noisy after long use.

6.6.7.1 Typical examples of ball and roller bearings

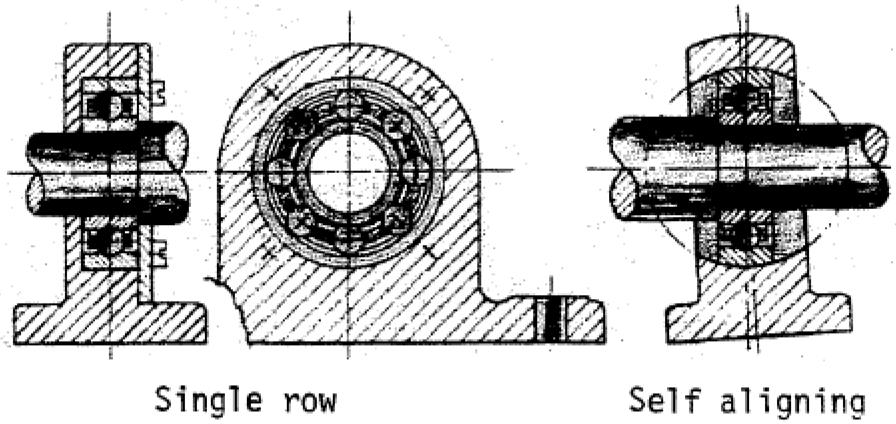
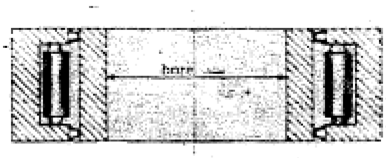
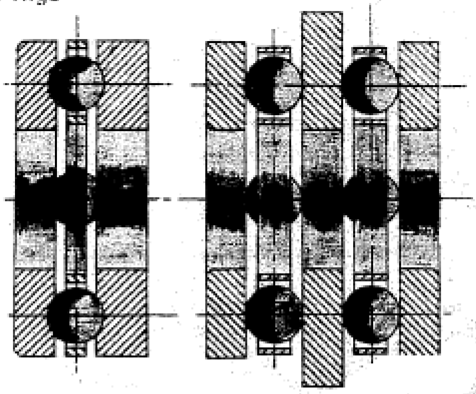


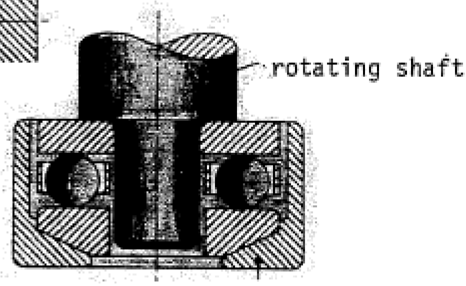
Figure 6.17

Needle roller bearing

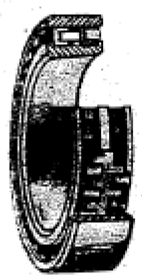


Thrust bearings

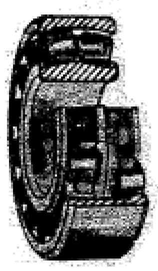




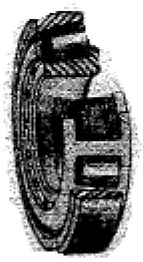
Single thrust bearing with spherical seating



Cylindrical roller



Spherical roller bearing



Tapered roller bearing

Figure 6.18

6.7 Velocities and bearing pressures (maximum allowed)

Type of bearing	Type of lubrication	Pressure on projected area kPa	Velocity m/min
Turbines, pumps journals 25 to 250 mm	ring-lubricated annular bearings	480	1 280

in diameter			
generators, motors, journals 50 to 400 mm in diameter	ring lubricated	700-900	73-366
reduction gears, marine turbines	pressure-lubricated annular bearings oil cooler	1 035	550-1 830
diesel engine main bearing	forced feed	3 450	280

Table 6.2

6.8 Bearing losses

Power loss $= \mu QV$

where μ = coefficient of friction.

Q = load on bearing in newtons

V = peripheral velocity in metres per second.

Worked Example 6.1

A solid shaft, 70 mm in diameter, transmits 125 kW at 440 r/min.

Calculate:

1. the torque transmitted
2. the shear stress in the shaft

A collar on this shaft must resist an axial force of 2 225 N.

Calculate the bearing pressure between the collar and the bearing if the outside diameter of the collar is taken as twice the diameter of the shaft.

Solution:

1. Torque transmitted

$$\text{Power} = \frac{2\pi T_{mean} N}{60}$$

$$T_{mean} = \frac{Power \times 60}{2 \times \pi \times N}$$

$$= \frac{125 \times 10^3 \times 60}{2 \times \pi \times 440}$$

$$= 2713\ Nm$$

2. Shear stress in the shaft

Assume

$$T_{mean} = T_{max}$$

$$T = \frac{\pi d_s{}^3}{16} \tau$$

$$\tau = \frac{T \times 16}{\pi d_s{}^3}$$

$= \frac{2713\ Nm \times 16}{\pi \times (0{,}07\ m)^3}$

$= 40{,}28\ MPa$

3. Bearing pressure between collar and bearing

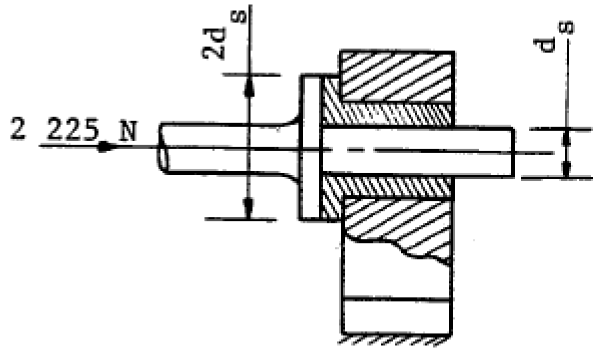


Figure 6.19

Worked Example 6.2

A solid shaft transmitting 53 kW at 140 r/min has a load of 18 kN applied at the journal. The maximum allowable shear stress due to torque must not exceed 65 MPa, and the maximum bearing load allowed on the journal projected area is 1 MPa. If the ratio of length of journal to journal diameter is 2 to 1, find the diameter and length of journal.

Solution:

Calculate the diameter and length of journal:
Consider shearing first:

Power $= \frac{2\pi T_{mean} N}{60}$

$T_{mean} = \frac{Power \times 60}{2\pi N}$

$= \frac{53 \times 10^3 \times 60}{2 \times \pi \times 140}$

$= 3615\ Nm$

$= 3{,}615\ kNm$

Assume

$T_{mean} = T_{max}$

$T = \frac{\pi {d_s}^3}{16} \tau$

$\therefore d_s = \sqrt[3]{\frac{T \times 16}{\pi \times \tau}}$

$= \sqrt[3]{\frac{3615\ Nm \times 16}{\pi \times 65 \times 10^6\ N/m^2}}$

$= 0{,}06566\ m$

say $d = 66\ mm$

Length $l = 2 \times d_s$

$= 2 \times 66$

$\therefore \quad = 132\ mm$

Consider bearing pressure:

$$Pressure = \frac{Force}{Projected\ area}$$

$$\therefore P_b = \frac{Q}{l \times d}$$

but $l = 2d_s$

$$\therefore P_b = \frac{Q}{2d_s \times d_s}$$

$$P_b = \frac{Q}{2d_s^2}$$

$$\therefore d_s^2 = \frac{18 \times 10^3 N}{2 \times 1 \times 10^6\ N/m^2}$$

$$= \sqrt{0{,}009\ m^3}$$

$$= 0{,}09486\ m$$

say $d = 95\ mm$

Length $l = 2d_s$

$$= 2 \times 95$$

$$= 190\ mm$$

$\therefore$ Safe sizes are: $= 190\ mm\ d_s = 95\ mm$

Worked Example 6.3

The load on a crankshaft journal is 11,5 kN, and the crankshaft transmits a torque of 4,2 kNm. If the diameter and length of the journal are 75 mm and 150 mm, respectively, find the shear stress and bearing pressure.

Solution:

$$Torque = \frac{\pi {d_s}^3}{16} \times \tau$$

$$\therefore \tau = \frac{16 \times T}{\pi \times d^3}$$

$$= \frac{16 \times 4200\ Nm}{\pi \times (0{,}075)^3\ m^3}$$

$$= 51 \times 10^6\ N/m^2$$

$$\tau = 51\ MPa$$

Calculate the bearing pressure:

$$Pressure = \frac{Force}{Projected\ area}$$

$$= \frac{11{,}5 \times 10^3 N}{0{,}075\ m \times 0{,}15\ m}$$

$$= 1 \times 10^6\ N/m^2$$

$$= 1\ MPa$$

Worked Example 6.4

The two journals of a shaft carry a flywheel of mass 10 160 kg midway between them. If the coefficient of friction is 0,075 and the speed 70 r/min, find the work done per minute and the power absorbed by friction.
Diameter of journals = 300 mm.

Solution:

$$\begin{aligned}
Total\ load\ on\ bearings\ (Q) &= 10\ 160\ kg \times 9{,}81\ m/s^2 \\
&= 99669{,}6\ newtons \\
Total\ friction\ force\ (F) &= \mu \times Q \\
&= 0{,}075 \times 99669{,}6 \\
&= 7475{,}22\ newtons \\
Distance\ moved\ in\ metres\ per\ minute &= \pi \times d \times N \\
&= \pi \times 0{,}3 \times 70 \\
&= 65{,}97\ metres\ per\ minute
\end{aligned}$$

$$\begin{aligned}
Work\ done\ per\ minute &= Friction\ force\ \times\ distance\ moved\ per\ minute \\
&= 7475{,}22\ N \times 65{,}97\ m \\
&= 493140{,}26\ Nm\ per\ minute \\
Power\ absorbed\ by\ friction &= friction\ force\ \times\ V \\
&= \mu \times Q \times V \\
&= 7475{,}22\ N \times \frac{65{,}97}{60} m/s \\
&= 7097{,}72\ watts \\
&= 8219\ kW
\end{aligned}$$

Worked Example 6.5

A shaft is supported by two bearings being one metre apart. A flywheel having a mass of 10 000 kg is situated on the shaft at a point 0,25 m from the left end support. The shaft has a diameter of 250 rnm and each journal has n length of 375 mm.

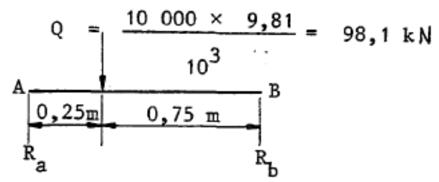
If the shaft rotates at 250 r/min and μ = 0,075 calculate -

1. the reaction at each journal;
2. the work done per minute at each journal;
3. the total work done per minute;
4. the power absorbed by friction at each journal;
5. the total power absorbed by friction;
6. the frictional torque in the shaft;
7. the shear stress in the shaft owing to the frictional torque; and
8. the pressure at each journal in MPa.

Solution:

1. Reaction at each journal:

$$\begin{aligned}
R_b \times 1 &= 98{,}1 \times 0{,}25 \\
R_b &= 73{,}575\ kN
\end{aligned}$$



$Moments\ about\ R_b$

$R_a\ x\ 1 = 98{,}1 \times 0{,}75$

$R_a = 73{,}575\ kN$

2. Work done per minute at each journal

$Work\ done\ at\ A = friction\ force \times distance\ moved\ in\ m/min$

$= \mu \times R_a \times \pi dN$

$= 0{,}075 \times 73{,}575\ kN \times c \times 0{,}25\ m \times 250\ r/min$

$= 1\ 083{,}48\ kNm\ per\ minute$

$Work\ done\ at\ B = 11 \times R_b \times \pi dN$

$= 0{,}075 \times 24{,}525\ kN \times \pi dN \times 0{,}25\ m \times 250\ r/min$

$= 361{,}16\ kNm\ per\ minute$

3. Total work done per minute

$Total\ work\ done = work\ done\ by\ A + work\ done\ by\ B$

$= 1\ 083{,}48\ kNm\ per\ minute + 361{,}16\ kNm\ per\ minute$

$= 1\ 444{,}64\ kNm\ per\ minute$

Alternatively:

$Total\ work\ done\ per\ minute = Total\ friction\ force \times distance\ moved\ in\ m/min$

$= \mu \times Q \times \pi dN$

$= 0{,}075 \times 98{,}1\ kN \times \pi \times 0{,}25\ m \times 250\ r/min$

$= 144{,}64\ kNm\ per\ minute$

4. Power absorbed by friction at each journal:

$Power\ at\ A = Work\ done\ per\ second$

$= \frac{1\ 083{,}48}{60}$

$= 18{,}058\ kW$

$Power\ at\ B = \frac{361{,}16}{60}$

$= 6{,}019\ kNm/s$

$= 6{,}019\ kW$

5. Total power absorbed by friction:

$Total\ power = power\ at\ journal\ A + power\ at\ journal\ B$

$= 18{,}058\ kW + 6{,}019\ kW$

$= 24{,}077\ kW$

Alternatively:

$Total\ power\ absorbed\ by\ friction = Total\ friction\ force \times V$

$= \mu \times Q \times \frac{\pi dN}{60}$

$= 0{,}75 \times 98{,}1 \times \pi \times 0{,}25\ m \times \frac{250}{60}\ kNm/s$

$= 24077\ kW$

6. Frictional torque in the shaft:

$Total\ power = \frac{2\pi NT}{60}$

$$T = \frac{60 \times Power}{2\pi N}$$
$$T = \frac{60 \times 24{,}077}{2 \times \pi \times 250}$$
$$T = 0{,}9197\ kNm$$
$$T = 919{,}7\ Nm$$

7. Shear stress in the shaft:

$$T = \frac{\pi}{16}\tau d^3$$
$$\tau = \frac{16 \times T}{\pi \times d^3}$$
$$\tau = \frac{16 \times 919{,}7\ Nm}{\pi \times (0{,}25)^3 m^3}$$
$$\tau = 299775\ N/m^2$$
$$\tau = 299{,}775\ kN/m^2$$

Say τ = $0{,}3\ MPa$ to overcome friction only

8. Pressure at each journal:

$$Pressure\ at\ A = \frac{R_a}{area\ of\ journal\ A}$$
$$= \frac{73{,}575 \times 10^3 N}{0{,}25\ m \times 0{,}375\ m}$$
$$= 784800\ N/m^2$$
$$= 0{,}7848\ MPa$$

$$Pressure\ at\ A = \frac{R_b}{area\ of\ journal\ B}$$
$$= \frac{24{,}525 \times 10^3 N}{0{,}25\ m \times 0{,}375\ m}$$
$$= 261600\ N/m^2$$
$$= 0{,}2616\ MPa$$

Worked Example 6.6

A horizontal shaft is subjected to an axial thrust of 60 kN taken up by a collar. Mean friction diameter is 150 mm, width 25 mm measured radially. Coefficient of friction 0,06.

Find the work absorbed per minute if the speed is 60 r/min. Also find the power.

Solution:

$$Friction\ force = \mu \times Q$$
$$= 0{,}06 \times 60 \times 10^3$$
$$Distance\ moved = \pi \times d \times N$$
$$= \pi \times 0{,}15 \times 60$$
$$Work\ absorbed = \pi \times Q \times \pi \times d \times N$$
$$= 0{,}06 \times 60 \times 10^3\ N \times \pi \times 0{,}15\ m \times 60 r/min$$
$$= 101{,}8 \times 10^3\ Nm/min$$
$$Power\ absorbed\ by\ friction = \frac{101800}{60}$$

$$= 1{,}7 \times 10^3 W$$
$$= 1{,}7\ kW$$

Activity 6.1

1. A crankshaft journal is loaded to the extent of 20 kN and the crankshaft transmits a torque of 5 kNm. Find the size of the journal. The maximum shearing stress must not exceed 60 MPa, and the maximum bearing pressure is 1 MPa on the projected area of the journal. Assume $l = 2{,}5\,d$.
2. The two journals of a shaft carry a flywheel of mass 8 500 kg midway between them. If speed is 70 r/min and the coefficient of friction is 0,09, find the work done per minute and the power absorbed in friction. The journals have a diameter of 270 mm.
3. A horizontal shaft is subjected to an axial thrust of 80 kN taken up by a collar. The mean friction diameter is 200 mm, the width 30 mm measured radially, and the coefficient of friction is 0,08. Find the work absorbed by friction per minute if the speed is 40 r/min. Also find the power required to overcome friction.
4. A crankshaft journal loaded to the extent of 20 kN has a journal diameter of 75 mm. The maximum shearing stress must not exceed 60 MPa. Assume $l = 2{,}5\,d$, and calculate:
   (i) the torque transmitted by the crankshaft;
   (ii) the bearing pressure.

Self-Check

I am able to:	Yes	No
• Describe friction		
• Describe lubrication		
o Lubricants		
o Lubricating methods		
• Describe bearings		
• Describe velocities and bearing pressures		
• Describe bearing and losses		
• Describe he working pressure for bearings and crossheads		
• Describe the size of solid and hollow shafts to transmit a given power (pure torsion)		

If you have answered ‘no’ to any of the outcomes listed above, then speak to your facilitator for guidance and further development.

Module 7

Keys and Keyways

Learning Outcomes
On the completion of this module the student must be able to:

- Describe keys
    - Gib-head
    - Feather
    - Woodruff
- Describe finding the size of keys
- Describe standard key sizes
- Describe taper pins and splines
- Describe the design of keys
- Describe the design of a splint shaft
- Describe keyways
- Compare keys and cotters

7.1 Introduction

Keyways are cut into shafts and pulleys so that keys can be fitted to ensure a positive drive. The pulley keyway is lined up with the keyway in the shaft. The general dimension of the key is expressed in terms of the shaft diameter.

7.2 Keyway

This is a recess in a shaft or hub to accommodate a key.

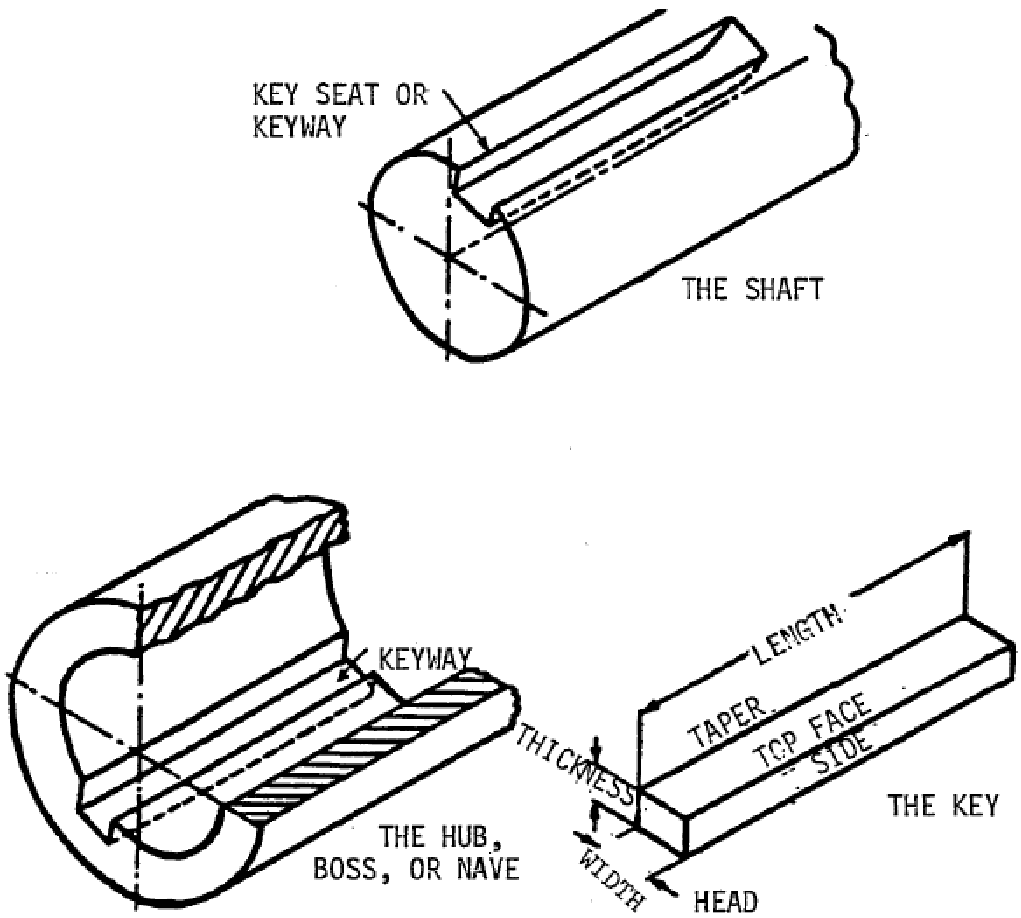


Figure 7.1

Keyways are cut out with milling machines, shaping machines or key seaters. If a horizontal milling machine is used, the resulting keyway will look like the one in **Figure 7.2**.

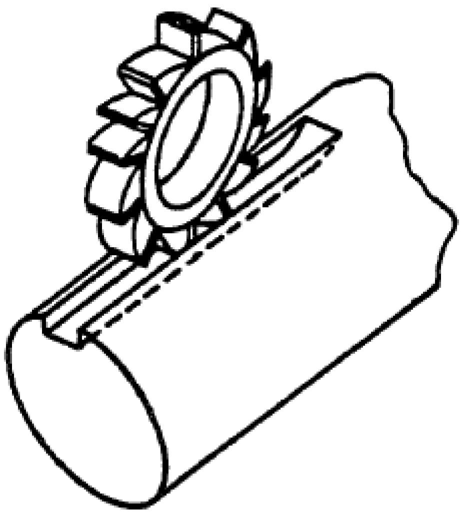


Figure 7.2

If a vertical milling machine is used, the resulting keyway will look like the one in **Figure 7.3**.

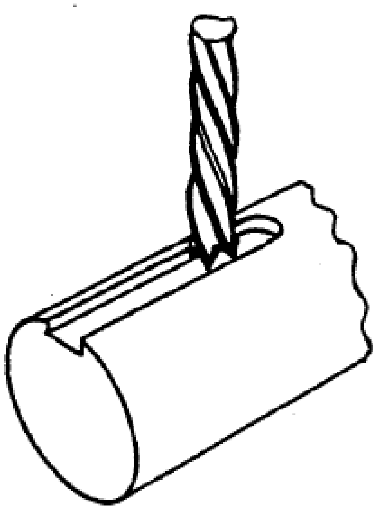


Figure 7.3

In both cases, the end of the milled slot has the same profile as the cutter.

7.3 Keys

A key is defined as a piece of metal inserted between the joint of two parts to prevent relative movement, or as a piece of metal inserted in an axial direction between a shaft and hub to prevent relative rotation.

	Did you know? Keys are always made of steel because they are subjected to considerable crushing and shearing stresses.

7.3.1 Different types of keys

There is a wide variety of keys, designed for light and heavy duties, for parallel and tapered shafts, to allow axial movement of the hub along the shaft and to prevent relative rotation or to prevent both axial movement and relative rotation between the shaft and the hub.

Keys may be divided into five classes:

- sunk
- saddle
- tangent
- round
- splined

7.3.1.1 Sunk keys

These keys are divided into three classes.

- Parallel sunk keys
- taper-sunk keys
- Woodruff sunk keys

- Parallel sunk key

This key may be rectangular or square section, uniform in width and thickness throughout. Ends may be squared or rounded.

	Note: A parallel key is taperless and is used where the pulley, gear, or other mating piece is required to slide along the shaft over the key and then to be fastened with a screw.

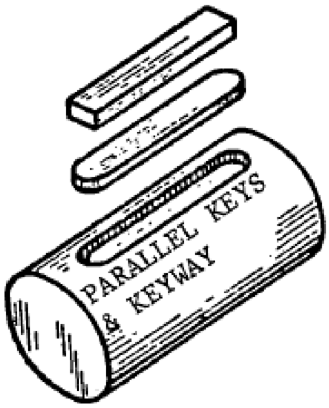


Figure 7.4

Feather key

A feather is a particular kind of parallel key which transmits a turning moment and also axial movement. It is fastened to shaft or the hub, the key being a sliding fit in the keyway of the moving piece.

Figure 7.5 shows a feather key screwed to the shaft.

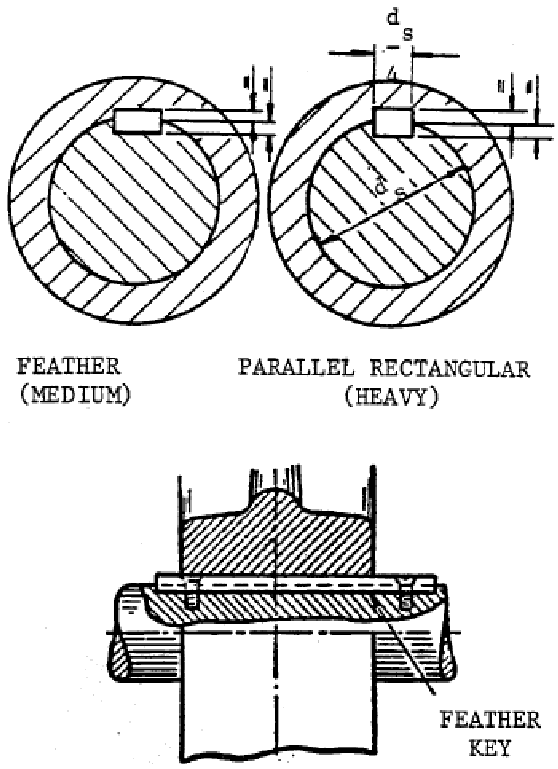


Figure 7.5

In the double-head feather key, the key has a projection at each end to prevent its axial movement in the hub.

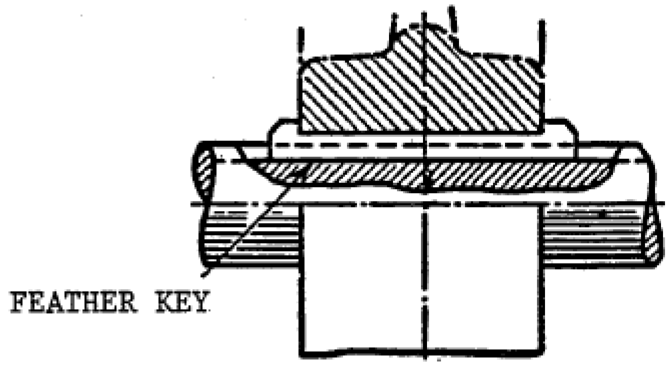


Figure 7.6

Figure 7.7 shows a single-head feather, fitted to the encircling member by means of a screw.

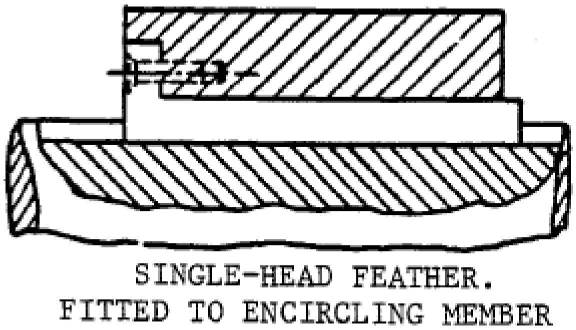


Figure 7.7

Figure 7.8 shows a peg feather key. The peg on a peg feather key usually fits into a hole on the hub or other part of the sliding piece.

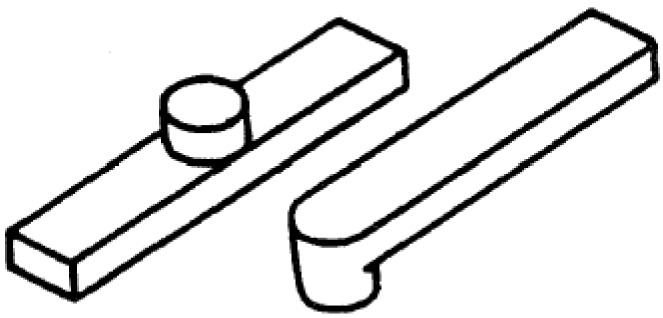


Figure 7.8

Standard proportions for parallel sunk key
Keys may fail either by shearing or by crushing, as already stated.

Equating the moment of the total shear strength of a key to the moment of the compressive strength, and assuming that the crushing stress of the material is twice the shearing stress (ie $\sigma_c = 2\tau$), it is found that theoretically keys should be made square.

This is the approximate cross-section of a feather which is, from its nature, only a running fit in the keyway. In practice, the breadth W (width) of a key is made approximately equal to $\frac{d_s}{4}$ (where d_s equals the diameter of the shaft) and the depth T (thickness) of a key is made approximately equal to

$$= \frac{2}{3}W$$
$$= \frac{2}{3} \times \frac{d_s}{4}$$
$$= \frac{d_s}{6} \text{ so}$$

that keys should rarely fail in shear. The failure is, in fact, generally due to deformation of the key or the shaft through the forces which tend to turn the key over, particularly when those forces alternate in direction.

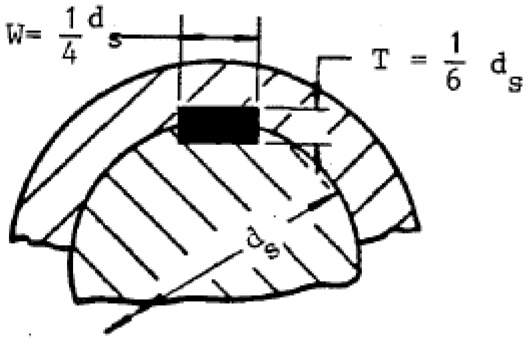


Figure 7.9

A rectangular key is recessed halfway into the boss and halfway into the shaft when measured at the side and not when measured on the centre line.

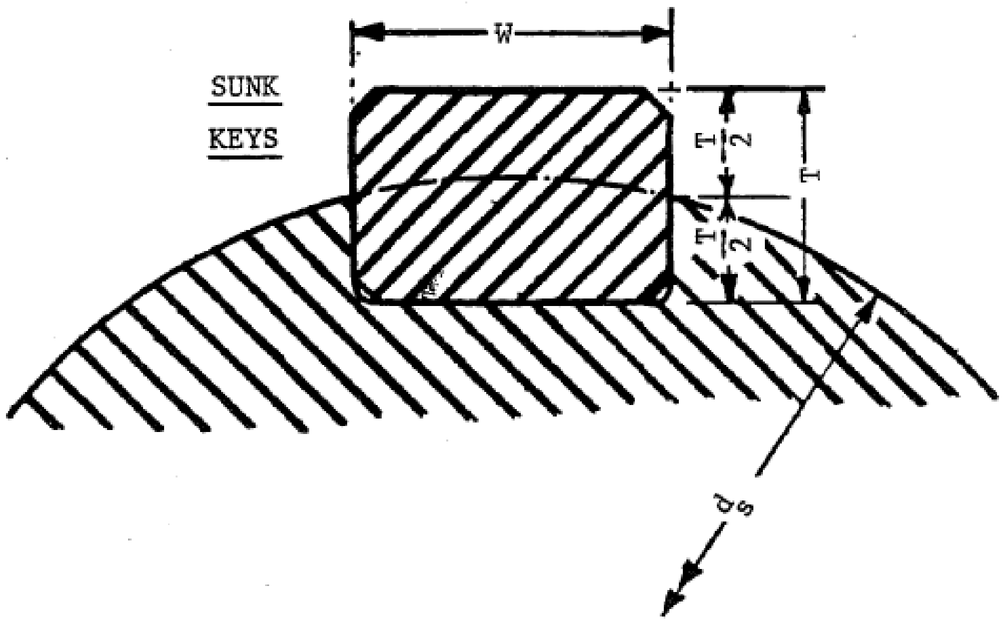


Figure 7.10

The weakening effect of the keyway upon the shaft must be taken into account. The deeper the key in the shaft, the weaker the shaft becomes.

The shaft may be enlarged in the neighbourhood of the keyway, say 25 % of its original diameter.

This also facilitates the entry of the gibheaded key; it is, however, an unnecessary refinement in shafts of normal design, since the cost of forging the enlargement may be greater than the increased cost of a shafts that is slightly bigger in diameter.

Similarly a wheel boss may be enlarged in the neighbourhood of the key.

- Taper-sunk key
  This is a key of rectangular or square section, uniform in width, tapered in thickness. The ends may be squared or rounded. For rigid fastening, the key is made so that it is a good fit on its top and bottom faces and is also a good fit sideways.

  This type of key requires a parallel keyway of uniform depth in the shaft and one of slightly varying depth to suit the taper of the key in the boss of the wheel.

  The standard proportions for a taper key are the same as the proportions for a parallel key, but T (thickness) is measured at the larger end of the taper. The taper is 1 in 100.

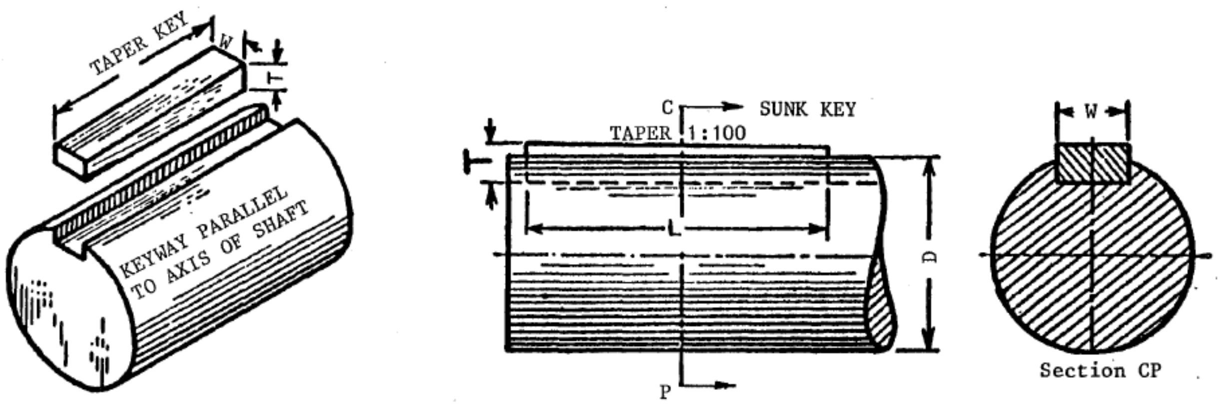


Figure 7.11

Keys are sometimes provided with gib-heads to facilitate withdrawal; a wedge is driven between the face of the boss and the head.

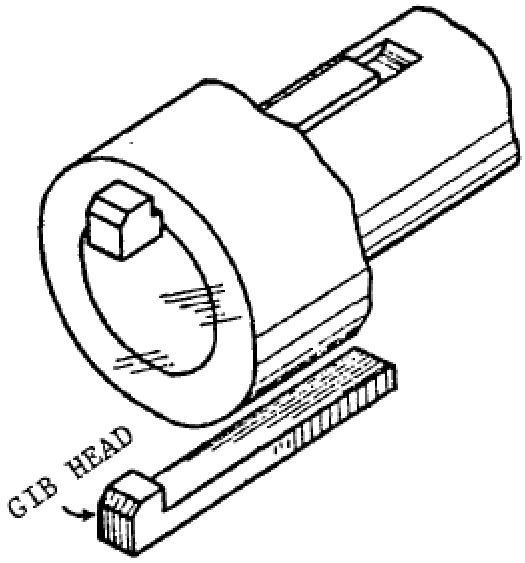


Figure 7.12

The following proportions relating to **Figure 7.12** are suggested for rectangular gib-headed keys:

$$W = \text{standard width} = \frac{d}{4}$$

$$T = standard\ thickness\ at\ large\ end$$

$$= \frac{2}{3}W = \frac{d}{6}$$

$$B = 1\frac{3}{4}T$$

$$C = T$$

$$A = 1\frac{1}{2}T$$

$$Angle\ of\ chamber\ = \ 45°$$

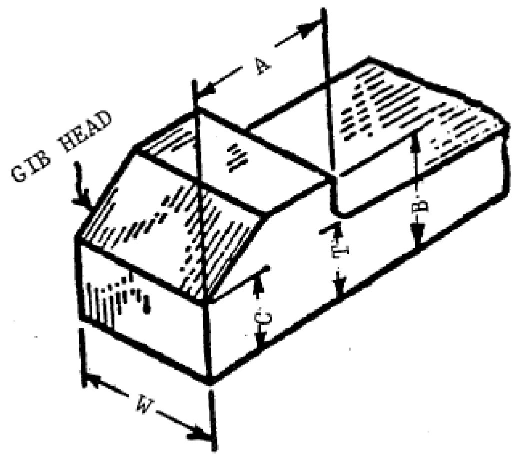


Figure 7.13

- Woodruff sunk key
  The Woodruff key is an easily adjustable sunk key. As will be seen from **Figure 7.14**, it is segmental, being part of a cylindrical disc capable of tilting in a recess milled in the shaft by a cutter having the same curvature as the disc from which the key is made.

  This key is largely used on machine-tool and automobile work.

  Its main advantages are:
  - It accommodates itself to any taper in the hub, nave, or boss of the mating piece
  - it is useful on tapering shafts ends

  The disadvantages are:
  - The singular depth of the keyway weakens the shaft
  - it cannot be used as a feather

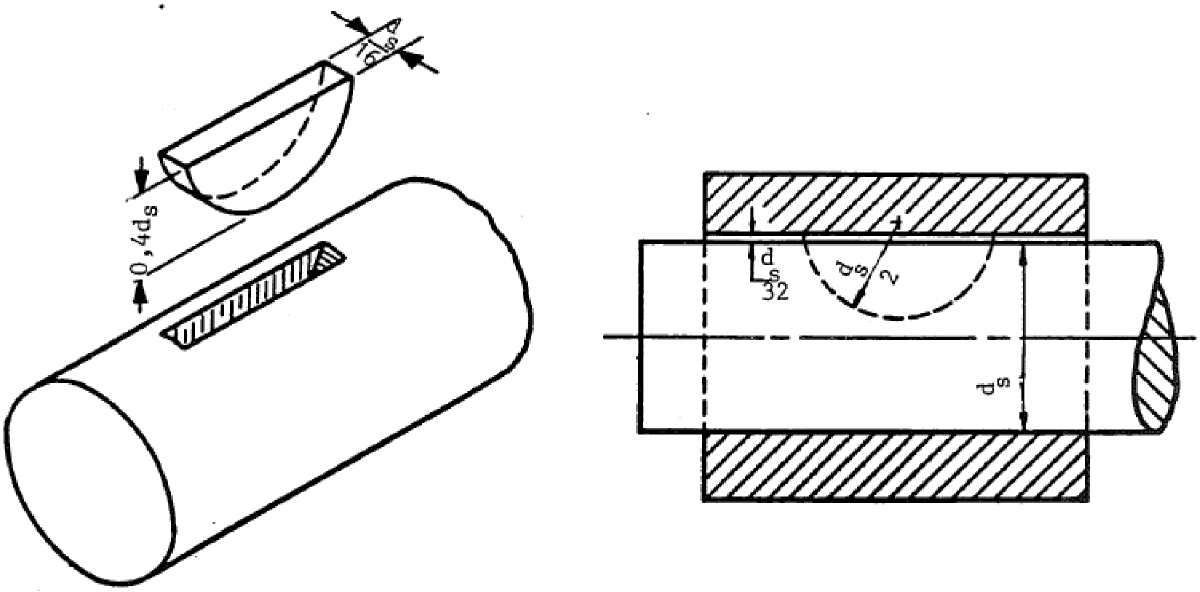


Figure 7.14

7.3.1.2 Saddle keys

These are keys of uniform width, but taper in thickness. They can be in two forms:

- flat saddle keys
- hollow saddle keys

- Flat saddle key

  This is a taper key to fit a keyway in the hub and a flat on the shaft. It is apt to slip round the shaft under heavy load, and thus is suitable for light duty only.

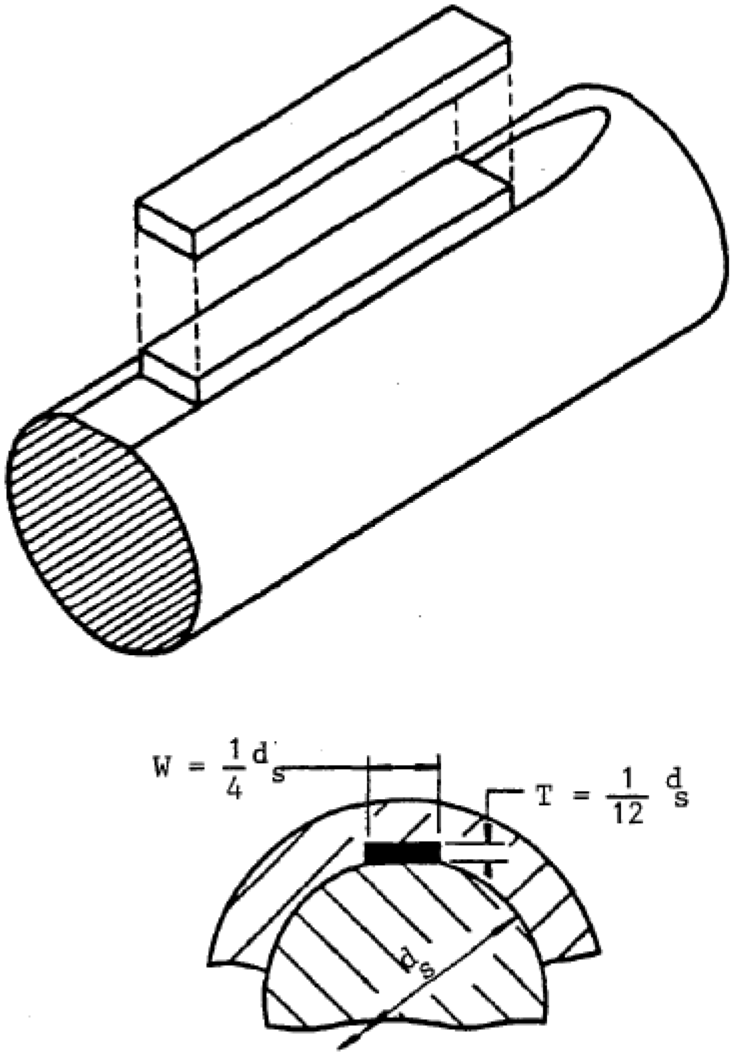


Figure 7.15

- Hollow saddle key

  This, too is a taper key made to fit a keyway in the hub, the bottom of the key being shaped to fit the curved surface of the shaft. It is suitable for light duty for it holds only by friction.

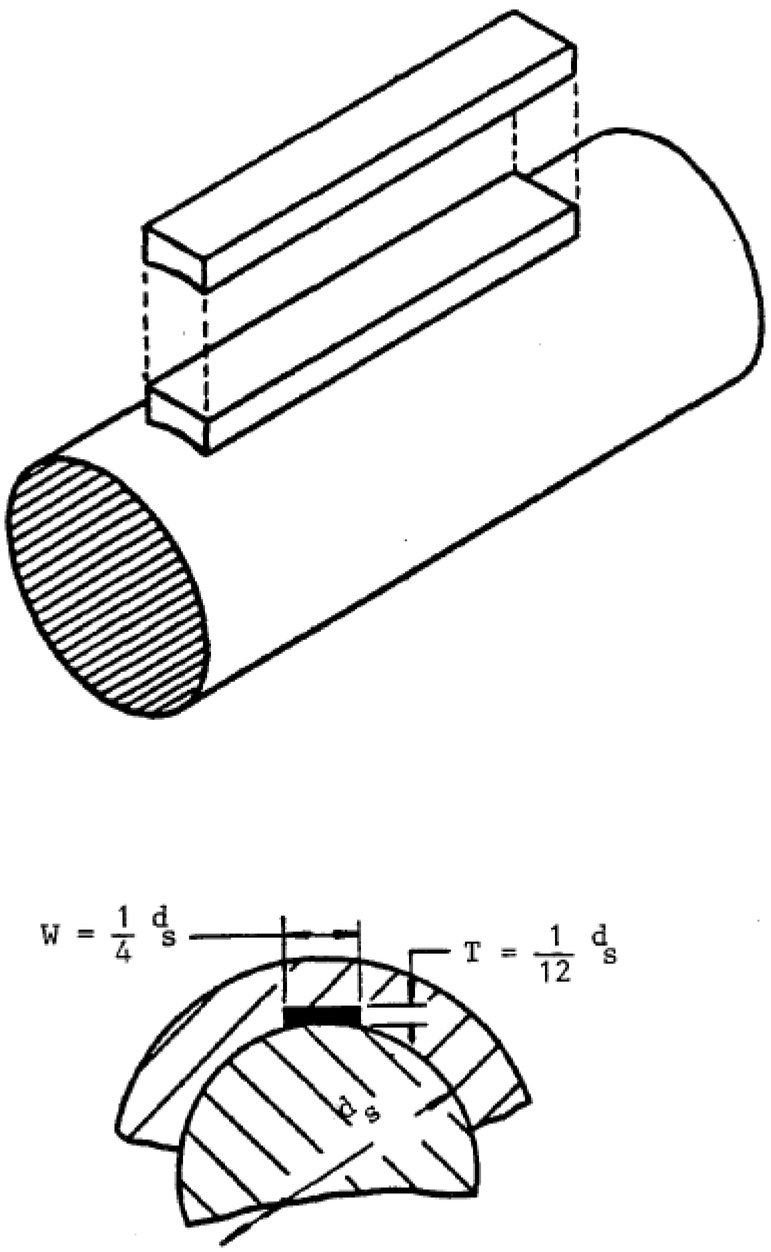


Figure 7.16

7.3.1.3 Tangential keys

When transmitting heavy power or when fitting a large flywheel to shafts, it may be necessary to use more than one key. Flywheels about 3 metres in diameter or more should be secured by two keys at right angles, or by two tangent keys at 120°.

This ensures at least three bearing points and eliminates all risk of rocking. When two or more keys are employed, an allowance must be made in the design on account of the difficulty of getting all keys to fit simultaneously.

This allowance will vary according to the type of work and the general character of the workshop appliances, but in general an assumption of an overload of from 50% to 75% is sufficient to cover all imperfections in manufacture and assembly.

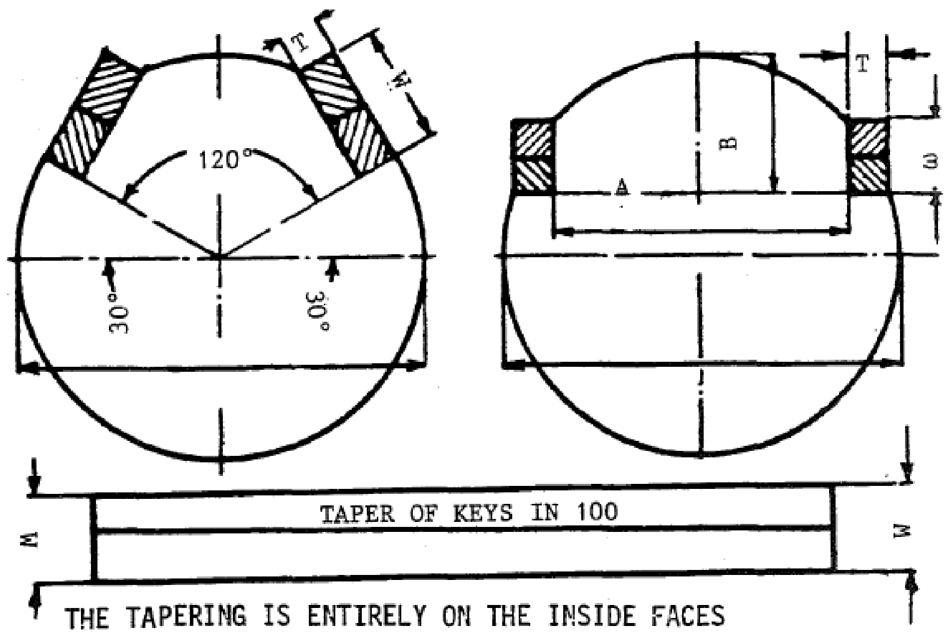


Figure 7.17

For drawing purposes, the following proportions may be used:

$T = 0{,}1\ d_s$	$\omega = 0{,}276\ d_s$	$B = 0{,}345\ d_s$
$W = 0{,}3\ d_s$	$A = 0{,}75\ d_s$	

where d_s is the diameter of the shaft. **Figure 7.18** shows a Lewis key. This is also a form of tangential key. A single key is used as compared with the double keys in the tangential type.

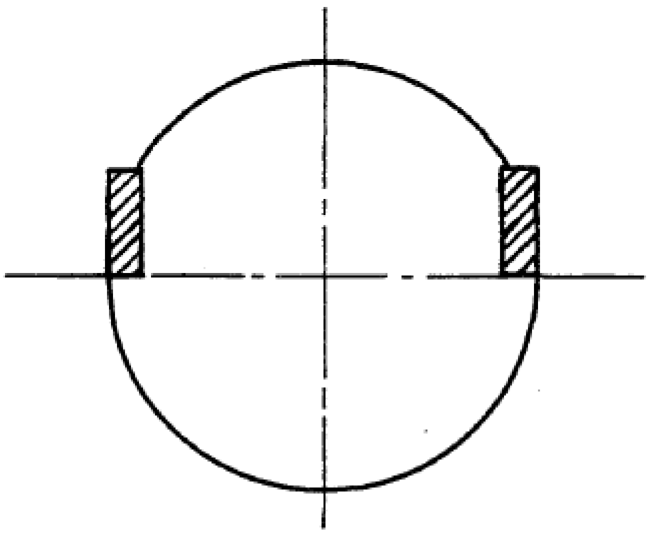


Figure 7.18

7.3.1.4 Round keys
These keys may be parallel or tapered. The parallel one is called a key, while the tapered one is a taper pin.

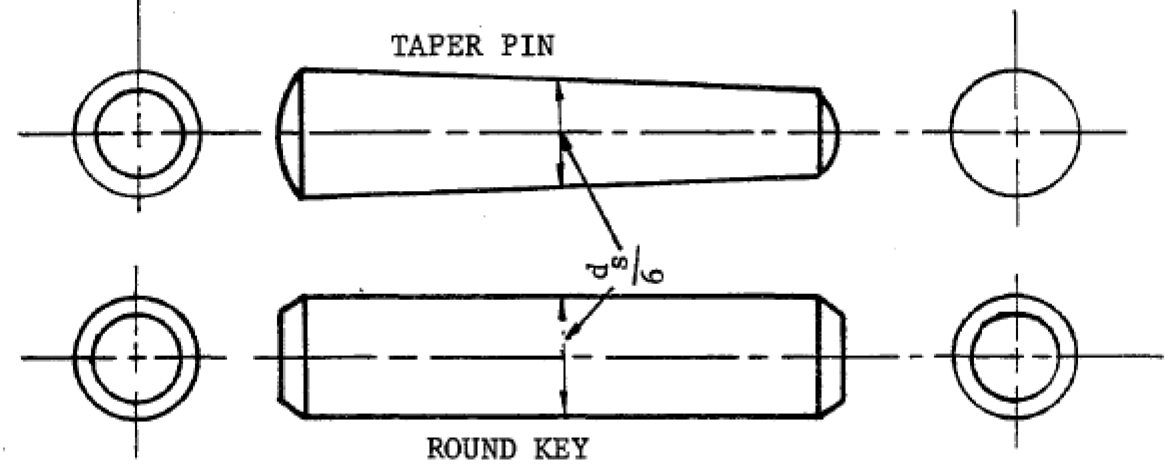


Figure 7.19

They are fitted into a hole drilled partly in the shaft and partly in. the hub. They are used to built-up crankshafts and are also used extensively where the parts are shrunk on. The diameter is usually made $\frac{1}{6}$ the diameter of the shaft.

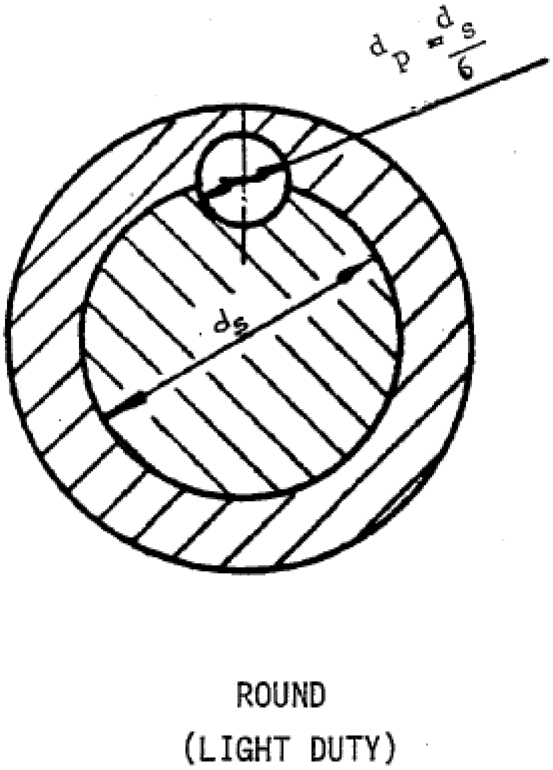


Figure 7.20

Figure 7.21 shows two methods of fixing a hub or collar ring to a shaft, using a taper pin. Standard taper pins have a taper of 1 in 48 on the diameter.

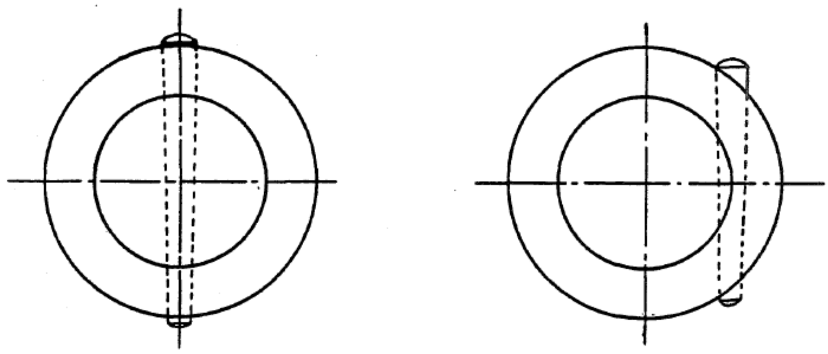


Figure 7.21

7.3.1.5 Splines:

Figure 7.22 shows a single spline shaft. The key or spline is integral (in one piece) with the shaft.

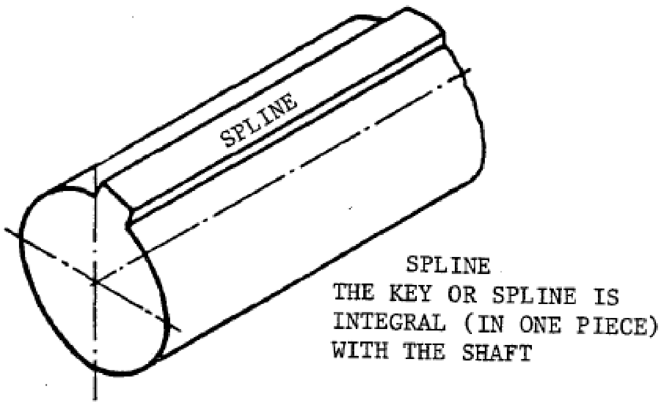


Figure 7.22

If a shaft is carrying very heavy loads, it should be obvious that the load is transferred to the hub via the key. This means that the power that any shaft or hub can transmit is limited by the strength of the key or single spline.

If heavy loading is expected, the shaft and hub will be multi-splined. The number of splines will depend on the load to be carried the greater the number of splines, the greater the permissible loading. Splined shafts are largely used in automobile work.

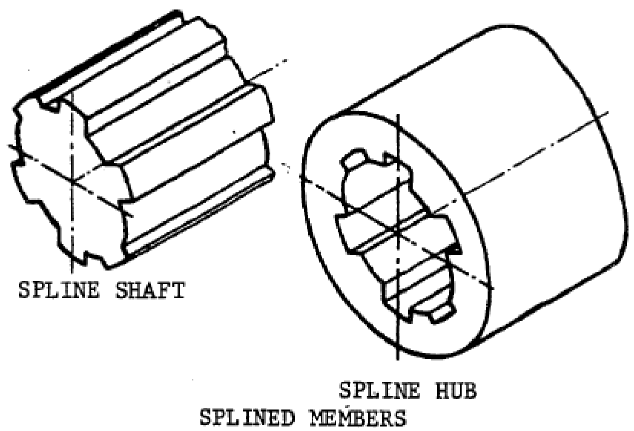


Figure 7.23

7.4 Comparison between keys and cotters

Keys are generally driven in parallel to the axes of shafts which are subjected to torsional, or twisting, stress.

Cotters are generally driven in at right angles to the connected rods or hubs, which are subjected to compressive or tensile stress along their axes. Briefly, keys are usually employed to transmit a twisting moment, whereas cotters are generally used in tension or compression only.

7.5 Design of keys

In finding the sizes of the keys, it is usual to equate the torque on the shaft to the torque on the key.

The key may fail in two ways:

- by shearing
- by crushing

7.5.1 Shearing of key

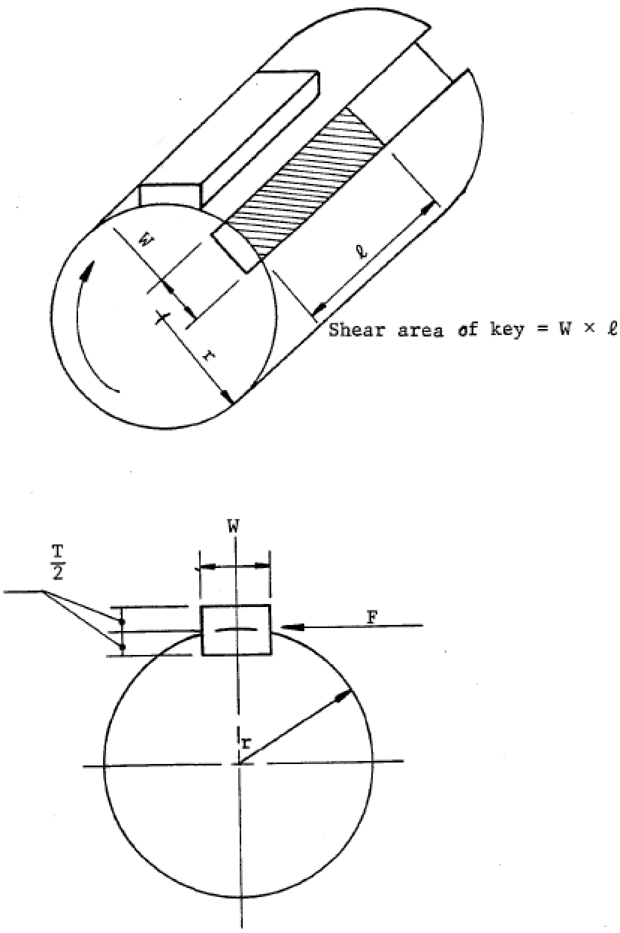


Figure 7.24

$$Torque\ on\ key\ (due\ to\ shearing) = Force \times radius$$

$$T_{key} = F \times r \text{ but } Force = Stress \times Area$$

$$Radius = \frac{Shaft\ diameter}{2}$$
$$= Stress \times Shear\ Area \times \frac{Shaft\ diameter}{2}$$
$$= T_{key} \times W \times l \times \frac{d_s}{2}$$
$$Torque\ on\ shaft = \frac{Power \times 60}{2\,\pi\,N} \text{ also}$$
$$Torque\ on\ shaft = \frac{\pi d_s{}^3}{16} \times \tau_{shaft}$$

Equate the torque on the shaft to the torque on the key.

$$Torque\ on\ shaft = Torque\ on\ key \text{ (due to shear)}$$
$$\frac{Power \times 60}{2\,\pi\,N} = T_{key} \times W \times l \times \frac{d_s}{2}$$

or

$$\frac{\pi d_s{}^3}{16} \times \tau_{shaft} = T_{key} \times W \times l \times \frac{d_s}{2}$$

7.5.2 Crushing of key

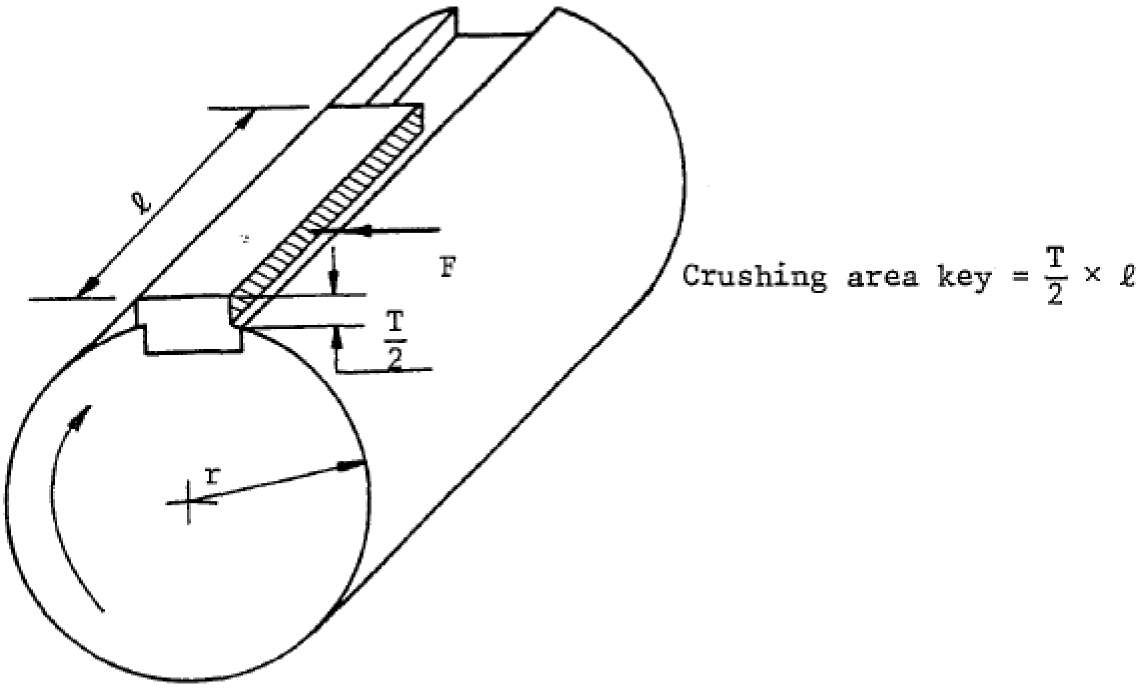


Figure 7.25

$$Torque\ on\ key\ (due\ to\ crushing) = Force \times radius$$
$$\text{but } Force = Stress \times Area$$
$$Radius = \frac{Shaft\ diameter}{2}$$

$$T_{key} = F \times r$$
$$= Stress \times Crushing\ Area \times \frac{Shaft\ diameter}{2}$$
$$Q_{c_{key}} = \frac{T}{2} \times l \times \frac{d_s}{2}$$

Now again

$$Torque\ on\ shaft = Torque\ on\ key \text{ (due to crushing)}$$

$$\frac{Power \times 60}{2\,\pi\,N} = Q_{c_{key}} \times \frac{T}{2} \times l \times \frac{d_s}{2}$$

or

$$\frac{\pi {d_s}^3}{16} \times \tau_{shaft} = Q_{c_{key}} \times \frac{T}{2} \times l \times \frac{d_s}{2}$$

where

d_s = diameter of shaft (m)
W = width of key (m)
T = thickness of key (m)
l = length of key (m)
τ_{shaft} = allowable shear stress in shaft materia 1 (Pa)
$T_{\ key}$ = allowable shear stress in key material (Pa)
σ_c = crushing stress in key material (Pa):
P = Power in watts

The usual method adopted is to work out the width of the key and ·the thickness of the key from empirical formulae and the shaft diameter. These are then used in (a) and (b) to find the length of the key required.

Alternatively, the tables for sizes of keys included in this lecture may be used to find the width and thickness, after which the length may be found as above.

1	2	3	4	5	6	7
Shaft		**Key**	**Keyway**			
diameter		**section**	**Width**	**Depth**		
Over	**to**	**W x T**	**Shaft and hub**		**Hub**	
					Parallel	**Taper**
6	8	2 X 2	2	1,2	1,0	0,5
8	10	3 X 3	3	1,8	1,4	0,9
10	12	4 X 4	4	2,5	1,8	1,2
12	17	5 X 5	5	3,0	2,3	1,7
17	22	6 X 6	6	3,5	2,8	2,2
22	30	8 X 7	8	4,0	3,3	2,4
30	38	10 X 8	10	5,0	3,3	2,4
38	44	12 X 8	12	5,0	3,3	2,4
44	50	14 X 9	14	5,5	3,8	2,9
50	58	16 X 10	16	6,0	4,3	3,4
58	65	18 X 11	18	7,0	4,4	3,4
65	75	20 X 12	20	7,5	4,9	3,9

75	85	22 X 14	22	9,0	5,4	4,4
85	95	25 X 14	25	9,0	5,4	4,4
95	110	28 X 16	28	10,0	6,4	5,4
110	130	32 X 18	32	11,0	7,4	6,4
130	150	36 X 20	36	12,0	8,4	7,1
150	170	40 X 22	40	13,0	9,4	8,1
170	200	45 X 25	45	15,0	10,4	9,1
200	230	50 X 28	50	17,0	11,4	10,1
230	260	56 X 32	56	20,0	12,4	11,1
260	290	63 X 32	63	20,0	12,4	11,1
290	330	70 X 36	70	22,0	14,4	13,1
330	380	80 X 40	80	25,0	15,4	14,1
380	440	90 X 45	90	28,0	17,4	16,1
440	500	100 X 50	100	31,0	19,5	18,1

Table 7.1

Worked Example 7.1

A shaft, 114 mm in diameter, transmits 75 kW at 100 r/min. The key in the driving pulley is 100 mm long. How wide should it be if the shear stress in the material is not to exceed 70 MPa?

Solution:

$$Torque\ on\ shaft = Torque\ on\ key \text{ (due to shearing)}$$

$$\frac{P \times 60}{2\,\pi\,N} = T_{\ key} \times W \ \times l \times \frac{d_s}{2}$$

$$\frac{75\times10^3\times60}{2\times\pi\times100} Nm = 70 \times 10^6 \frac{N}{m^2} \times W \times 0{,}1m \times \frac{0{,}114\ m}{2}$$

$$7162\ Nm = 399000 \times W\ N$$

$$\therefore W = \frac{7162\ Nm}{399\times10^3 N}$$

$$= 0{,}0179\ m$$

Say 18 mm wide.

Worked Example 7.2

A square key is to be used to key a gear to a 36mm shaft. The hub length of the gear is 64 mm. Both shaft and key are made of the same material having the same allowable shear stress. What are the minimum dimensions of the sides of the square key?

Solution:

$$Torque\ on\ shaft = Torque\ on\ key \text{ (due to shear)}$$

$$\frac{\pi {d_s}^3 \tau_{shaft}}{16} = T_{\ key} \times W \ \times l \times \frac{d_s}{2} \text{ but } T_{\ key} = \tau_{shaft}$$

$$\frac{\pi(0{,}036\ m)^3\tau_{shaft}}{16} = \tau_{shaft} \times W \times 0{,}064\ m \times \frac{0{,}036\ m}{2}$$

$$W = \frac{\pi(0{,}036\ m)^3\tau_{shaft}}{16} \times \frac{2}{0{,}064\ m \times 0{,}036\ m}$$

$$= 0{,}00795\ m$$

Say 8 mm wide by 8 mm thick

Worked Example 7.3

A feather key is 13 mm wide by 10 mm deep, and is to transmit 680 Nm of torque from a shaft 38 mm in diameter. The steel key has an allowable stress in tension and compression of 110 MPa and an allowable stress in shear of 55 MPa. Determine the required length of the key.

Solution:

Length required to resist shear:

$$Torque\ on\ shaft = Torque\ on\ key \text{ (due to shear)}$$

$$\tau_{shaft} = T_{\ key} \times W \times l \times \frac{d_s}{2}$$

$$680\ Nm = 55 \times 10^6 N/m^2 \times 0{,}013 \times l \times \frac{0{,}038\ m}{2}$$

$$l = \frac{680\ Nm \times 2}{55 \times 10^6 N/m^2 \times 0{,}013 \times 0.038\ m}$$

$$= 0{,}05\ m$$

$$l = 50\ mm$$

Length required to resist compression:

$$Torque\ on\ shaft = Torque\ on\ key \text{ (due to crushing)}$$

$$\tau_{shaft} = Q_{c_{key}} \times \frac{I}{2} \times l \times \frac{d_s}{2}$$

$$680\ Nm = 110 \times 10^6 N/m^2 \times \frac{0{,}01\ m}{2} \times l \times \frac{0{,}038\ m}{2}$$

$$l = \frac{680\ Nm \times 2 \times 2}{110 \times 10^6 N/m^2 \times 0{,}01 \times 0.038\ m}$$

$$= 0{,}065\ m$$

$$l = 65\ mm$$

Use a key of 65 mm in length.

Worked Example 7.4

A feather key is 13 m wide by 10 mm deep, and is to transmit 680 Nm of torque from a shaft 38 mm in diameter. The steel key has an allowable stress in tension and compression of 110 MPa and an allowable stress in shear of 55 MPa. Determine the required length of the key.

Solution:

Length of key required to resist shear:

$$Torque\ on\ shaft = Torque\ on\ key \text{ (due to shear)}$$
$$\tau_{shaft} = T_{\ key} \times W \times l \times \frac{d_s}{2}$$
$$680\ Nm = 55 \times 10^6 N/m^2 \times 0{,}01 \times l \times \frac{0{,}038\ m}{2}$$
$$l = \frac{680\ Nm \times 2}{55 \times 10^6 N/m^2 \times 0{,}01 \times 0.038\ m}$$
$$= 0{,}065\ m$$
$$l = 65\ mm$$

Length of key required to resist crushing:

$$Torque\ on\ shaft = Torque\ on\ key \text{ (due to crushing)}$$
$$\tau_{shaft} = Q_{c_{key}} \times \frac{T}{2} \times l \times \frac{d_s}{2}$$
$$680\ Nm = 110 \times 10^6 N/m^2 \times \frac{0{,}013\ m}{2} \times l \times \frac{0{,}038\ m}{2}$$
$$l = \frac{680\ Nm \times 2 \times 2}{110 \times 10^6 N/m^2 \times 0{,}013 \times 0.038\ m}$$
$$= 0{,}050\ m$$
$$l = 50\ mm$$

Use a key of 65 mm in length.

Worked Example 7.5

A shaft of 40 mm in diameter is transmitting a torque of 500 Nm by means of a pin key as shown. The material of the pin has an allowable shear stress of 68 MPa. Find the size of the pin.

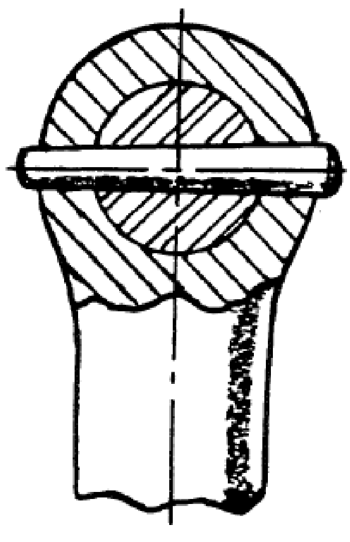


Figure 7.26

Solution:

$$Torque\ on\ shaft = Torque\ on\ pin$$
$$T_{shaft} = Force \times radius$$
$$= Shear\ stress \times Shear\ area \times radius$$
$$T_{shaft} = \tau \times \frac{2\pi {d_p}^2}{4} \times \frac{d_s}{2}$$
$$500\ Nm = 68 \times 10^6 N/m^2 \times \frac{2\pi {d_p}^2}{4} \times \frac{0{,}040\ m}{2}$$
$${d_p}^2 = \frac{500\ Nm\ 4\times 2}{2\times\pi\times 0{,}040\ m\times 68\times 10^6 N/m^2}$$
$$d_p = \sqrt{2{,}341 \times 10^{-4} m^2}$$
$$= 0{,}015\ 3\ m$$

Say $d_p = 16\ mm$ in diameter

7.6 Design of a spline shaft

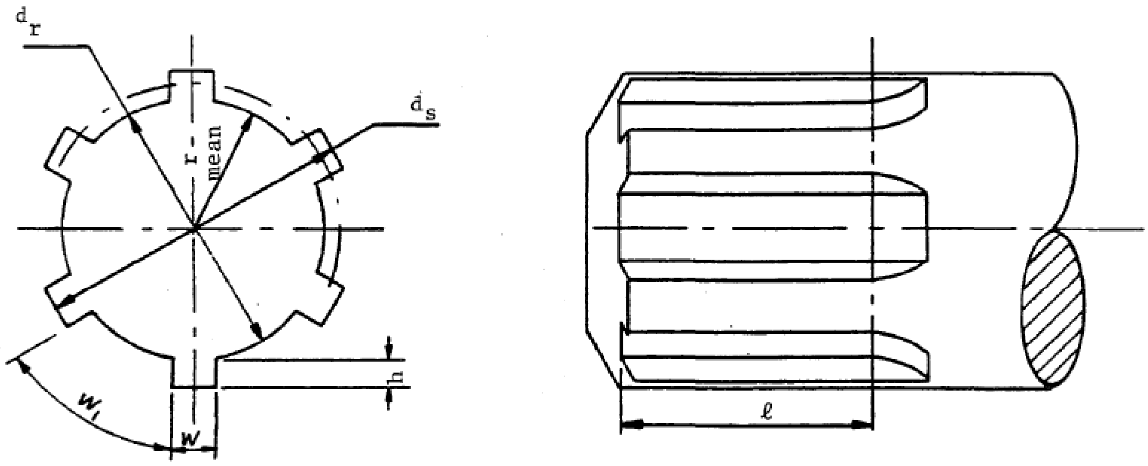


Figure 7.27

Splined connections as shown above are used to permit relative axial movement between the shaft and hub of the connected member. The splines are keys made integrally with the shaft and usually are four, six or ten in number.

The splines are usually made with straight sides or cut with an involute profile. When there is relative axial movement in the splined connection, the side pressure on the splines should be limited to about 7 MPa.

The torque capacity of a splined connection is:

i. $T = p \times A \times r_m$
ii. $d_s = d_r + 2h$
iii. $d_r = d_s - 2h$
iv. $h = \frac{d_s - d_r}{2}$
v. $If\ W = W_1\ then\ W = W_1 = \frac{\pi d_r}{2n}$

where

p = permissible pressure on the splines < 7 MPa
A = total area of the splines, square metres
$A = \frac{1}{2}(d_s - d_r) \times (number\ of\ splines) \times l$
d_s = shaft diameter [m]
d_r = the root diameter of the splined shaft [m]
h = the height of splines [m]
W = the width of splines [m]
l = effective length of splines [m]
n = the number of splines on the shaft
W_1 = the width of the keyways [m]
r_m = mean radius [m]
$= \frac{d_s+d_r}{4}$
T = torque [Nm]

Worked Example 7.6

A splined connection in an automobile transmission consists of 10 splines cut in a shaft of 58 mm in diameter. The height of each spline is 5,5 mm and the keyways in the hub are 45 mm long.

Determine:

a. the power that may be transmitted at 2 500 r/min if the allowable normal pressure on the splines is limited to 5 MPa.
b. the force required to slide the hub axially under full load ($\mu = 0{,}1$).

Solution:

$$d_r = d_s - 2h$$
$$= 0{,}058\ m - 0{,}0055\ m$$
$$= 0{,}047\ m$$
$$A = \frac{1}{2}(0{,}058\ m - 0{,}047\ m) \times 10 \times 0{,}045\ m$$
$$= 2{,}475 \times 10^{-3} m^2$$
$$r_m = \frac{d_s+d_r}{4}$$
$$= \frac{0{,}058\ m - 0{,}047\ m}{4}$$
$$= 0{,}02625\ m$$
$$T = p \times A \times r_m$$
$$= 5 \times 10^6 N/m^2 \times 2{,}475 \times 10^{-3} m^2 \times 0{,}0265\ m$$
$$= 324{,}8\ Nm$$

1.
$$Power = \frac{2\,\pi\,T\,N}{60}$$
$$= \frac{2 \times \pi \times 324{,}8 \times 2500}{60}$$
$$Power = 85{,}03\ kW$$

2. $force\ on\ splines = p \times A$

$= 5 \times 10^6 N/m^2 \times 2{,}475 \times 10^{-3} m^2$

$= 12375\ N$

$sliding\ force = force\ on\ splines \times friction\ coefficient$

$= 12375 \times 0{,}1$

$= 1237{,}5\ N$

Note

$sliding\ force = friction\ force$

Worked Example 7.7

A splined connection is required for the transmission of an automobile having an engine developing a maximum power of 84 kW at 1 200 r/min.

The maximum torque exceeds the mean torque by 45%. There are 10 splines cut in a shaft. The width of the splines is equal to 0,5 times the width of the grooves. The shear stress in the shaft due to twisting must not exceed 45 MPa.

Calculate:
(i) the maximum torque transmitted by the shaft
(ii) the core diameter of the shaft
(iii) the width of the splines
(iv) the width of the grooves
(v) the outside diameter of shaft
(vi) the height of the splines
(vii) the shear stress in the splines

<u>Solution:</u>

(i) maximum torque transmitted

$$T_{mean} = \frac{Power \times 60}{2\ \pi\ N}$$

$$= \frac{84 \times 10^3 \times 60 Nm}{2 \times \pi \times 1200}$$

$$= 668{,}5\ Nm$$

$$T_{max} = T_{mean} \times \frac{145}{100}$$

$$= 668{,}5\ Nm \times \frac{145}{100}$$

$$T_{max} = 969{,}3\ Nm$$

(ii) the core diameter of the shaft

$$T_{max} = \frac{\pi {d_r}^3}{16} \times \tau$$

$${d_r}^3 = \frac{T_{max} \times 16}{\pi \times \tau}$$

$$= \frac{969{,}3\ Nm\ 16}{\pi \times 45 \times 10^6 Nm^2}$$

$d_r = \sqrt[3]{1097 \times 10^{-4} m^3}$
$= 0{,}0479$
Say $d_r = 48\ mm$ in diameter

(iii) the width of the splines

$W = 0{,}5W_1 \qquad \therefore W_1 = \frac{W}{0{,}5}$
$W_1 = 2\ W$
$Circumference\ of\ core\ diameter = \pi\ d\ r$
Also: $circumference = n(w + W_1)$
$= n(W + 2W)$
$= 3\ n\ W$
$\therefore \pi d_r = 3\ n\ W$
$W = \frac{\pi d_r}{3\ m}$
$= \frac{\pi \times 0{,}048\ m}{3 \times 10}$
$= 0{,}005\ m$
$W = 5\ mm$

(iv) the width of the grooves

$W_1 = 2\ W$
$= 2 \times 5\ mm$
$W_1 = 10\ mm$

(v) the outside diameter of shaft

Assume $p = 7\ MPa$ and $l = 60\ mm$
$T = a \times p \times r_m$
$T = \frac{1}{2}(d_s - d_r) \times \pi \times l \times p \times \frac{(d_s + d_r)}{4}$
$969{,}1 = \frac{1}{2}(d_s - 0{,}048\ m) \times 10 \times 0{,}06 \times 7 \times 10^6 \times \frac{(d_s + 0{,}048)}{4}$
$\frac{969{,}3}{10 \times 0{,}06 \times 7 \times 10^6} = = \frac{1}{2}(d_s - 0{,}048\ m) \times = \frac{1}{4}(d_s - 0{,}048)$
$2308 \times 10^{-4} = 0{,}125 \times (d_s{}^2 - 0{,}048^2)$
$d_s{}^2 - 0{,}048^2 = 1{,}8464 \times 10^{-3}$
$d_s{}^2 = 1{,}8464 \times 10^{-3} + 0{,}048^2$
$d_s = \sqrt{4{,}1504 \times 10^{-3}}$
$= 0{,}0644$
Say $d_s = 64\ mm$ in diameter

(vi) height of the splines

$h = \frac{d_s - d_r}{2}$
$= \frac{64\ mm - 48\ mm}{2}$
$h = 8\ mm$

(vii) shear stress in the splines

$Torque\ on\ shaft = Torque\ on\ splines$

$$\tau_{shaft} = \tau_{splines} \times n \times W \times l \times \frac{d_r}{2}$$

$$969{,}3\ Nm = \tau_{splines} \times 10 \times 0{,}005\ m \times 0{,}06\ m \times \frac{0{,}048\ m}{2}$$

$$\tau_{splines} = \frac{969{,}3\ Nm \times 2}{10 \times 0{,}005\ m \times 0{,}06\ m \times 0.048\ m}$$

$$= 13{,}5\ MPa$$

Activity 7.1

1. Define a key.
2. Keys are divided into five classes. Name each class.
3. When will a feather key be used in a design?
4. A standard Woodruff key for a shaft (64 mm in diameter) is to be used. Calculate the size of key needed.
5. Write down the standard proportions for a gib-headed key.
   If this gib-headed key is going to be used on a shaft having a diameter of 100 mm, what will the size of the gib-headed key be?
6. A pin key is used to secure a lever to a shaft with a diameter of 50 mm. If the torque transmitted is 960 N.m and the material of the pin has an allowable stress of 78 MPa, find the size of the pin required.
7. A shaft is 38 mm in diameter, and transmits a torque of 645 Nm. The allowable stress in tension and compression for the key is 100 MPa and the allowable shear stress in the key is 60 MPa. Use the tables provided to determine the width and thickness of the key, and then determine the required length of the key.
8. A shaft of 48 mm in diameter has six splines cut into it, the splines being 40 mm long. The depth of each spline is 7 mm. The allowable normal pressure on the splines is limited to 4,5 MPa. For a speed of 2 200 r/min, determine the power that may be transmitted.
9. A motor-car transmission consists of a splined connection having 10 splines cut in a shaft of 60 mm. The keyways in the hub are 45 mm long, and the power transmitted by the splines is 90 kW when the transmission is rotating at 2 500 r/min. ·If the allowable normal pressure on the splines is limited to 5 MPa, determine the required height of the splines.

Self-Check

I am able to:	Yes	No
• Describe keys		
o Gib-head		
o Feather		
o Woodruff		
• Describe finding the size of keys		
• Describe standard key sizes		
• Describe taper pins and splines		

• Describe the design of keys		
• Describe the design of a splint shaft		
• Describe keyways		
• Compare keys and cotters		
If you have answered 'no' to any of the outcomes listed above, then speak to your facilitator for guidance and further development.		

Module 8

Couplings

Learning Outcomes
On the completion of this module the student must be able to:

- Describe shaft couplings
  - Flange couplings
  - Strength of the coupling
  - Marine coupling
  - Flexible couplings
  - Muff coupling
- Describe claw coupling
  - Standard proportions
  - Design
- Describe universal joint or Hooke's coupling
  - Variation in angular velocity of driving shaft
  - Determine maximum and minimum angular velocities
- Calculate the size of bolts for a flange coupling, given the number of bolts and pitch circle diameter to transmit the power at a given speed by equating the torque on the shaft to the torque on the bolts

8.1 Introduction

Couplings are basically used to join shafts which allows power to be transmitted, or to join pipes. Such a coupling must be able to transmit the maximum torque generated in the shaft. Couplings are divided into two groups: solid and flexible.

8.2 Shaft coupling

Shaft couplings are used to join two shafts which are required to transmit rotary motion. They may also, in special circumstances, be used as belt pulleys.

8.3 Flange couplings (rigid type)

These are used for shafts of which the diameter exceeds about 75 mm. The flanges may be made of cast iron and keyed to each shaft or forged integrally with the shaft.

The flanges are connected by fitted bolts. Solid flange couplings are more expensive, and are used only when considerations of space and mass affect the design.

?	**Did you know?** Cast iron couplings may be made plain, but are preferably shrouded, that is, the flanges are recessed to shield the heads and nuts of the bolts.

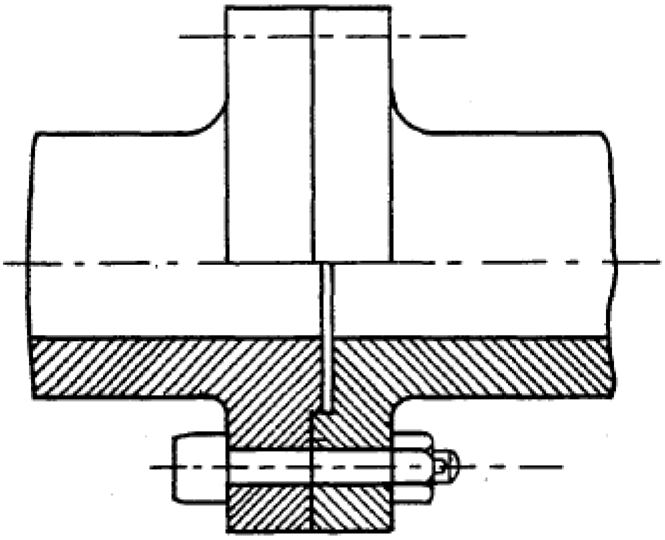


Figure 8.1

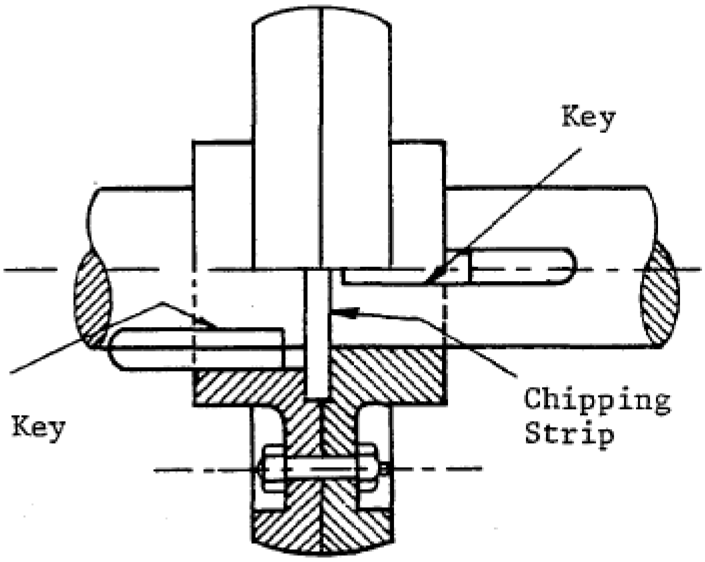


Figure 8.2

Each flange should be forced onto the shaft by hydraulic pressure, and then trued up in a lathe in order to ensure accuracy of alignment.

One shaft may enter the coupling of the other about 8 mm, or a projection ± 6 mm on one coupling may fit into a recess on the other.

This, however, increases the difficulty of assembling and dismounting the shafting.

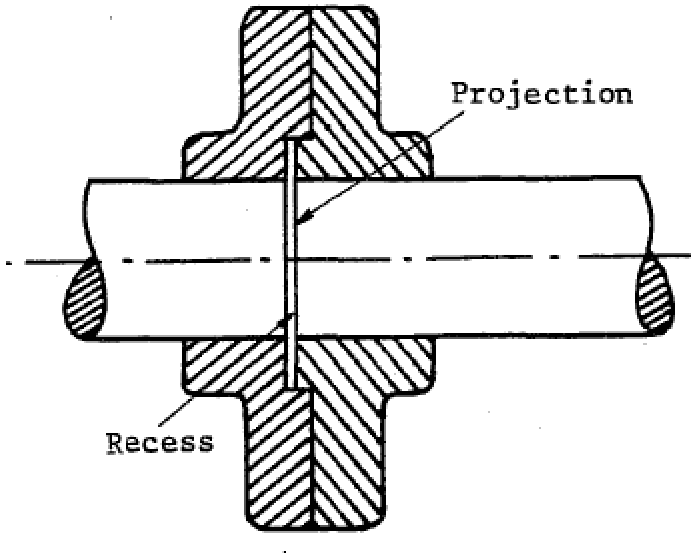


Figure 8.3

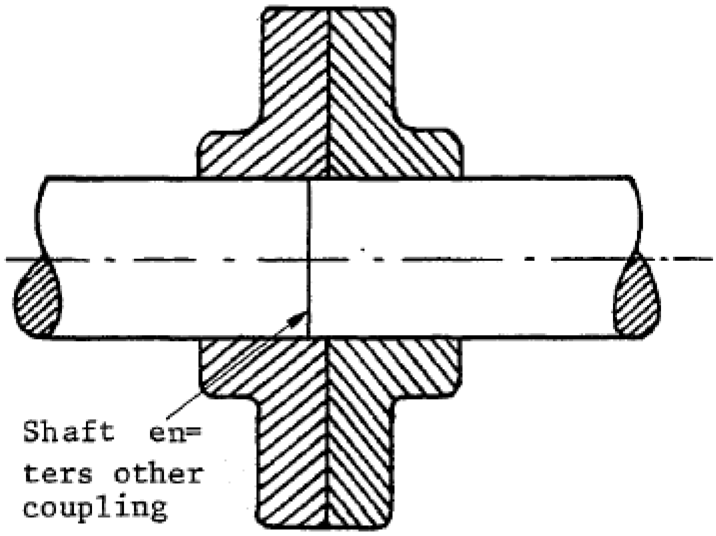


Figure 8.4

The bolts connecting the flanges are in shear. They should be fitted bolts, and the holes should be reamed for accuracy. The keys in the couplings should fit at the sides and not at the top and bottom.

8.4 Standard Proportions for Flange Couplings

8.4.1 Plain Flange Coupling

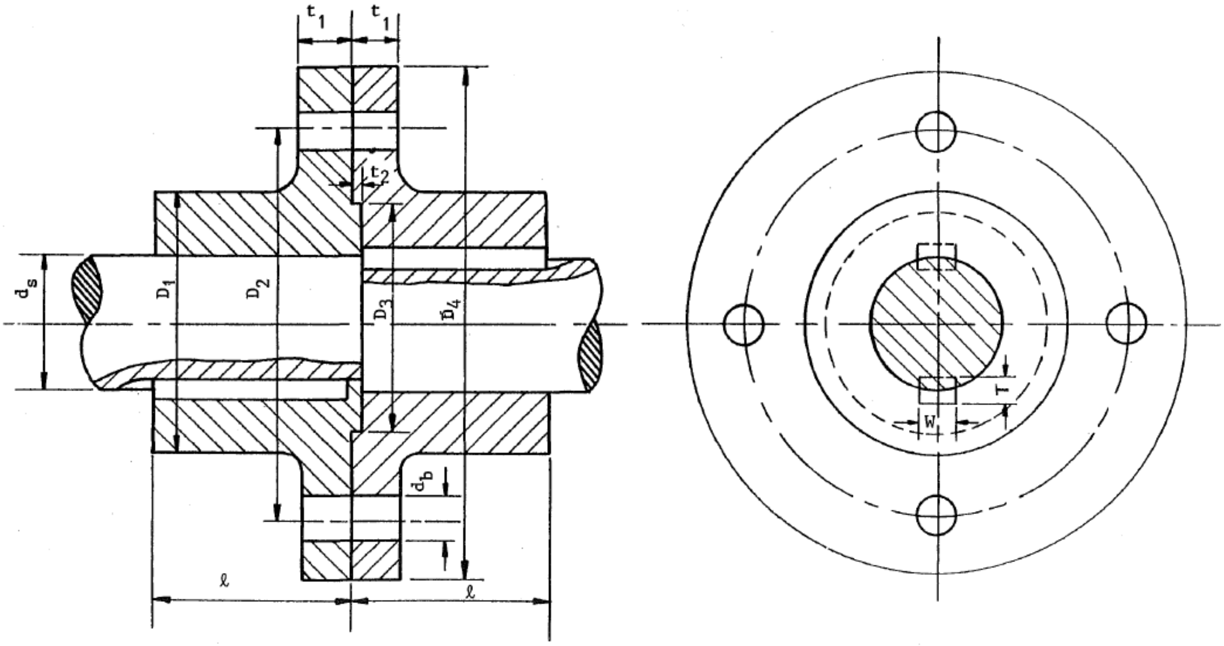


Figure 8.5

$D_1 = 2d_s$

$D_2 = 3d_s$

$D_3 = 1{,}75\ d_s$

$D_4 = 3d_s + 3d_b$

$d_b = 0{,}25\ d_s$

$l = 1{,}5\ d_s$

$t_1 = 0{,}5\ d_s$

$t_2 = 5\ mm\ to\ 10\ mm$

$n = 0{,}02\ d_s + 3$ (to the nearest number)

Key sizes $W = \frac{d_s}{4}$

$T = \frac{d_s}{6}$

Note:
d_s must be in mm. n = number of coupling bolts.

8.4.2 Shrouded Flange Coupling

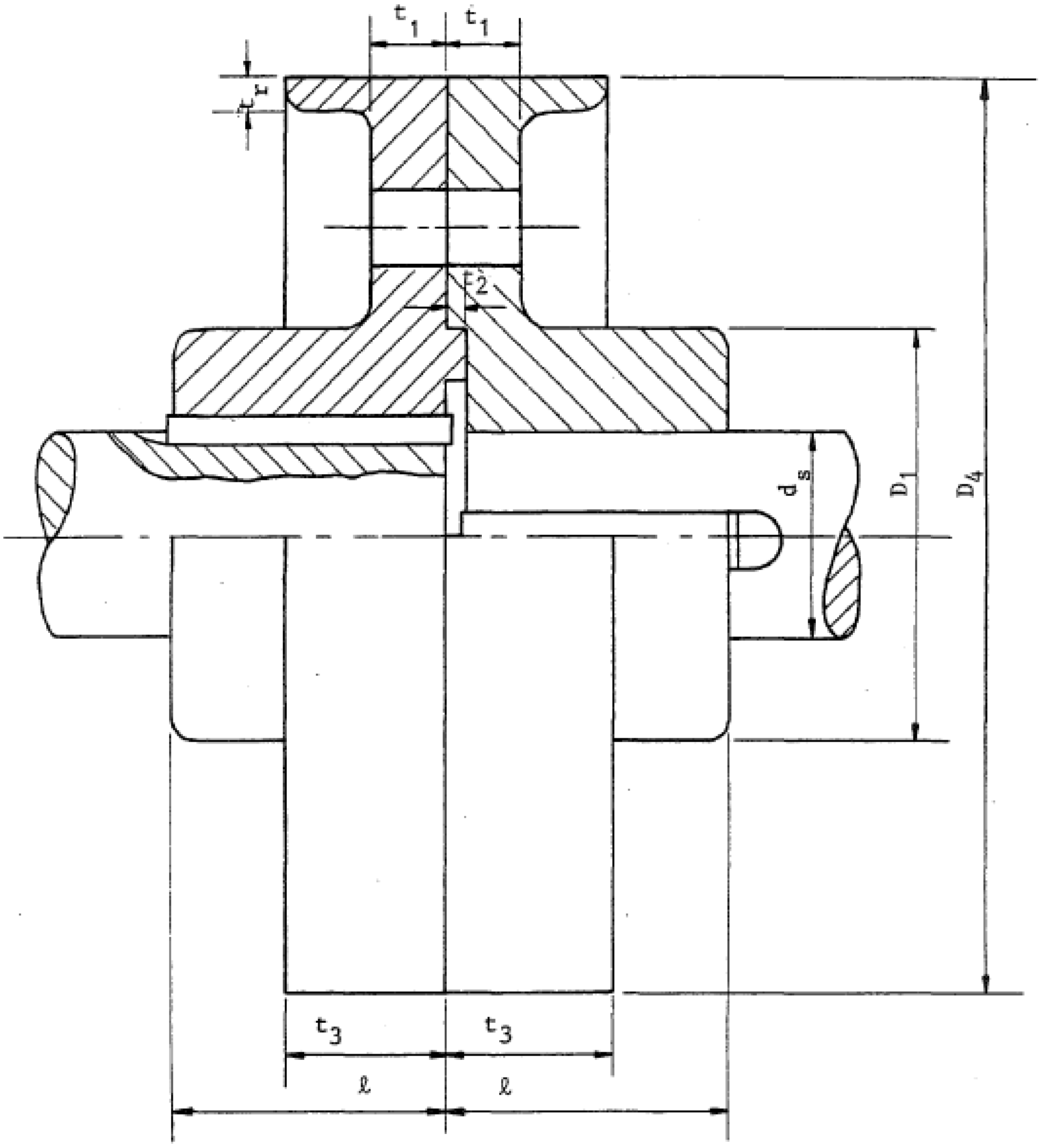


Figure 8.6

Standard proportions are the same as for **Figure 8.5**, and the width of the rim (t_3) $= 0{,}5,\ d_s + 13$ mms $t_r = 5\ mm\ to\ 10\ mm.$

Rotation of the bolts during the process of screwing home the nuts is prevented by means of a small snug, or pin, inserted close to the head of the bolt.

In order to protect the threads when driving home the bolts, the diameter of the threaded portion of the bolts may be made 3 mm to 6 mm less than the main diameter.

The bolts themselves often have tapered shanks, and they are designed for a shear stress of about 27,6 MPa. The low value is used to allow for bending.

8.5 Strength of coupling

The actual forces acting are, in most cases, incalculable and for this reason the design of the shaft coupling is based largely on the results of experience.

As has already been stated, the connecting bolts, which theoretically are subject only to shear, may likewise by subjected to forces due to bad alignment of the shaft or even to the bending of the shaft if the bearings are too far apart.

In ordinary couplings, the moment of the strength of the bolts in shear should theoretically be equal to the moment of resistance of the shaft to twisting.

Therefore:

$$Torque\ on\ bolts = Torque\ on\ shaft$$
$$force \times radius = \frac{\pi}{16}\tau {d_s}^3$$
$$\tau_b \times \frac{\pi}{4}{d_b}^2 \times n \times \frac{D_2}{2} = \frac{\pi}{16}\tau {d_s}^3$$

Where:

- d_b = diameter of bolts
- n = number of bolts
- D_2 = pitch circle diameter of bolts
- d_s = diameter of shaft
- τ_b = shear stress in bolts
- τ = shear stress in shaft

From this equation the size of the bolts can be calculated if their number is assumed in accordance with the standard proportion formula.

8.6 Marine coupling

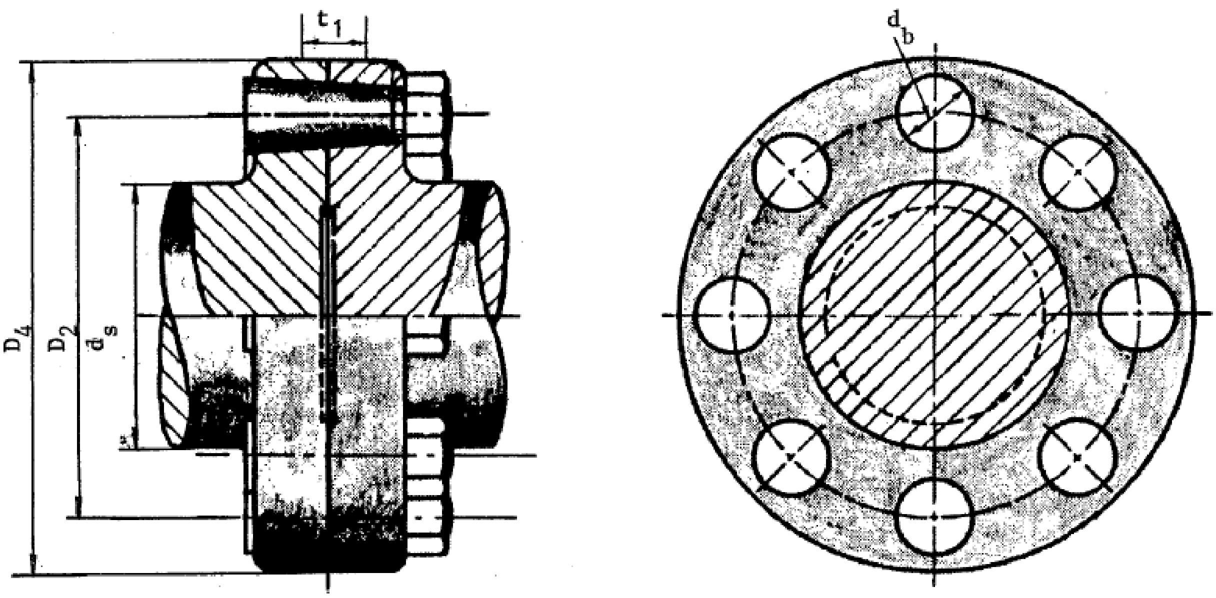


Figure 8.7

In marine couplings, the number of bolts usually are 6, 8, 9 or 12. In the case of large shafts, these bolts are tapered, being 3 mm in 100 mm in length, and the bolts are sometimes made with, but often without heads.

It is necessary to obtain the diameter of the coupling bolts before the dimensions of the coupling can be found, and for this purpose the pitch circle diameter of the bolts (D_2) may be taken as equal to $1{,}6\ d_s$. Outside diameter of the flanges $(D_4) = d_s + 3d_b + 50\ mm.$

Thickness of flange (t_1) = 0,3 ds
Number of bolts (n) = $\frac{d_s}{75} + 2$

Note:
The size of the locating disc varies from 125 mm to 150 mm in diameter and 25 mm to 48 mm in thickness.

8.7 Flexible couplings

These are frequently used when the permanent correct alignment of the shafting cannot be ensured. A direct drive usually is more economical than a drive by means of a belt or bearing.

It has, of course, the disadvantage that four bearings must be in correct alignment. It is difficult to maintain this adjustment, and for this reason flexible couplings are fitted. These also possess, a certain amount of resilience, which gives a cushioning effect when fluctuating loads are being transmitted to the generator, and reduces the liability to shaft fracture.

Note:
On small drives operating at moderate speeds, a flexible coupling may have rubber, leather or other fabric as the flexible element.

These couplings are, however, bulky for larger powers. In this case the flexible element may consist of a plate, say 8 mm thick, bolted to a flange on the driving shaft on the inner circumference and bolted likewise on the outer circumference. Such a coupling is shown in **Figure 8.8**.

A simple type of coupling which permits-a small longitudinal movement of the shafts is shown in **Figure 8.9**, The holes in one flange are bushed in order that the bolts may have a suitable bearing surface.

The bolts must be strong on account of the bending stresses they are subjected to.

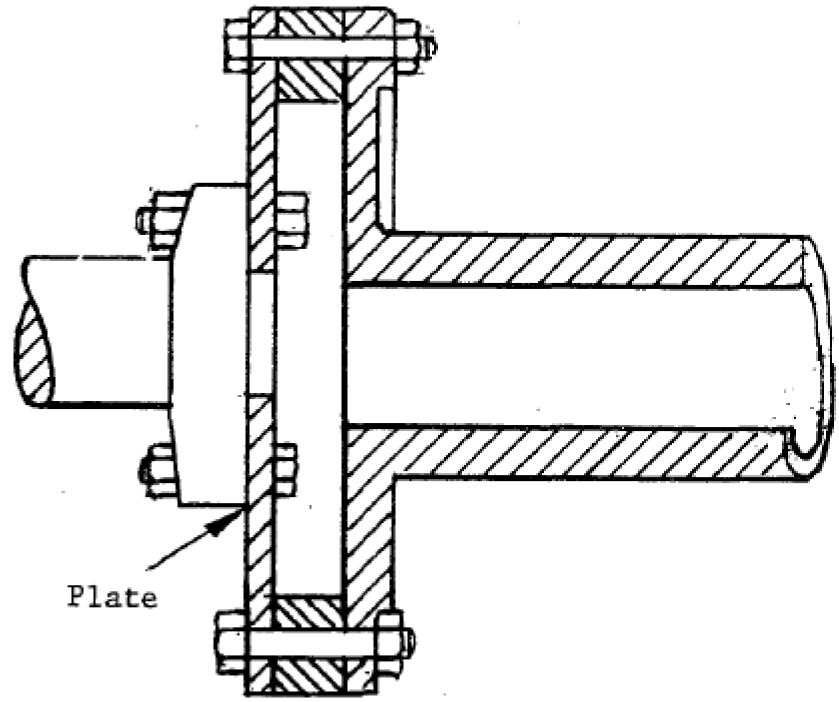


Figure 8.8

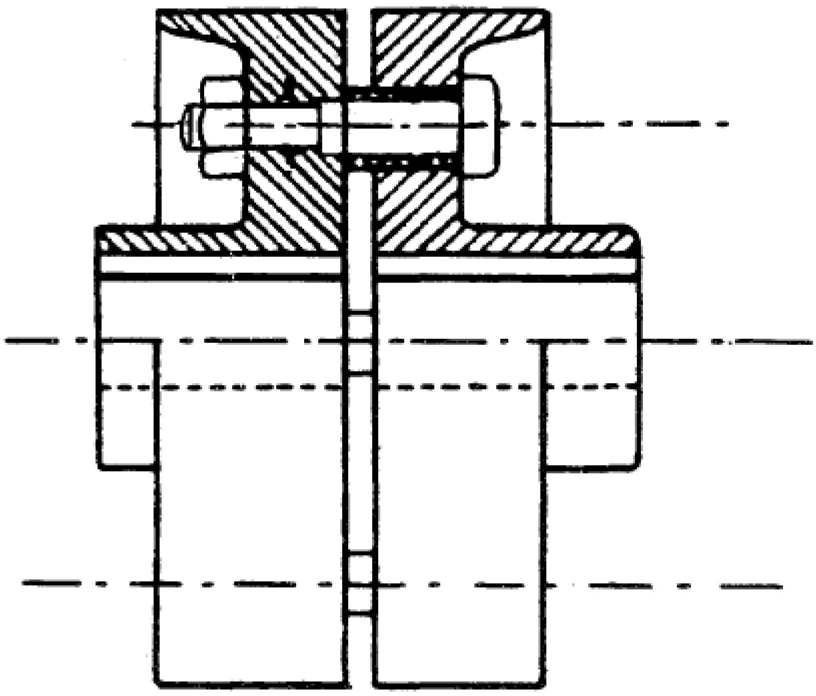


Figure 8.9

8.8 Muff coupling

A muff coupling is easily removed, and occupies little space. A split muff coupling has the hole for the shaft bored with a piece of .shim clamp between the halves. A parallel key which extends the full length of the coupling is used. Two keys may also be used.

Most of the torque is carried by friction, the key being used for extra safety. It is quite common to design the coupling by assuming that the key takes half the torque and friction the other half.

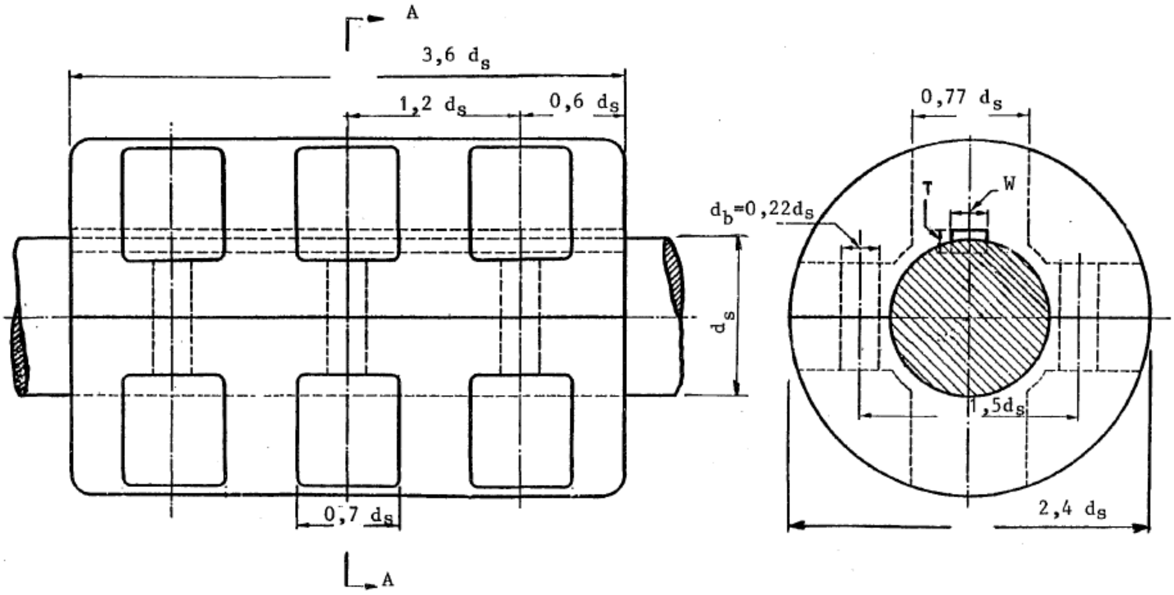


Figure 8.10 Split muff coupling

Worked Example 8.1

Two shafts, each 76 mm in diameter, are to be connected by a flange coupling; the diameter of the bolt circle is 128 mm. Find the diameter and number of bolts required. The Stress in the shaft is 69 MPa and in the bolts 62 MPa.

Solution:

According to standard proportions

$$\begin{aligned} n &= 0{,}02 + 3 \\ &= 0{,}02 \times 76 + 3 \\ &= 1{,}52 + 3 \\ &= 4{,}52\ bolts \end{aligned}$$

The nearest even number is 4 bolts.

$$\begin{aligned} Torque\ on\ bolts &= Torque\ on\ shaft \\ \therefore \frac{\pi {d_b}^2}{4} \times n \times \tau_b \times \frac{D_2}{2} &= \frac{\pi {d_s}^3 \times \tau}{16} \\ \therefore \pi \times {d_b}^2 \times 62 \times 10^6\ N/m^2 \times \frac{0{,}128\ m}{2} &= \frac{\pi \times (0{,}076)^3 m^3 \times 69 \times 10^6\ N/m^2}{16} \\ \therefore {d_b}^2 &= \frac{(0{,}076)^3 m^3 \times 69 \times 10^6\ N/m^2 \times 2}{62 \times 10^6\ N/m^2 \times 0{,}128\ m \times 16} \\ d_b &= \sqrt{0{,}000477\ m^2} \\ &= 0{,}02184\ m \\ &= 21{,}84\ mm \\ \text{say } d_b &= 22\ mm \end{aligned}$$

Worked Example 8.2

Design a shaft and flange coupling to transmit 373 kW at 300 r/min.
Shear stress for shaft equals 69 MPa.
Shear stress for bolts equals 55 MPa.
Shear stress for key equals 83 MPa.
Crushing stress for bolts equals 27,6 MPa.
Crushing stress for key equals 207 MPa.

Solution:

$$T = \frac{Power \times 60}{2\,\pi\,N}$$

$$= \frac{373 \times 10^3 \times w \times 60}{2 \times \pi \times 300\ r/min}$$

$$T = 11873\ Nm$$

also

$$T = \frac{\pi {d_s}^3}{16} \times \tau$$

$$11873\ Nm = \frac{\pi \times {d_s}^3 \times 69 \times 10^6\ Nm}{16}$$

$$\therefore d_s = \sqrt[3]{\frac{11873 Nm \times 16}{\pi \times 69 \times 10^6\ Nm}}$$

$$= 0{,}09568\ m$$

$$= 95{,}68\ mm$$

use $d_s = 100\ mm$ standard shaft

Length of key

(a) Consider shearing

$$Torque\ on\ shaft = Torque\ on\ key \text{ (due to shear)}$$

$$T = W\ \times l \times T_{key} \times \frac{d_s}{2}$$

$$11873\ Nm = 0{,}028\ m \times l \times 83\ \times 10^6 N/m^2 \times \frac{0{,}01\ m}{2}$$

$$l = \frac{11873\ Nm \times 2}{0{,}28 \times 83 \times 10^6 N/m^2 \times 0{,}1\ m}$$

$$l = 0{,}102\ m$$

$$Length = 102\ mm$$

(b) Now consider crushing

$$Torque\ on\ shaft = Torque\ on\ key \text{ (due to crushing)}$$

$$T = \frac{T}{2} \times l \times \sigma_c \times \frac{d_s}{2}$$

$$11873\ Nm = \frac{0{,}016\ m}{2} m \times l \times 207 \times 10^6 N/m^2 \times \frac{0{,}1}{2} m$$

$$l = \frac{11873\ Nm \times 2 \times 2}{0{,}016\ m \times 207 \times 10^6 N/m^2 \times 0{,}1\ m}$$

$$l = 0{,}143\ m$$

$$Length = 143\ mm$$

Since we always design for safety, use a key length of 143 mm.

Assume 4 bolts for this size of shaft. Pitch circle diameter.

$$D_2 = 3d_s \text{ standard proportion}$$
$$= 3 \times 100$$
$$= 300\ mm$$

$$Torque\ on\ bolts = Torque\ on\ shaft$$
$$\therefore \frac{\pi {d_b}^2}{4} \times n \times \tau_b \times \frac{D_2}{2} = 11\ 873\ Nm$$
$$\therefore {d_b}^2 = \frac{11\ 873\ Nm \times 4 \times 2}{\pi \times 4 \times 55 \times 10^6\ N/m^2 \times 0{,}300\ m}$$
$$d_b = \sqrt{0{,}000458\ m^2}$$
$$= 0{,}0214\ m$$
$$d_b = 21{,}4\ mm$$
$$\therefore d_b = \text{Standard bolts of } 21{,}4\ mm \text{ diameter}$$

∴ Use 4 – 24 mm bolts

Crushing of bolts in flanges

$$Torque = Force \times radius \times number\ of\ bolts$$
$$\therefore Force = \frac{Torque}{Radius \times n}$$
$$= \frac{11873\ Nm \times 2}{0{,}3\ m \times 4}$$
$$= 19788\ N$$
$$p = 19788\ kN$$

But stress $= \frac{Force}{Area}$

$$27{,}6 \times 10^6\ N/m^2 = \frac{19788\ N}{t_1 \times d_b}$$
$$\therefore t_1 = \frac{19\ 788\ N}{27{,}6 \times 10^6\ N/m^2 \times 0{,}024\ m}$$
$$= 0{,}02987\ m$$
$$= 29{,}87\ mm$$

Use a flange thickness of 30 mm

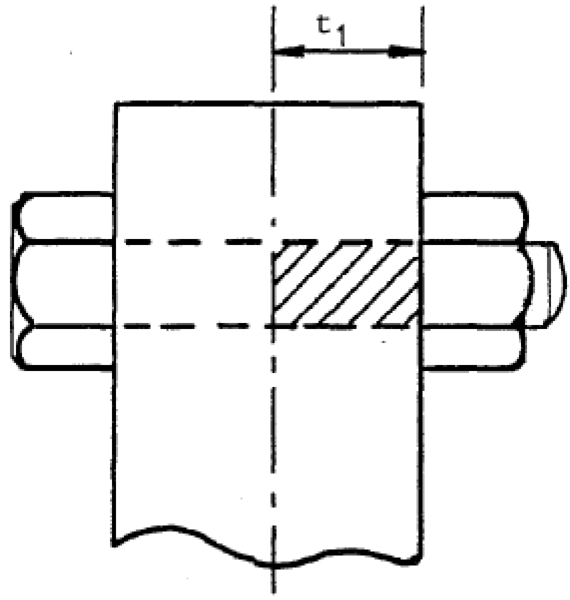


Figure 8.11

Other dimensions may be obtained from standard proportions. See **Figure 8.5**.

Worked Example 8.3

A brushed-pin type of flexible coupling for connecting an electric motor and a centrifugal pump shaft has to be designed.

The power to be transmitted is 18 kW at 1 000 r/min.

The diameters of the motor and pump shaft are 50 mm and 45 mm, respectively. A pitch circle diameter for the pins of 152 mm will be used.

There are six pins of minimum diameter 12 mm, these values being chosen to keep the necessary dimensions of the rubber bushes reasonably low.

Solution:

$$T = \frac{Power \times 60}{2\,\pi\,N}$$

$$= \frac{373 \times 10^3 \times w \times 60}{2 \times \pi \times 300\; r/min}$$

$$T = 11873\; Nm$$

also

$$Torque = Force \times Perpendicular\ distance$$

$$T = P \times R$$

$$172\; Nm = P \times \frac{0{,}152\; m}{2}$$

$$P = \frac{172\; Nm \times 2}{0{,}152\; m}$$

$$P = 2263\; N$$

$$Force\ on\ each\ pin = \frac{Total\ force}{Number\ of\ pins}$$
$$= \frac{2263\ N}{6}$$
$$= 377\ N$$

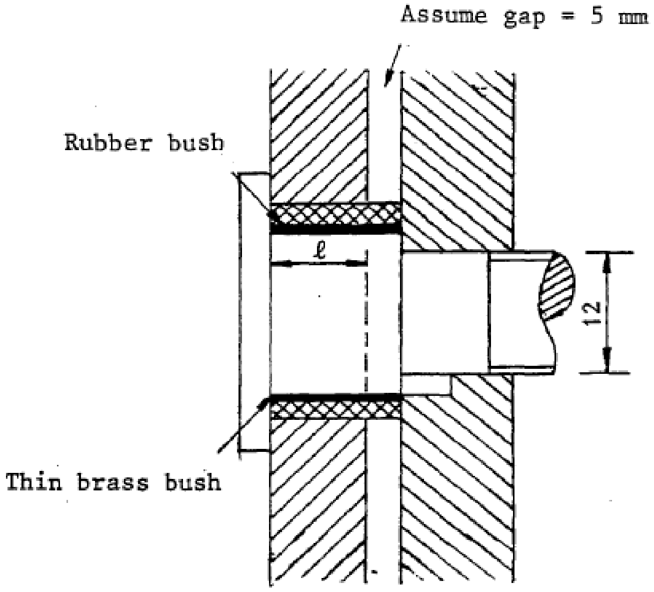


Figure 8.12

As can be seen from **Figure 8.12**, the portion of the pin of least diameter (12 mm) is threaded to be secured into the coupling by using a standard nut and washer.

Assume that the enlarged portion of the pin is 18 mm in diameter. On the latter slides a thin brass sleeve, 2 mm thick, carrying the rubber bush, say 8 mm in thickness for the rubber bush; its overall dimensions will be $18 + 4 + 16 = 38\ mm$.

Let i be the necessary bearing length of the bush; then its projected area is $0{,}038 \times l\ m^2$.

A suitable value for bearing pressure, having regard to the life and durability of the rubber, will be 310 kPa, although higher values are sometimes specified.

$$Bearing\ pressure = \frac{Force}{Projected\ Area}$$
$$P_b = \frac{P}{l \times 0{,}038\ m}$$
$$310 \times 10^3\ N/m^2 = \frac{377\ N}{l \times 0{,}038\ m}$$
$$l = \frac{377\ N}{310 \times 10^3 N/m^2 \times 0{,}038 m}$$
$$= 0{,}032$$
$$l = 32\ mm$$

Stresses in the pins

The portion of the pin that is 12 mm in diameter should be a tapping fit in the coupling disc, since a clearance would set up bending stresses, which might reach unduly high values.

The threaded portion should be as short as possible, so that the shear can be taken by the unthreaded "neck".

This shear stress due to the torque will be:

$$\tau = \frac{Force}{Area\ of\ neck} = \frac{377\ N}{\frac{\pi}{4} \times {d_b}^2 m^2}$$
$$= \frac{377 \times 4\ N}{\pi \times (0{,}012)^2 m^2}$$
$$= 3{,}33\ MPa$$

Note:
Although this value is low, other and more serious stresses will be present.

Since the pin is not rigidly held in the left-hand face and the rubber is compressible, the force P at the enlarged portion will cause bending in the pin.

Its effect on the 12 mm portion will be reduced by the longitudinal resistance of the shoulder to bending, but in estimating the bending stress, this resistance will be neglected.

Assuming a uniform distribution of load P along the bush, the arm of the bending moment will be

$$arm = \left(\frac{1}{2} \times 32\right) + 5$$
$$arm = 21\ mm$$

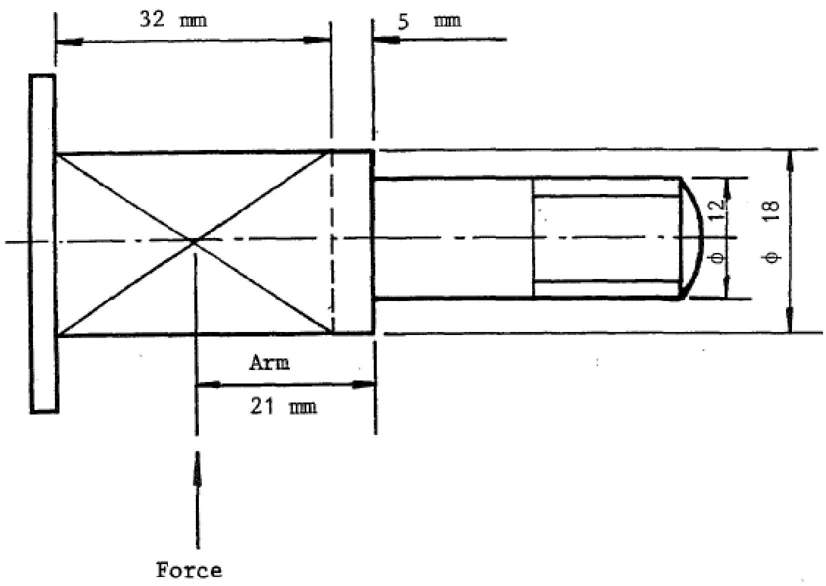


Figure 8.13

(**Figure 8.13** shows the force acting on the pin)

Now bending stress

$$M = \sigma \times Z$$

$$Force \times distance = \sigma \times \frac{\pi}{32} {d_b}^3$$

$$\sigma = \frac{Force \times Distance \times 32}{\pi \times {d_b}^3}$$

$$\sigma = \frac{377\ N \times 0{,}021\ m\ 32}{\pi \times (0{,}012)^3\ m^3}$$

$$\sigma = 46{,}7\ MPa$$

Since the coupling rotates in one direction only, this is not an alternating stress; but some fluctuation of these tensile and compressive stress (due to bending) will occur as the motor or pump outputs vary.

Note:
Care should be taken in assembling that the nuts are not tightened down unnecessarily.

General

The remaining proportions of the coupling may now be fixed, as shown in **Figure 8.14**. The outer diameter of the hubs is twice the shaft diameter, and the length corresponds to that for the standard rigid type.

The overall diameter and width of flanges are fixed by the dimensions of the bushes and the pitch circle diameter of the pins.

The halves of the coupling, which are of cast iron in this case, may be of steel in special circumstances, and should be machined all over to ensure balance, particularly when required to run at high speeds.

Each half is secured to its shaft by a standard sunk key 64 mm long, by 13 mm wide, by 9 mm deep.

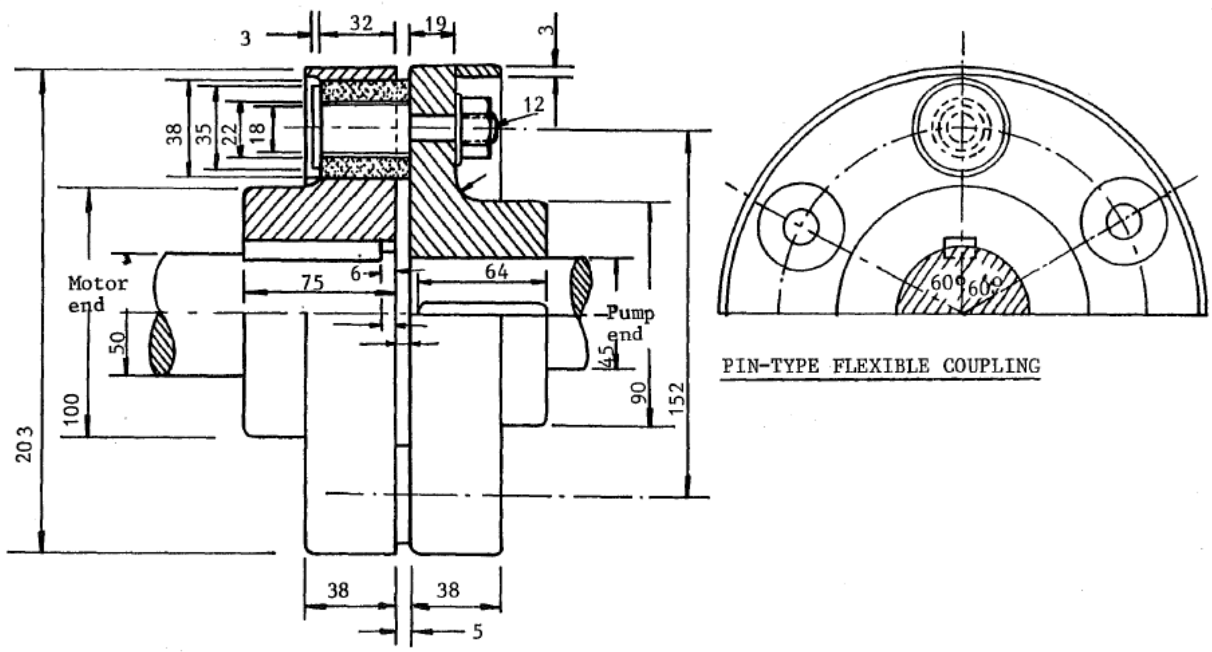


Figure 8.14

The brass sleeve and rubber bush are slipped on the pin from the smaller end, and fit against the thin 35 mm head at the opposite end. Thus, if necessary, the pin and its assembly may be withdrawn to the left after removing the nut, or the left-hand half coupling may be withdrawn over the bushes.

Since a clearance of 5 mm is left between the faces of the two halves of the coupling, there is no rigid connection between them, the drive taking place through the medium of compressible rubber bushes.

8.9 Claw coupling

A claw coupling is a type of clutch, but as it can not slip it can only be engaged when the shafts are stationary or when shock does not matter. This coupling is designed for disengagement at will.

The coupling may have 2, 3 or 4 claws. The fixed half of the coupling is pressed on and keyed to one shaft (B), and the other half is arranged to slide along a feather on the shaft (A); the two halves engage by means of projecting claws on the faces.

The sliding half is operated by a lever, the forked end of which fits the groove around this part of the coupling. Shaft (A) enters the half B for a short distance, and is supported by it.

When the sliding half is disengaged, shaft (A) remains stationary while the bush on the fixed half revolves around it. A lubricator of the Stauffer type is usually fitted to supply grease to the bush.

Dimensions of a coupling for 50 mm shafts are given in **Figure 8.15**. For diameters of 75 mm upwards, two feathers, opposite to one another, are provided for the sliding half.

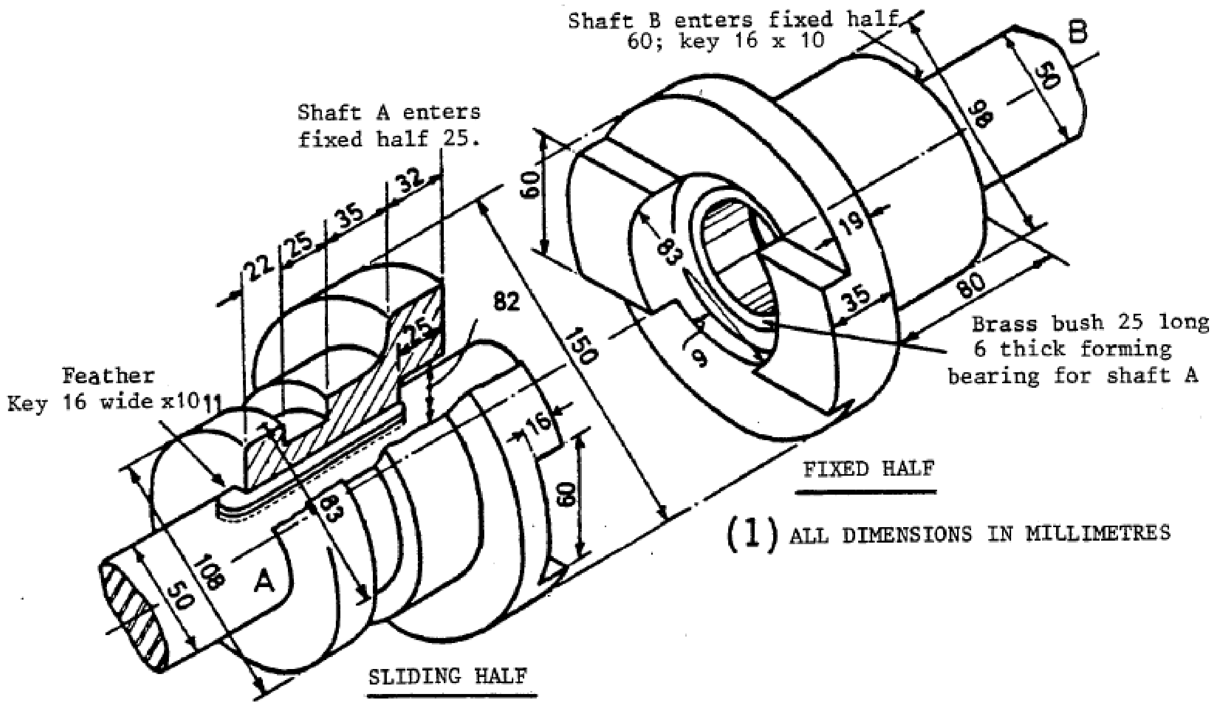


Figure 8.15

Figure 8.16 shows a claw coupling in the engaged position and **Figure 8.17** shows a claw coupling in the disengaged position.

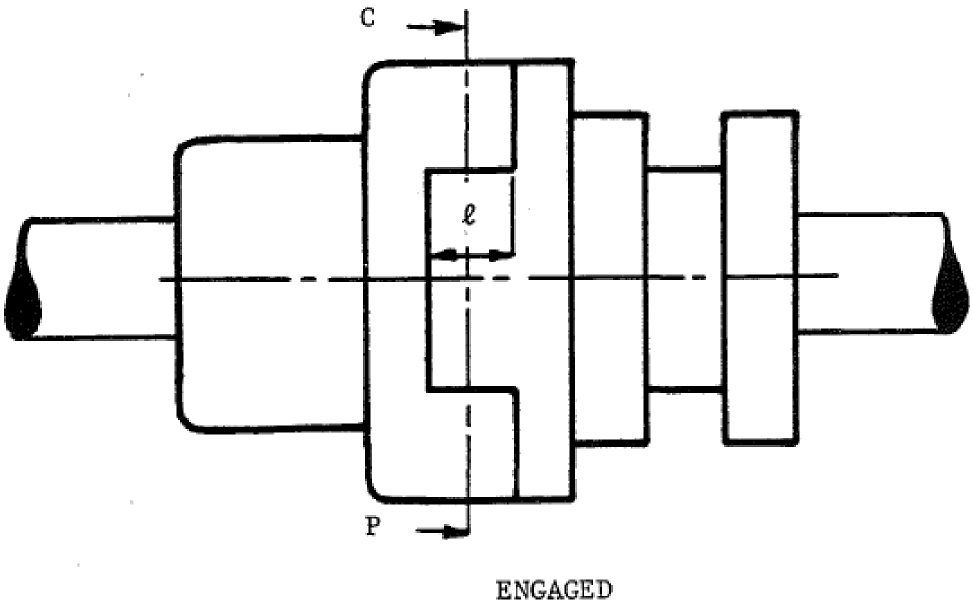


Figure 8.16

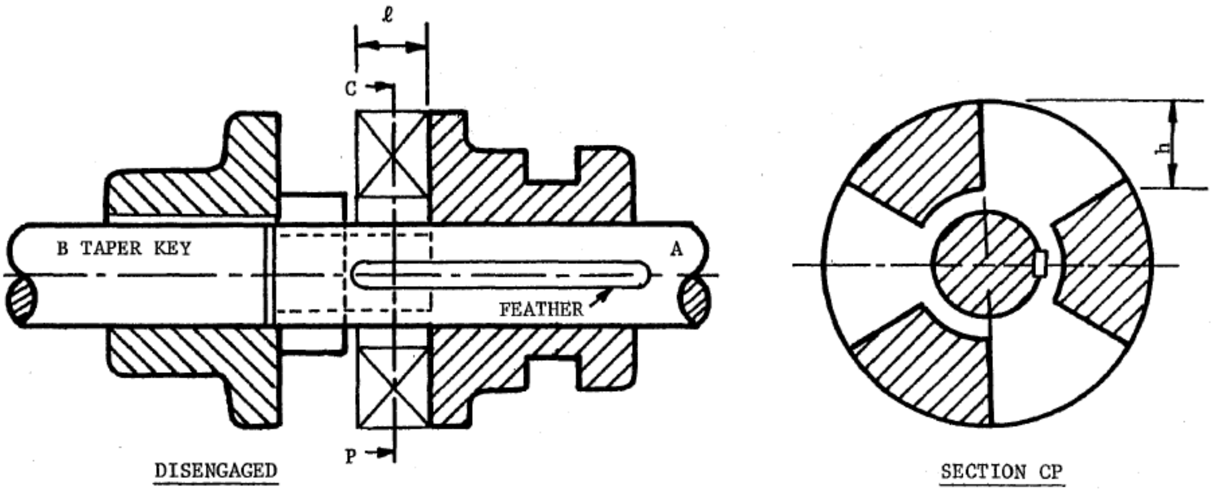


Figure 8.17 Claw coupling

8.10 Standard proportions for a claw coupling

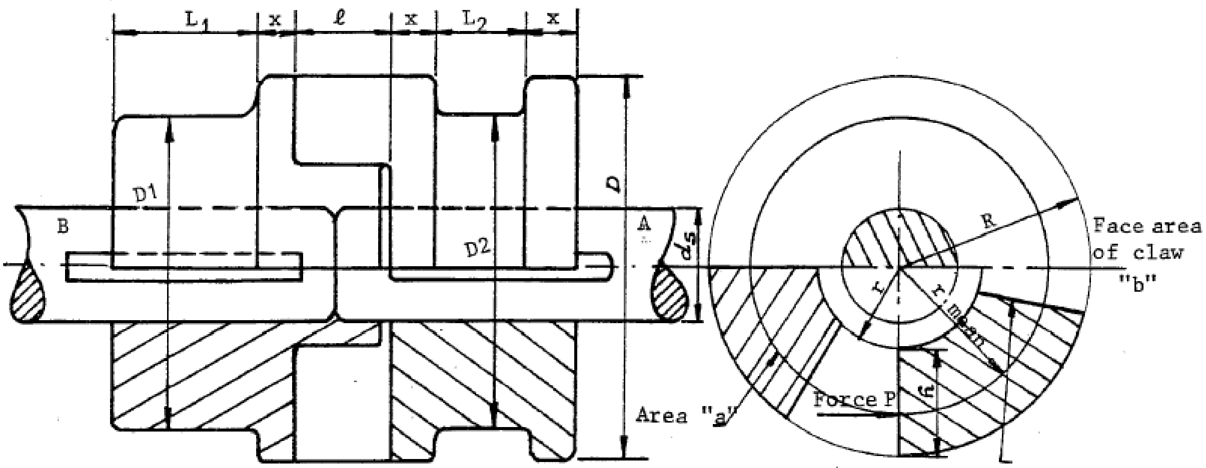


Figure 8.18

Unit	$(U) = d_s + 25$	$l = 0{,}55 \times U$
	$D = 2{,}1 \times U$	$L_1 = 0{,}6 \times U$
	$D_1 = 1{,}6 \times U$	$L_2 = 0{,}5 \times U$
	$D_2 = 1{,}5 \times U$	$X = 0{,}3 \times U$
	$r = 0{,}5 \times U$	

8.11 Claw coupling design

8.11.1 Consider shearing of the claws

$$Torque\ on\ the\ shaft = Torque\ on\ the\ claws$$

$$\frac{\pi d_s^3}{16} \times \tau_{shaft} = Total\ force\ on\ claws \times r_{mean}$$

$$r_{mean} = \frac{R+r}{2} \text{ or } \frac{D+d}{4}$$ where $D = outside\ diameter\ of\ claws$ and $d = inside\ diameter\ of\ claws$

$$Total\ force\ on\ claws = \frac{\pi {d_s}^3 \tau_{shaft}}{16} \times \frac{1}{r_{mean}}$$

$$Force\ (P)\ per\ claw \quad \frac{\pi {d_s}^3 \tau_{shaft}}{16} \times \frac{4}{(D+d)} \times \frac{1}{n}$$

Where n = $number\ of\ claws$

$$= \frac{\pi {d_s}^3 \tau_{shaft}}{4n(D+d)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

Now

$$Total\ shear\ force = (Shear\ stress \times Shear\ area\ of\ claws)$$

$$= \tau_{claws} \times \frac{\pi}{4}(D^2 - d^2)$$

$$Shear\ force\ (P)\ per\ claw = \tau_{claws} \times \frac{\pi}{4}(D^2 - d^2) \times \frac{1}{2n}$$

$$= \tau_{claws} \times \frac{\pi}{8n}(D^2 - d^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Equating equation (1) and (2)

$$\frac{\pi {d_s}^3 \tau_{shaft}}{4n(D+d)} = \tau_{claws} \times \frac{\pi}{8n}(D^2 - d^2)$$

$$(D^2 - d^2)(D + d) = \frac{2{d_s}^3 \tau_{shaft}}{\tau_{claws}}$$

8.11.2 Consider crushing of the claws

$$Torque\ on\ the\ shaft = Torque\ on\ the\ claws$$

$$\frac{\pi {d_s}^3}{16} \times \tau_{shaft} = Total\ force\ on\ claws\ \times r_{mean}$$

$$\therefore Total\ force\ on\ claws = \frac{\pi {d_s}^3}{16} \times \tau_{shaft} \times \frac{1}{r_{mean}}$$

$$= \frac{\pi {d_s}^3}{16} \times \tau_{shaft} \times \frac{4}{(D+d)}$$

$$Force\ (P)\ per\ claw = \frac{\pi {d_s}^3 \tau_{shaft}}{4(D+d)} \times \frac{1}{n}$$

Now

$$Total\ crushing\ force = (Crushing\ stress \times Face\ area\ of\ claws)$$

$$= \sigma_{c_{claws}} \times l \times h \times n$$

Where $h = height\ of\ claw$ and $l = length\ of\ claw$

$$h = R - r \text{ or } \frac{D-d}{2}$$

$$Total\ crushing\ force = \sigma_{c_{claws}} \times l \times \frac{D-d}{2} \times n$$

$$Crushing\ force\ (P) per\ claw = \tau_{claws} \times \frac{\pi}{8n}(D^2 - d^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Equating equation (1) and (2)

$$\frac{\pi {d_s}^3 \times \tau_{shaft}}{4n(D+d)} = \sigma_{c_{claws}} \times \frac{l \times D - d}{2}$$

$$D^2 - d^2 = \frac{\pi {d_s}^3 \tau_{shaft}}{2 \times n \times l \times \sigma_{c_{claws}}}$$

Worked Example 8.4

Two 60mm-diameter shafts are to be connected by a claw coupling. There are three claws on each half of the coupling with an external diameter of twice the internal diameter. The depth of the claws is 32 mm. The shear stress in the shaft due to torsion is 62 MPa.

Calculate:

i. the torque transmitted by the shaft
ii. the internal and external diameter of the claws, if the allowable shear stress in the claws must not exceed 6 MPa and the compressive stress must not exceed 10 MPa
iii. the minimum length of the key in one half of the coupling if the shear stress in the key must not exceed 48 MPa, and the compressive stress in the key must not exceed 142 MPa
iv. the power transmitted at 240 r/min, if the angle of twist is not to exceed 2 degrees over a length of 3 metres of the shaft. The modulus of rigidity is 83 GPa

Solution:

i. The torque transmitted by the shaft

$$Torque = \frac{\pi D^3}{16} \times \tau$$
$$= \frac{\pi (0{,}06)^3 m^3}{16} \times 62 \times 10^6\ N/m^2$$
$$= 2629{,}5\ Nm$$
$$T = 2{,}6295\ kNm$$

ii. Internal and external diameters of the claws

Shearing

$$(D^2 - d^2)(D + d) = \frac{2{d_s}^3 \tau_{shaft}}{\tau_{claws}} \qquad D = 2d$$
$$[(2d)^2 - d^2][2d + d] = \frac{2 \times (0{,}06)^3 m^3 \times 62 \times 10^6\ N/m^2}{6 \times 10^6\ N/m^2}$$
$$[4d^2 - d^2][3d] = 0{,}004464\ m^3$$
$$3d^3 \times 3d = 0{,}004464\ m^3$$
$$d^3 = \frac{0{,}004464\ m^3}{9}$$
$$d = \sqrt[3]{4{,}96 m^3}$$
$$= 0{,}079\ m$$
$$\text{say } d = 80\ mm$$
$$D = 80\ mm \times 2$$
$$D = 160\ mm$$

Crushing

$$D^2 - d^2 = \frac{\pi {d_s}^3 \tau_{shaft}}{2 \times n \times l \times \sigma_{c_{claws}}}$$

$$(2d)^2 - d^2 = \frac{\pi \times (0{,}06m)^3 \times 62 \times 10^6\ N/m^2}{2 \times 3 \times 0{,}032\ m \times 10 \times 10^6\ N/m^2}$$

$$4d^2 - d^2 = 0{,}0219\ m^2$$

$$d^2 = \frac{0{,}0219\ m^2}{3}$$

$$d = \sqrt{0{,}007304\ m^2}$$

$$= 0{,}0855$$

Say $d = 86\ mm$

$$D = 2 \times 86\ mm$$

$$D = 172\ mm$$

Use the following dimensions for safety:

$$D = 172\ mm$$

$$d = 86\ mm$$

iii. The minimum length of the key due to shearing

$$Torque\ on\ shaft = Torque\ on\ key$$

$$T_{shaft} = \tau_{key} \times W \times l \times \frac{d_s}{2}$$

Since the width of the key is unknown, we assume it according to standard proportions

$$\therefore Width = \frac{d_s}{2}$$

$$\therefore T = \tau_{key} \times \frac{d_s}{4} \times l \times \frac{d_s}{2}$$

$$T = \frac{{d_s}^2 l \times \tau_{key}}{8}$$

$$\therefore l = \frac{T \times 8}{{d_s}^2 \times \tau_{key}}$$

$$= \frac{2629{,}5\ Nm \times 8}{(0{,}06)^2 m^2 \times 48 \times 10^6 N/m^2}$$

$$= 0{,}1218\ m$$

$$\therefore l = 122\ mm$$

The minimum length of the key due to crushing

$$Torque\ on\ the\ shaft = Torque\ on\ the\ key$$

$$\tau_{shaft} = \sigma_{c_{keys}} \times \frac{T}{2} \times l \times \frac{d_s}{2}$$

but $T = \frac{d_s}{6}$ according to standard proportions

$$\therefore T = \frac{d_s}{6 \times 2} \times l \times \sigma_{c_{key}} \times \frac{d_s}{2}$$

$$T = \frac{{d_s}^2 \times l \times \sigma_c}{24}$$

$$2629{,}5\ Nm = \frac{(0{,}06)^2 m^2 \times l \times 142 \times 10^6 N/m^2}{24}$$

$$l = \frac{2629{,}5\ Nm \times 24}{(0{,}06)^2 m^2 \times 142 \times 10^6 N/m^2}$$

$$= 0{,}1235\ m$$

$$l = 124\ mm$$

Use a key with a length of 124 mm.
Alternate method to find length of key.
Assume key sizes according to **Table 7.1**.

$\therefore width = 18\ mm$ and $thickness = 11mm$ for a 60 m diameter shaft

Shearing

$$T_{shaft} = \tau_{key} \times W \times l \times \frac{d_s}{2}$$

$$l = \frac{T_{shaft} \times 2}{W \times \tau_{key} \times d_s}$$

$$= \frac{2629{,}5\ Nm \times 2}{0{,}018\ \text{m} \times 48 \times 10^6 N/m^2 \times 0{,}06\ m}$$

$$= 0{,}102\ m$$

$$\therefore l = 102\ mm$$

Crushing

$$\tau_{shaft} = \sigma_{c_{keys}} \times \frac{T}{2} \times l \times \frac{d_s}{2}$$

$$2629{,}5\ Nm = \frac{\tau_{shaft} \times 2}{W \times \tau_{key} \times d_s}$$

$$l = \frac{2629{,}5\ Nm \times 4}{0{,}011\ \text{m} \times 142 \times 10^6 N/m^2 \times 0{,}06\ m}$$

$$= 0{,}112\ m$$

$$l = 112\ mm$$

Use a key with a length of 112 mm

iv. The power transmitted at 240 r/min

$$\frac{\tau_{max}}{J} = \frac{G\theta}{l}$$

$$\tau_{max} = \frac{G \times \theta \times J}{l}$$

but $J = \frac{\pi {d_s}^4}{32}$

$$= \frac{G \times \theta \times \pi {d_s}^4}{32 \times l}$$

$\theta = 2°$

$= \frac{\pi \times 2}{180}$

$= 0{,}035\ radians$

$$= \frac{83 \times 10^9 N/m^2 \times 0{,}035 \times \pi \times (0{,}06\ m)^4}{32 \times 3\ m}$$

$$= 1232\ Nm$$

$$Power = \frac{2\pi NT}{60}$$

$$= \frac{2 \times \pi \times 240\ r/min \times 1232 Nm}{60}$$

$$= 30{,}96\ kW$$

Worked Example 8.5

Two shafts are to.be connected by a claw coupling. There are 3 claws on each half of the coupling - the external and internal diameters of which are 160 mm and 80 mm respectively. The shaft on which the coupling is positioned transmits 80 kW at 290 r/min.
The shear stress in the shaft due to torsion is 62 MPa.

Calculate:

i. the diameter of the shaft

ii. the length (depth)of the claws if the compressive stress in the claws is 12 MPa

Solution:

i. The diameter of the shaft

$$Power = \frac{2\pi NT}{60}$$

$$\therefore T = \frac{Power \times 60}{2 \times \pi \times N}$$

$$= \frac{80 \times 10^3 W \times 60}{2 \times \pi \times 290\ r/min}$$

$$= 2634\ Nm$$

$$T = \frac{2634\ kNm}{3}$$

$$\text{Also } T = \frac{\pi {d_s}^3}{16} \times \tau$$

$$\therefore {d_s}^3 = \frac{T \times 16}{\pi \times \tau}$$

$$= \frac{2631\ Nm \times 16}{\pi \times 62 \times 10^6 N/m^2}$$

$$= \sqrt[3]{0{,}0002162\ m^3}$$

$$= 0{,}06\ m$$

$$= 60\ mm$$

ii. The length (l) of the claws

$$D^2 - d^2 = \frac{\pi {d_s}^3 \times \tau_{shaft}}{2 \times n \times l \times \sigma_{c_{claws}}}$$

$$l = \frac{\pi {d_s}^3 \times \tau_{shaft}}{2 \times n \times \sigma_{c_{claws}} \times (D^2 - d^2)}$$

$$= \frac{\pi \times (0{,}06m)^3 \times 62 \times 10^6\ N/m^2}{2 \times 3 \times 12 \times 10^6\ N/m^2 \times [(0{,}16\ m)^2 - (0{,}08\ m)^2]}$$

$$= 0{,}0304$$

$$\text{say } l = 31\ mm$$

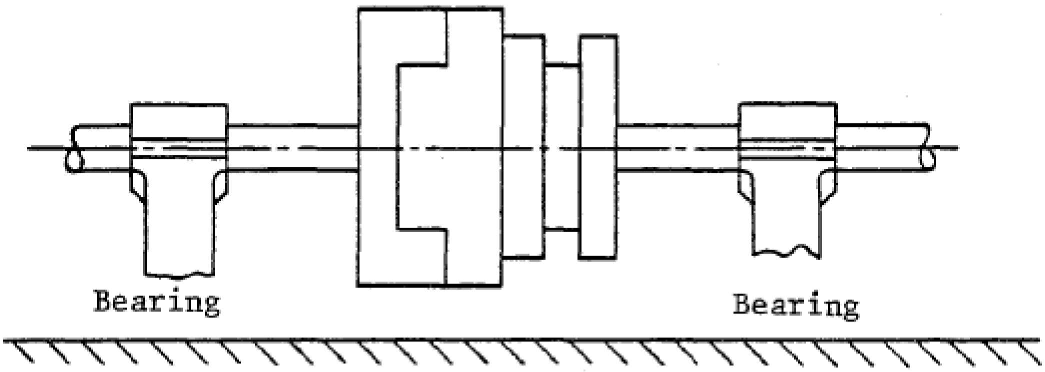
Worked Example 8.6

Figure 8.19 shows a claw coupling assembly. The shaft on which the coupling is positioned transmits 75 kW at 50 r/min. The shaft is supported by two bearings mounted on the floor. The claw coupling has four jaws - two on each face. The internal diameter is 0,6 times the external diameter. The depth of the claws is 25 mm. The outline and details of the operating lever are shown.

Calculate:

i. the diameter of the shaft (allowable stress due to shearing should not exceed 60 MPa)
ii. the shearing and bearing stresses induced in the key. Use a key measuring 32 mm x 18 mm x 250 mm
iii. the inside and outside diameters of the claws if the shear stress in the claws must not exceed 6,5 MPa and the pressure between claws is limited to 20 MPa

iv. the diameter of the shaft of the operating mechanism. (Assume effective length of the hand-operated lever to be 400 mm and the maximum force applied 300 N. Calculate for pure torsion and take allowable stress as 30 MPa.



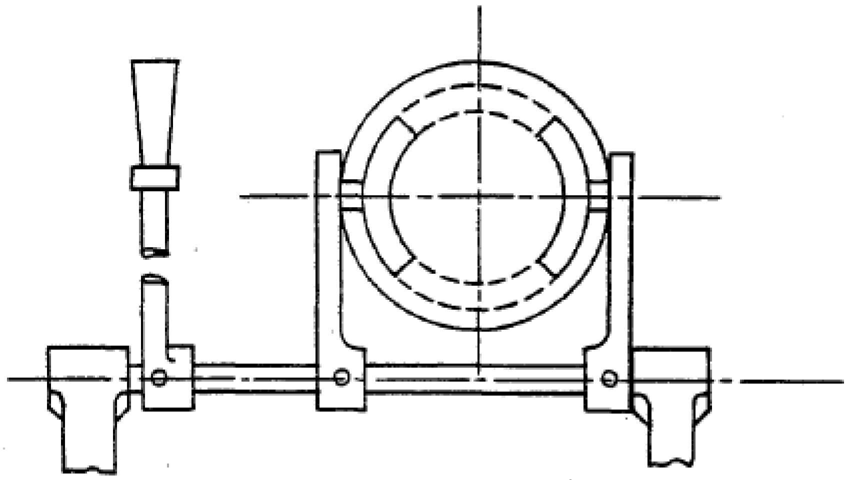


Figure 8.19

Solution:

i. Calculate the shaft diameter

$$Power = \frac{2\pi NT}{60}$$

$$\therefore T = \frac{Power \times 60}{2\ \pi\ N}$$

$$= \frac{75 \times 10^3 W \times 60}{2 \times \pi \times 50\ r/min}$$

$$= 14324\ Nm$$

$$T = \frac{14{,}324\ kNm}{3}$$

$$\text{Also } T = \frac{d_s{}^3}{16} \times \tau$$

$$\therefore d_s{}^3 = \frac{T \times 16}{\pi \times \tau}$$

$$= \frac{14{,}324\ 10^3 Nm \times 16}{\pi \times 60 \times 10^6 N/m^2}$$

$$d_s = \sqrt[3]{0{,}00122\ m^3}$$

$$d_s = 106\ mm$$

ii. Shear stress induced in the key

$$Torque\ on\ shaft = Torque\ on\ key$$

$$T_{shaft} = \tau_{key} \times W \times l \times \frac{d_s}{2}$$

$$\tau_{key} = \frac{T_{shaft} \times 2}{W \times l \times d_s}$$

$$= \frac{14{,}324 \times 10^3\ Nm \times 2}{0{,}032\ \text{m} \times 0{,}250 \times 0{,}106\ m}$$

$$\tau_{key} = 33{,}78\ MPa$$

Crushing stress induced in the key

$$Torque\ on\ shaft = Torque\ on\ key$$

$$\tau_{shaft} = \sigma_{c_{key}} \times \frac{T}{2} \times l \times \frac{d_s}{2}$$

$$\sigma_{c_{key}} = \frac{\tau_{shaft} \times 4}{T \times l \times d_s}$$

$$= \frac{14{,}324 \times 10^3\ Nm \times 4}{0{,}018\ \text{m} \times 0{,}25 \times 0{,}106\ m}$$

$$\sigma_{c_{key}} = 120{,}12\ MPa$$

iii. Inside and outside diameters of the claws

Shearing of claws

$$(D^2 - d^2)(D + d) = \frac{2{d_s}^3 \tau_{shaft}}{\tau_{claws}} \qquad d = 0{,}6\ D$$

$$[D^2 - (0{,}6D)^2][D + 0{,}6D] = \frac{2 \times (0{,}106)^3 \times 60 \times 10^6\ N/m^2}{6{,}5 \times 10^6\ N/m^2}$$

$$[D^2 - 0{,}36\ D^2][D + 0{,}6D] = 0{,}022\ m^3$$

$$0{,}64 D^2 \times 1{,}6\ D = 0{,}022\ m^3$$

$$1{,}024\ D^3 = 0{,}022\ m^3$$

$$D = \sqrt[3]{\frac{0{,}022\ m^3}{1{,}024}}$$

$$D = 278\ mm$$

$$d = 0{,}6 \times 278\ mm$$

$$d = 169\ mm$$

Crushing of claws

$$D^2 - d^2 = \frac{\pi {d_s}^3 \tau_{shaft}}{2 \times n \times l \times \sigma_{c_{claws}}}$$

$$D^2 - (0{,}6D)^2 = \frac{2 \times (0{,}106)^3 \times 60 \times 10^6\ N/m^2}{2 \times 2 \times 0{,}025\ m \times 20 \times 10^6\ N/m^2}$$

$$D^2 - 0{,}36\ D^2 = 0{,}1123\ m^2$$

$$0{,}64\ D^2 = 0{,}1123\ m^2$$

$$D = \sqrt{\frac{0{,}1123\ m^2}{0{,}64}}$$

$$D = 419\ mm$$

$$d = 0{,}6 \times 419\ mm$$

$$d = 252\ mm$$

Use claws having an outside diameter of 419 mm and an inside diameter of 252 mm.

iv. Diameter of the shaft of the operating mechanism

$$
\begin{aligned}
Torque &= Force\ \times radius \\
&= 300\ N \times 0{,}4\ m \\
&= \frac{120\ Nm}{3} \\
\text{Also } T &= \frac{\pi d_s{}^3}{16} \times \tau \\
\therefore d_s{}^3 &= \frac{T \times 16}{\pi \times \tau} \\
&= \frac{120\ Nm \times 16}{\pi \times 30 \times 10^6 N/m^2} \\
d_s &= \sqrt[3]{0{,}0000203\ \ m^3} \\
&= 0{,}02728\ m \\
d_s &= 28\ mm
\end{aligned}
$$

8.12 Universal joint or Hooke's coupling

When the axes of two shafts are inclined to one another, a form of coupling known as a Hooke's joint or coupling may be used. One advantage of the use of this coupling is that the angle between the axes of the shafts may be altered while the shafts are in motion.

Thus, in the mechanism of the motor car, universal joints must be introduced between the gear box and the back axle. In this manner the driving shaft can be out of line in any direction without interfering in any way with the transmission of the rotation from the gear box to the back axle.

Note:
As a rule, a universal joint does not work well if the angle a (see **Figure 8.20** below) is more than 45°, and the angle should preferably be limited to about 20° or 25°, except when the speed of rotation is slow and little power is transmitted.

8.12.1 Variation in Angular Velocity of Driving Shaft

Owing to the angularity between two shafts connected by a universal joint, there is a variation in the angular velocity of one shaft during a single revolution.

Thus, the angular velocity of the driven shaft will not be the same at all points of the revolution as the angular velocity of the driving shaft. In other words, if the driving shaft moves with a uniform motion, then the driven shaft will have a variable motion.

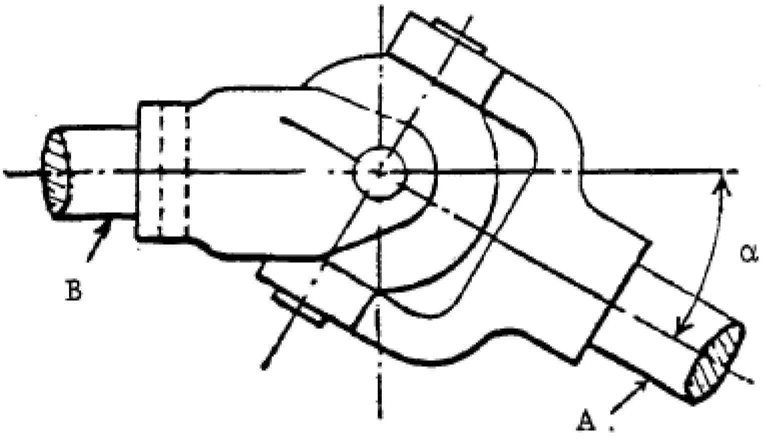


Figure 8.20

8.12.2 Determining Maximum and Minimum Angular -Velocities

If shaft A (see **Figure 8.20** above) runs at a constant speed, shaft B revolves at a minimum speed when shaft A occupies the position shown in the sketch, and the maximum speed of shaft B occurs when the fork of the driving shaft A has turned 90° from the position illustrated.

Let:

$$Maximum\ angular\ velocity\ of\ driven\ shaft = \omega_1$$
$$Minimum\ angular\ velocity\ of\ driven\ shaft = \omega_2$$
$$Angular\ velocity\ of\ driving\ shaft = \omega$$
$$Angle\ between\ the\ shafts = \alpha$$

Then $maximum\ angular\ velocity\ of\ driven = \omega_1 = \omega \times \frac{1}{\cos\alpha}$

and $minimum\ angular\ velocity\ of\ driven = \omega_2 = \omega \times \cos\alpha$

Also $\omega = \frac{2\pi N}{60}$

Where $\omega = Angular\ velocity\ in\ rad/s$

$N = Speed\ in\ r/min$

8.12.3 Double Hooke's joint

The driven shaft can be made to revolve at the same speed as the driver at all instants by the use of an intermediate shaft and two Hooke's joints, as shown in **Figure 8.21**.

The two forks at the ends of the intermediate shaft C must lie in the same plane and shafts A and B must be equally inclined to shaft C. There are thus two possible positions for shaft B.

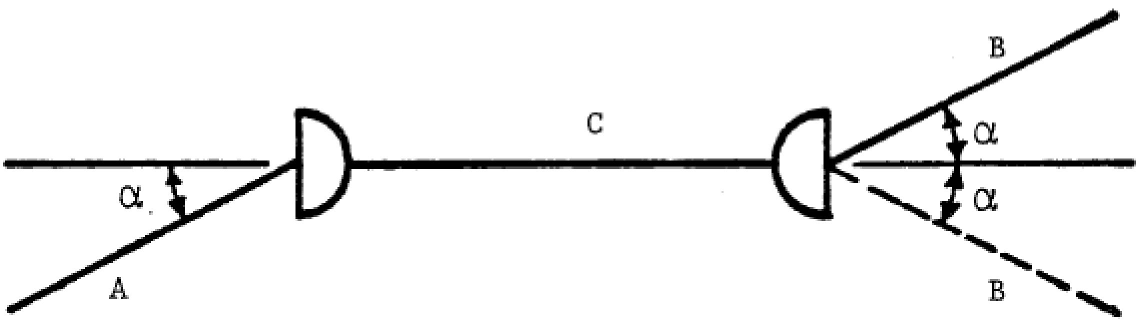


Figure 8.21

If the forks on C are incorrectly set at right angles [90°] , then ω_1 will fluctuate between

$\omega \cos^2 \alpha$ and $\frac{\omega}{\cos^2 \alpha}$

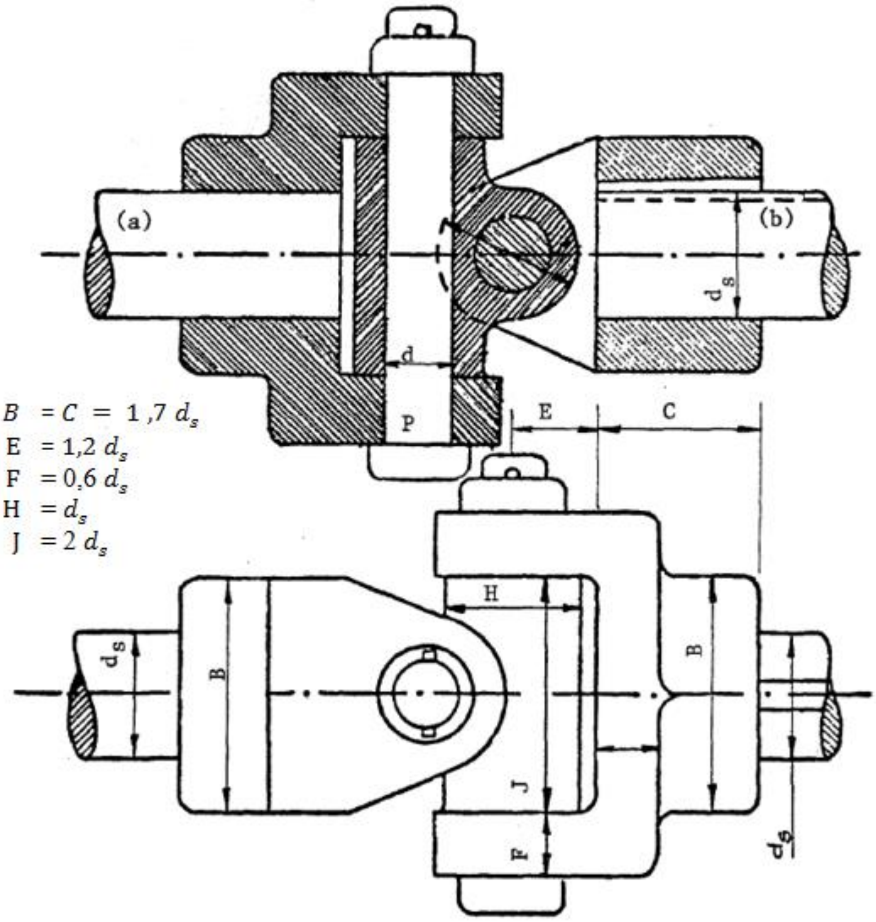


Figure 8.22

The shafts A and B are forked at the ends as shown in **Figure 8.22**. The joint is made by two pins P and P of equal size; these allow the forked ends to turn freely about. their axes.

If the power transmitted from a to b is known, the mean torque can be obtained:

$$Power = \frac{2\pi NT\ mean}{60}$$

As the pin P is in double shear, the diameter of the pin d can be calculated from:

$$Torque\ on\ pin = Torque\ on\ shaft$$

$$Force \times radius = \frac{\pi}{16}\tau_s {d_s}^3 \text{ and } Force = stress \times area$$

$$2 \times \frac{\pi}{4} d^2 \times \tau_p \times \frac{J}{2} = \frac{\pi}{16}\tau_s {d_s}^3 \qquad \text{but } J = 2d_s$$

$$\text{or } 8d^2\tau_p = \tau_s {d_s}^2 \qquad \therefore \frac{J}{2} = d_s$$

In **Figure 8.23** the shafts are keyed into similar forked ends, which engage with pins arranged on opposite sides of a centre cross-piece. As shown in **Figure 8.24**, one pin passes right through the cross-piece and the projections at each end form the " journals" for one forked end.

The short pins are arranged at right angles to the long pin; they are driven in and held either by set screws, bearing on recessed portions, or by tapered pins passing through cross-piece and pin.

The long pin may be locked in position in the same way, but in the smaller sizes it is assumed to secure sufficiently without pinning.

Another arrangement is shown in **Figure 8.25**. Four similar pins are screwed into the forked ends and locked in position by means of thin. check nuts. Plain ends projected into the cross-piece and form trunnion bearings.

Thus arranged, the pins can easily be taken out for inspection or dismantling.

Yet another arrangement consists of a cross-piece having four machined journals. On each journal there fits a set of pin rollers in a cage which is a press fit in the fork end of the rod.

This arrangement is a very compact one, and is used extensively in motor vehicles. The cages are pre-packed with lubricant and require very little maintenance.

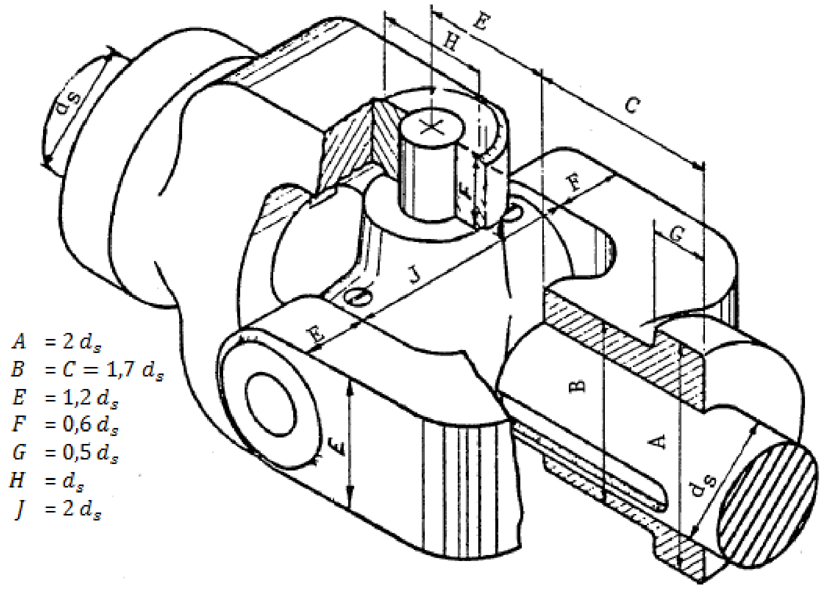


Figure 8.23

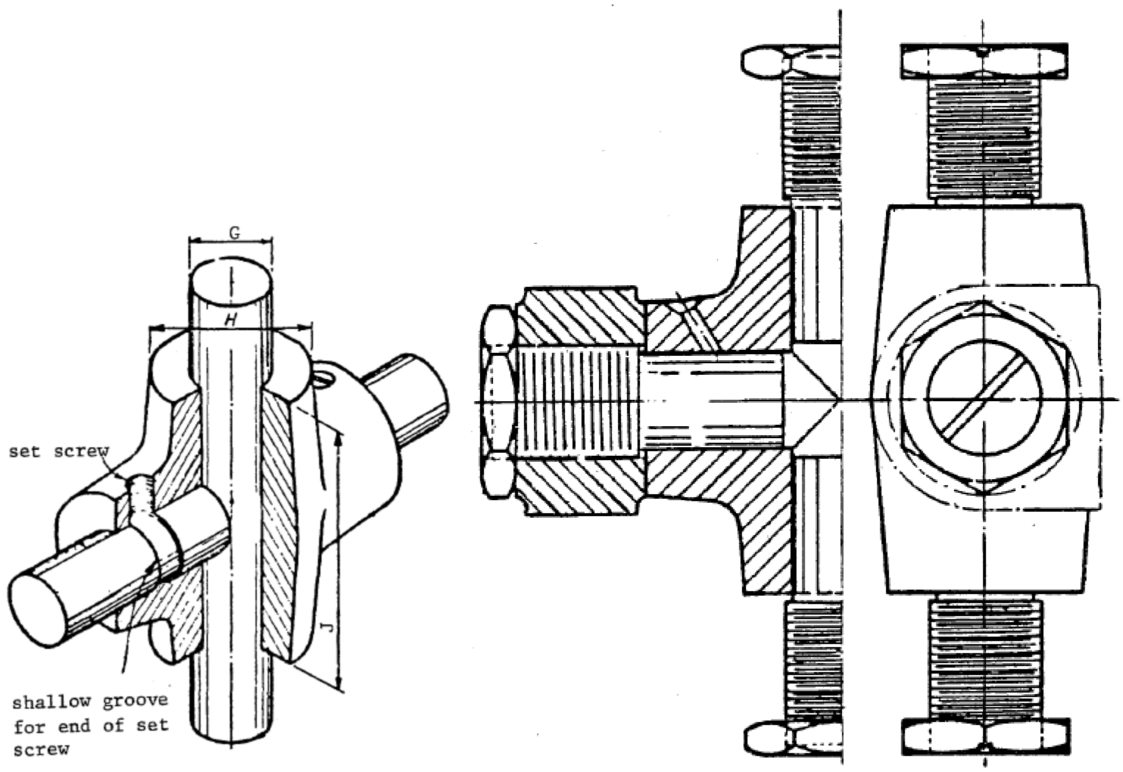


Figure 8.24

Another arrangement is drawn in **Figure 8.25**. This consists of two fork ends and two plates which are bolted together by four bolts. This type is easy to take apart for maintenance but tend to be bulkier than other types.

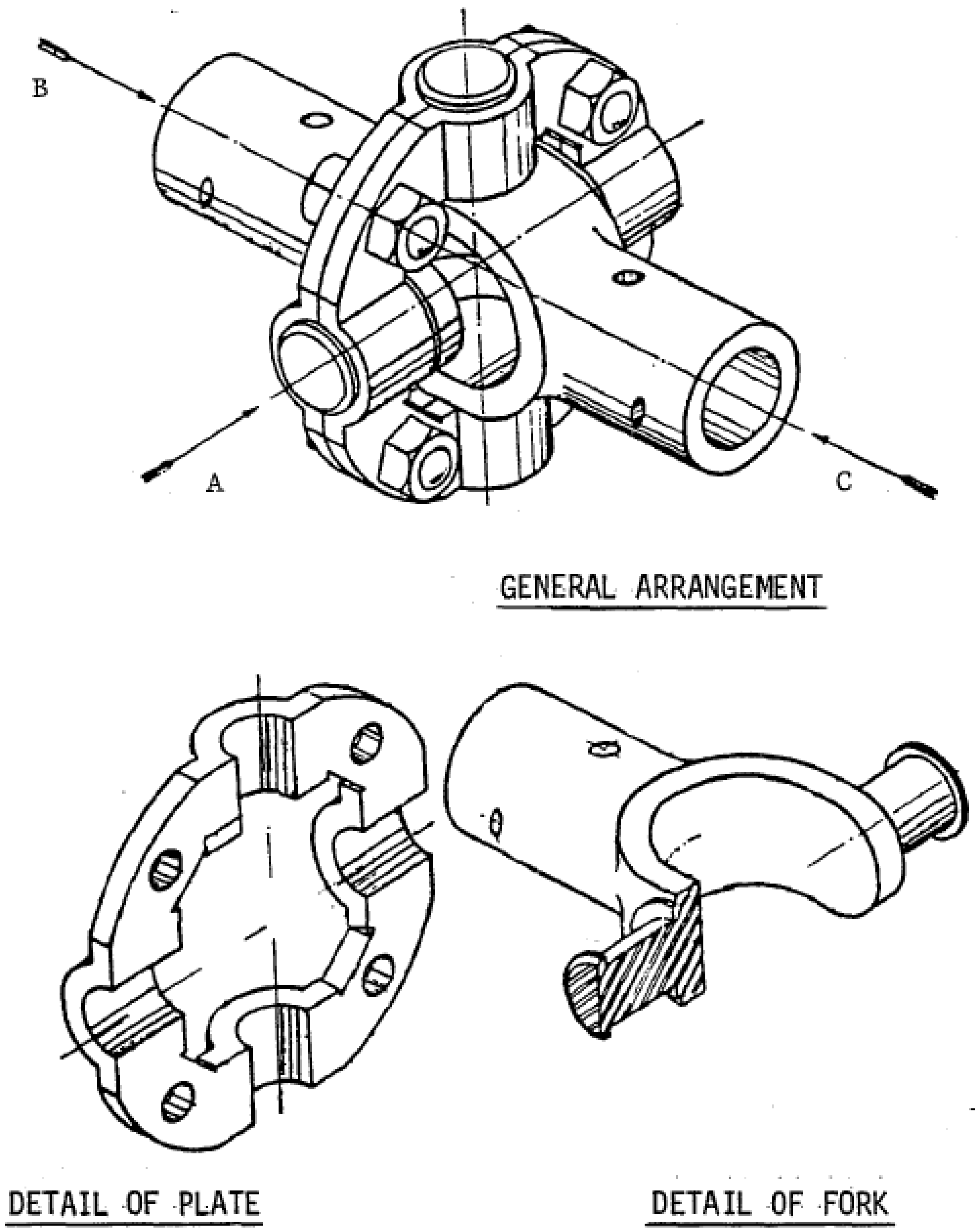


Figure 8.25

Worked Example 8.7

A Hooke's coupling connects two shafts which are inclined at an angle of 20°. The speed of the driving shaft is 1 000 r/min. Find the maximum and minimum angular velocities of the driven shaft.

Solution:

$$\text{Maximum speed } \omega_1 = \omega \sec \alpha$$

$$\text{but } \omega = \frac{2\pi N}{60}$$

$$= \frac{2\pi \times 1000}{60}$$

$$= 104{,}7\ rad/s$$

$$\therefore \omega_1 = 104{,}7 \times \frac{1}{\cos 20°}$$
$$= 104{,}7 \times 1{,}064$$
$$\therefore \omega_1 = 111{,}4\ rad/s$$
Maximum speed $\omega_2 = \omega \cos 20°$
$$= 104{,}7 \times 0{,}94$$
$$= 98{,}42\ rad/s$$

Worked Example 8.8

Assume that two universal joints of which the shafts are inclined at 20° are fitted incorrectly at 90°. The speed of the driving shaft is 1 000 r/min.

Find the maximum and minimum velocities of the driven shaft.

Solution:

$$\omega = \frac{2\pi N}{60}$$
$$= \frac{2\pi \times 1000}{60}$$
$$= 104{,}7\ rad/s$$
Maximum value of $\omega_1 = \frac{\omega}{\cos^2 \alpha}$
$$= \frac{\omega}{\cos^2 20°}$$
$$= \omega \sec^2 20°$$
$$= \frac{104{,}7}{(0{,}94)^2}$$
$$= 118{,}5\ rad/s$$
Maximum speed $\omega_2 = \omega \cos^2 20°$
$$= 104{,}7 \times \cos^2 20°$$
$$= 104{,}7 \times (0{,}94)^2$$
$$= 92{,}51\ rad/s$$

Worked Example 8.9

Figure 8.26 shows two views of a universal coupling designed to transmit 11 kW at 200 r/min.

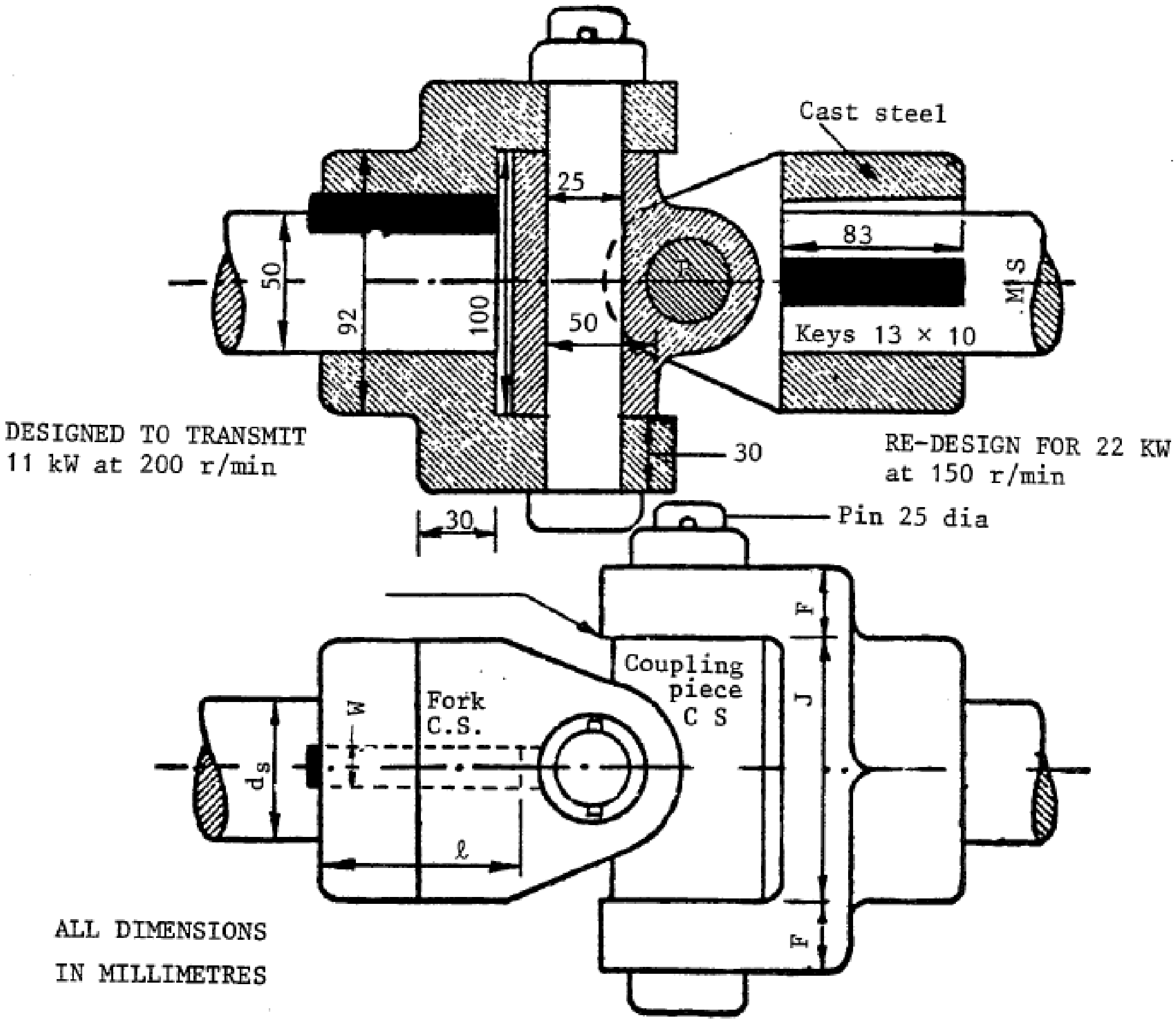


Figure 8.26

From the sizes given you are required to determine:

1. Shaft torque
2. shear stress in keys
3. compressive stress in keys
4. shear stress on pins
5. bearing stress on pins in fork
6. shear stress in shaft

Then redesign for 22 kW at 150 r/min; not exceeding the stress calculated above. The distance between forks (J) is in each case $2d_s$.

Solution:

1. Shaft torque

$$Power = \frac{2\pi NT}{60}$$

$$\therefore T = \frac{Power \times 60}{2\ \pi\ N}$$

$$= \frac{11 \times 10^3 \times 60}{2 \times \pi \times 200}$$

$$T = 525\ Nm$$

2. Shear stress in key

$Torque = Force \times radius$

but $Force = area \times stress$

also $area = W \times l$

$\therefore Torque = W \times l \times \tau_{key} \times \frac{d_s}{2}$ where $W = \frac{d_s}{4}$

$525\ Nm = 0{,}013\ m \times 0{,}083\ m \times \tau \times \frac{d_s}{2}$ $= \frac{50}{4}$

$\tau_{key} = \frac{525\ Nm \times 2}{0{,}013\ m \times 0{,}083\ m \times 0{,}05\ m}$ $=$

$12{,}5\ mm$

$\tau_{key} = 19{,}46 \times 10^6\ Pa$ $W = 13\ mm$

$= 19{,}46\ Pa$ $l = 0{,}083\ m$

3. Crushing stress in key

$Torque = Force \times radius$

but $Force = area \times stress$

also $area = \frac{T}{2} \times l$

$\therefore Torque = \frac{T}{2} \times l \times \sigma_c \times \frac{d_s}{2}$ but $T = \frac{d_s}{6}$

$\sigma_c = \frac{2 \times 525\ Nm}{0{,}005\ m \times 0{,}083\ m \times 0{,}05\ m}$ $\therefore \frac{T}{2} = \frac{d_s}{12}$

$= 50{,}6 \times 10^6\ N/m^2$ $= \frac{50}{12}$

$\sigma_c = 50{,}6\ MPa$ $= 4{,}2$

say $\frac{T}{2} = 5\ mm$

4. Shear stress in pins

The force acts at the shear line of the pin, ie between coupling piece and fork

$\therefore Radius = \frac{J}{2}\ mm$ $J = 2d_s$

$= \frac{0{,}100\ m}{2}$ $= 2 \times 5\ mm$

$= 50{,}6 \times 10^6\ N/m^2$ $= 100\ mm$

$R = 0{,}05\ m$

$Force = stress \times area$

$= T_{pin} \times 2 \times \frac{\pi}{4} d^2$ where $d = dia. of\ pin$

$2\ \ for\ double\ shear$

$Torque = Force \times radius$

$T_{shaft} = \tau \times 2 \times \frac{\pi}{4} d^2 \times R$

$\tau_{pin} = \frac{4 \times T_{shaft}}{2 \times \pi \times d^2 \times R}$

$= \frac{4 \times 525\ Nm}{2 \times \pi \times (0{,}025)^2 m^2 \times 0{,}05\ m}$

$= 10{,}7 \times 10^6\ Pa$

$\tau_{pin} = 10{,}7\ MPa$

5. Bearing Stress on Pins in Fork

$Torque = number\ of\ pins\ \times area \times stress \times radius$

$R = distance\ between\ forks + thickness\ of\ forks$

$= \frac{100+30}{2}$

$= 65\ mm$

$= 0{,}065\ m$

$\therefore Torque = 2 \times d \times F \times \sigma_c \times R$

$525\ Nm = 2 \times 0{,}025\ m\ \times 0{,}03\ m\ \times \sigma_c \times 0{,}065\ m$ $d = dia. of\ pin$ F

$thickess\ of\ fork$

$\sigma_c = \frac{525\ Nm}{2\times0{,}025\ m\ \times0{,}03\ m\ \times0{,}065\ m}$

$= 5{,}4 \times 10^6\ N/m^2$

$\sigma_c = 5{,}4\ MPa$

6. Shear stress in shaft

$T_{shaft} = \frac{\pi {d_s}^3}{16} \times \tau_{shaft}$

$\tau_{shaft} = \frac{16\times T_{shaft}}{\pi \times {d_s}^3}$

$= \frac{16\times525Nm}{\pi\times(0{,}05)^3 m^3}$

$= 21{,}4 \times 10^6\ Pa$

$= 21{,}4\ MPa$

Re-design for 22 kW at 150 r/min

$Power = \frac{2\pi NT}{60}$

$T = \frac{Power\times60}{2\ \pi\ N}$

$= \frac{22\times10^3\times60}{2\pi\times150}$

$= 1400\ Nm$

$T = 1{,}4\ kNm$

Shaft size

$T = \frac{\pi {d_s}^3}{16} \times \tau$

$d_s = \sqrt[3]{\frac{16\times T}{\pi\times\tau}}$

$= \sqrt[3]{\frac{16\ \times1400}{\pi\times21{,}4\times10^6}}$

$= 0{,}0693\ m$

$d_s = 70\ mm$

Key sizes

Width $W = \frac{d_s}{4}$

$= \frac{70}{4}$

$= 17{,}5$

say $W = 18\ mm$

Thickness $T = \frac{d_s}{6}$
$= \frac{70}{6}$
$= 11{,}5\ mm$
say $T = 12\ mm$

Length of key in shear

$$Torque = Force \times radius$$
$$T_{shaft} = W \times l \times \tau_{key} \times \frac{d_s}{2}$$
$$l = \frac{2 \times T_{shaft}}{W \times \tau_{key} \times d_s}$$
$$= \frac{2 \times 1400\ Nm}{0{,}018\ \mathrm{m} \times 19{,}46 \times 10^6\ N/m^2 \times 0{,}07\ m}$$
$$= 0{,}114\ m$$
$$l = 114\ mm$$

Length of key in crushing

$$\tau_{shaft} = \frac{T}{2} \times l \times \sigma_c \times \frac{d_s}{2}$$
$$l = \frac{4 \times T_{shaft}}{T \times \sigma_{c_{key}} \times d_s}$$
$$= \frac{4 \times 1400 Nm}{0{,}012\ \mathrm{m} \times 50{,}6 \times 10^6\ N/m^2 \times 0{,}07\ m}$$
$$= 0{,}132\ m$$
$$l = 132\ mm$$

∴ Use key measuring 18 mm x 12 mm x 132 mm

Diameter of pins in shear
Force acts on shear line of pin

$$Distance\ between\ forks = J = 2 \times d_s$$ See **Figure 8.26**
$$= 2 \times 70\ mm$$
$$= 140\ mm$$
$$\therefore Radius = \frac{140}{2}\ mm$$
$$= 70\ mm$$
$$= 0{,}07\ m$$

$$Force = stress \times area$$
$$= \tau_{pin} \times 2 \times \frac{\pi}{4} d^2$$
$$Torque = Force \times radius$$
$$T_{shaft} = \tau_{pin} \times 2 \times \frac{\pi}{4} d^2 \times radius$$
$$1400\ Nm = 10{,}7 \times 10^6 N/m^2 \times 2 \times \frac{\pi}{4} \times d^2 \times 0{,}07\ m$$
$$d = \sqrt{\frac{1400\ Nm \times 4}{10{,}7 \times 10^6 N/m^2 \times 2\pi \times 0{,}07\ m}}$$
$$= 0{,}034\ m$$
say $d = 35\ mm$

Thickness of fork

$$\begin{aligned}
Force &= stress \times area \\
&= \sigma_c \times number\ of\ pins \times area\ of\ one\ pin \\
&= \sigma_c \times 2 \times d \times F \qquad \text{See } \textbf{Figure 8.26} \\
Radius &= \frac{distance\ between\ forks + thickness\ of\ forks}{2} \\
&= \frac{0{,}14+F}{2} \\
R &= 0{,}07 + \frac{F}{2} \\
\therefore Torque &= force \times radius \\
T_{shaft} &= \sigma_c \times 2 \times d \times F \times \left(0{,}07 + \frac{F}{2}\right) \\
1400\ Nm &= 5{,}4 \times 10^6 N/m^2 \times 2 \times 0{,}035m \times F\left(0{,}07 + \frac{F}{2}\right) \\
\frac{1400\ Nm}{5{,}4 \times 10^6 N/m^2 \times 2 \times 0{,}035m} &= 0{,}07\,F + 0{,}5\,F^2 \\
0{,}0037 &= 0{,}07\,F + 0{,}5\,F^2 \\
0{,}5\,F^2 + 0{,}07\,F - 0{,}0037 &= 0 \\
\text{where } a &= 0{,}5 \\
b &= 0{,}7 \\
c &= -0{,}0037 \\
\\
F &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
&= \frac{-0{,}07 \pm \sqrt{(0{,}07)^2 - 4(0{,}5)(-0{,}0037)}}{2 \times 0{,}5} \\
&= -0{,}07 \pm 0{,}111 \\
&= 0{,}041\ m \qquad \text{we use the plus value only} \\
F &= 41\ mm
\end{aligned}$$

Activity 8.1

1. A solid shaft, 50 mm in diameter is required to transmit 18 kW with a stress due to twisting only of not more than 52 MPa. Calculate the operating speed in r/min and also the diameter of the 6 coupling bolts on a pitch circle diameter of 150 mm. The bolts have the same shear strength as the shaft.
2. Make a neat sketch of a solid flange coupling of which the shaft diameter is 100 mm. Use standard proportions to find the dimensions of the coupling.
3. The following particulars refer to a flange coupling:

Diameters of shaft to be connected	100 mm
Number of coupling bolts	6
Pitch circle diameter of the bolts	250 mm
Allowable shear stress in the bolts	9,5 MPa

If the shafts transmit 110 kW at 210 r/min, calculate the diameter of the coupling bolts.

4. A cylindrical, solid shaft is required to transmit 25 kW at 1 400 r/min from a motor. The shaft is coupled to the motor by means of a solid flange coupling.
   Calculate:
   i. The diameter of the shaft, if the stress in the shaft is not to exceed 10 MPa.
   ii. The number of the coupling bolts which have a diameter of 14 mm. The allowable shear stress in the bolts must not exceed 4,5 MPa. The bolts are spaced on a pitch circle diameter of 142 mm.
   iii. The thickness of the flanges due to crushing of the bolts and flanges. The allowable compressive stress for these bolts is not to exceed 1,8 MPa.

Activity 8.2

1. Make a neat sketch of a claw coupling, showing the two halves disengaged.
2. Make a neat sketch to show how the operating lever of a claw coupling is used to move the clutch faces together, and show a means of lubricating this mechanism.
3. Two 60mm-diameter shafts are to be connected by a claw coupling. There are three claws on each half of the coupling with an external diameter of 160 mm, internal diameter of 80 mm, and a depth of 32 mm. The shear stress in the shaft due to torsion is 62 MPa.
   Calculate:
   i. the shear and compressive stresses in the claws;
   ii. the minimum length of the key in one half of the coupling if the shear stress in the key must not exceed 50 MPa, and the compressive stress in the key must not exceed 148 MPa.
4. A shaft, on which a claw coupling is positioned, transmits 30 kW at 200 r/min. The shaft is supported by two bearings mounted on the floor.
   Calculate:
   i. the diameter of the shaft (allowable stress due to twisting should not exceed 55 MPa);
   ii. the diameter of the shaft of the operating mechanism. Assume the effective length of the hand-operating lever to be 380 mm and the maximum force. applied 270 N. Calculate for pure twisting and take allowable stress as 31 MPa;
   iii. size of pins of circu ·lar cross.-section fixing the operating handle and levers in position. (Calculate for shear, taking the allowable stress as 170 MPa)

Activity 8.3

1. Sketch a universal joint that has a cross-piece and bearing caps with needle bearings. Show clearly how the needle bearings are held in the fork ends of the rod.
2. Two shafts are connected to each other by means of a Hooke's coupling. The shafts are inclined at 30° to each other. The driving shaft rotates at a constant speed of 120 r/min, and transmits 60 kW,
   Calculate:
   i. the maximum angular velocity of the driven shaft
   ii. the minimum angular velocity of the driven shaft
   iii. the torque transmitted by the shaft
   iv. the diameter of the shaft if the stress in the shaft material must not exceed 45 MPa
3. The diameter of the shaft of the Hooke's joint (universal coupling) shown in **Figure 8.27**, is 80 mm. Evaluate the other sizes from values given on the drawing.

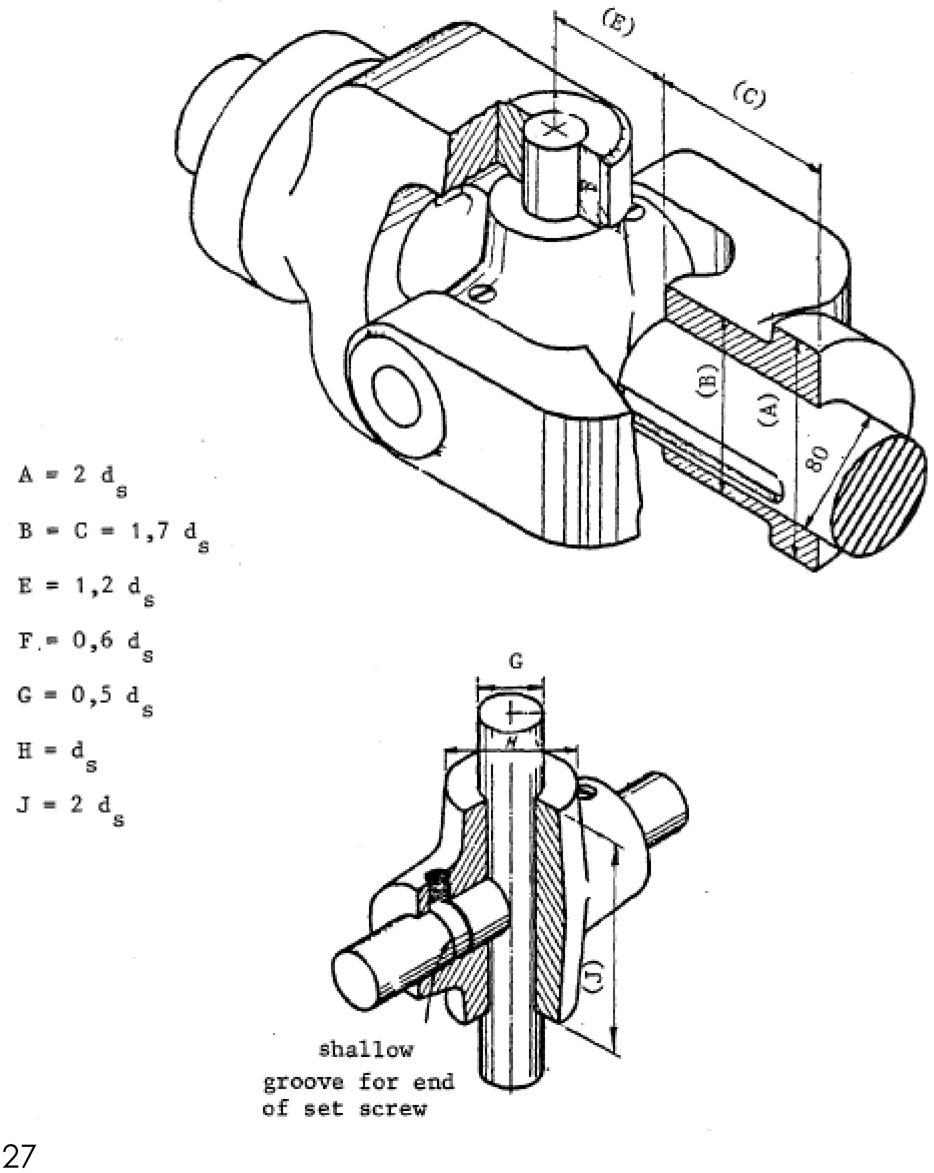


Figure 8.27

Self-Check

I am able to:	**Yes**	**No**
• Describe shaft couplings		
o Flange couplings		
o Strength of the coupling		
o Marine couplin		
o Flexible couplings		
o Muff coupling		
• Describe claw coupling		
o Standard proportions		
o Design		
• Describe universal joint or Hooke's coupling		
o Variation in angular velocity of driving shaft		
o Determine maximum and minimum angular velocities		
• Calculate the size of bolts for a flange coupling, given the number of bolts and pitch circle diameter to transmit the power at a given speed by equating the torque on the shaft to the torque on the bolts		
If you have answered 'no' to any of the outcomes listed above, then speak to your facilitator for guidance and further development.		

Module 9

Belts

Learning Outcomes
On the completion of this module the student must be able to:

- Describe belt drives
- Describe belt materials and the choice of belting material
- Describe belt joints
- Describe pulleys
- Describe the shapes of pulley rims
- Perform belt calculation
  - Angle of lap
  - Ration of tensions
- Describe the size of flat belt for transmitting a given power

9.1 Introduction

Belt drives are used to transmit power over long distances. Machines used to be fitted with flat belts because they were easy to repair and required little maintenance.

9.2 Belt drives

One of the principal features of belt driving, compared with other forms of gearing, for example toothed gearing or chain gearing, is the absence of noise.

Belts do not transmit shock, and under a sudden heavy load tend to slip on the pulley, perhaps thereby eliminating damage to the machine or job.

Did you know?
Belting is especially adapted to drives where the distance between the shafts is quite considerable.

9.2.1 Flat belts

They are not made continuous, but are cut to the required length and are then joined by cementing or by fasteners.

Flat belts are unsuitable for short drives, as the arc of contact of the belt on the pulley is too small' resulting in numerous slips.

9.2.2 Open belt drive
This type of belt drive is used where it is essential that both pulleys should rotate in the same direction.

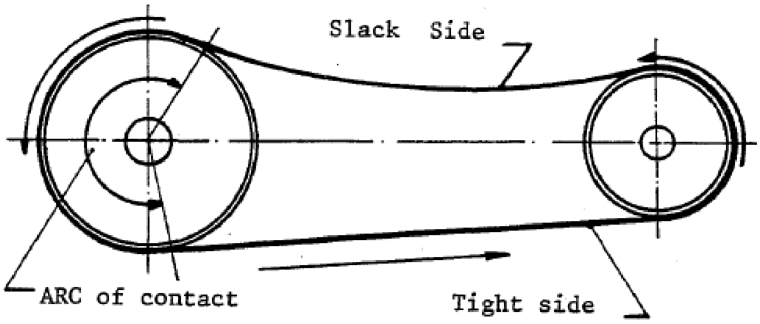


Figure 9.1 Open belt

9.2.3 Crossed belt drive
If the driven pulley has to revolve in an opposite direction to that of the driving pulley, the belt has to be crossed. However, wear is caused by the belt sides rubbing together. However, the efficiency is increased as a result of the larger arc of belt contact on the pulleys.

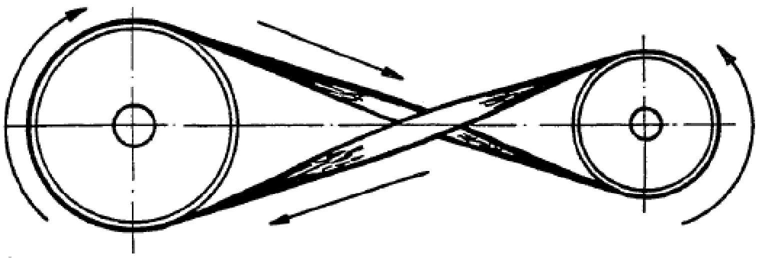


Figure 9.2

9.3 Belt materials
The following are the three most common forms of belting in present-day use:
- Leather
- Cotton and canvas
- Rubber and canvas

9.3.1 Leather belts
Steer hide leather are cut into suitable strips and joined together. Double belts are made by placing the flesh sides together and then cementing and stitching them.

9.3.2 Cotton and canvas belts
When they have to pass through belt shifting forks, the wear on the belt edges tends to cause the plies to come apart. The strength of this belt is about the same as that of leather belts.

9.3.3 Rubber and canvas belts
They are especially adapted to use in wet places, but are not reliable in the presence of oil and grease, as the plies tend to come apart.

9.4 Choice of belting material

The following points should be considered when selecting the most suitable belting material for a flat belt drive:

- Friction between belt and pulley.
- Conditions under which drive must operate, such as oil, dust, dampness.
- Size of pulleys and centre distance between them, in other words, will material bend or straighten easily.
- Will material have to withstand abnormal tension or stretch.

9.5 Belt joints

The aim, when joining up the ends of flat belts, should be to make the joints as strong as the belting, and to keep a uniform thickness.

9.5.1 Cemented and laced joints

The ends of the belt are usually joined in such a way that the joint can be easily broken or separated.

Think about it!
For thick belts, the joints are sometimes riveted to give extra strength.

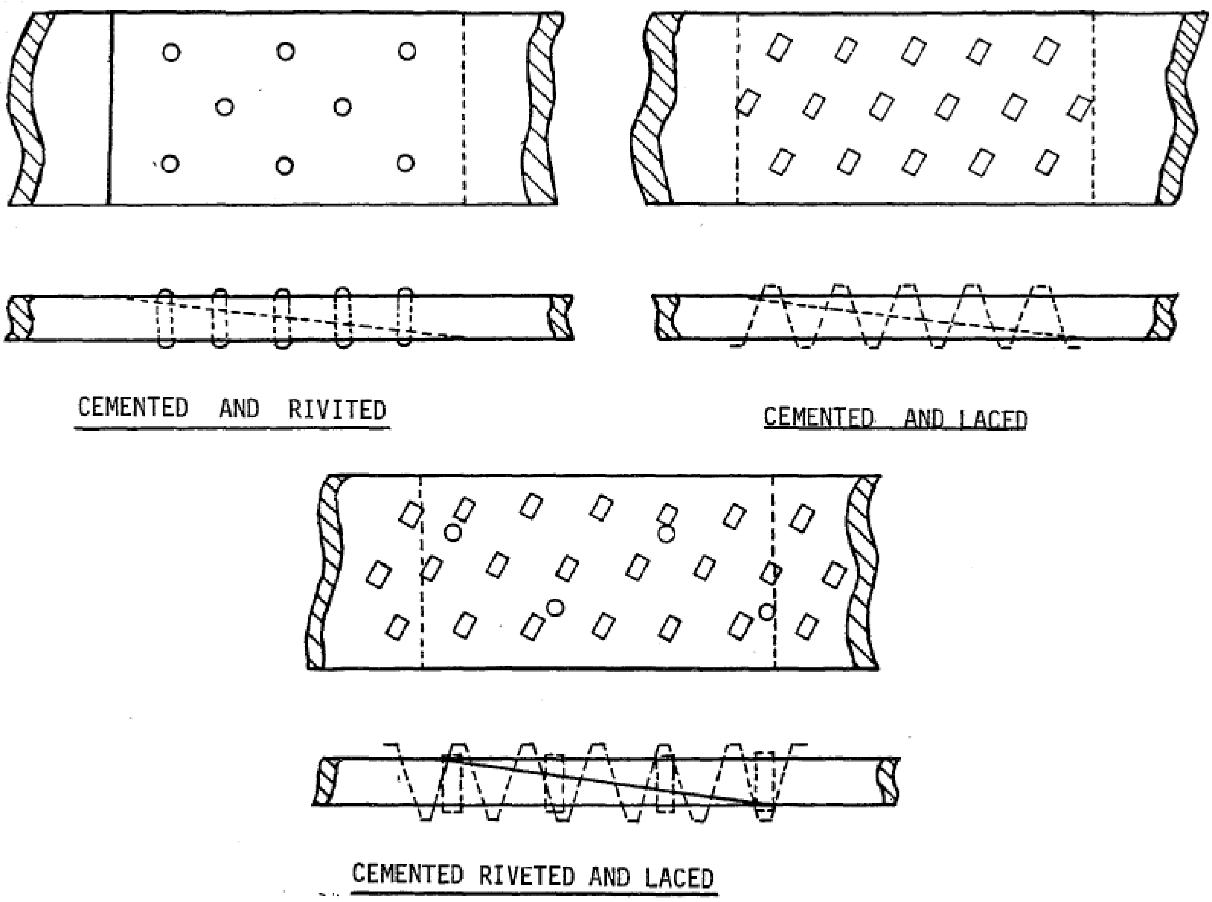


Figure 9.3

9.5.2 Metallic fasteners

A good-joint should be strong, flexible and light. The fastener should be cut off a little less than the width of the belt, to prevent dangerous metallic edges projecting beyond the side of the belt.

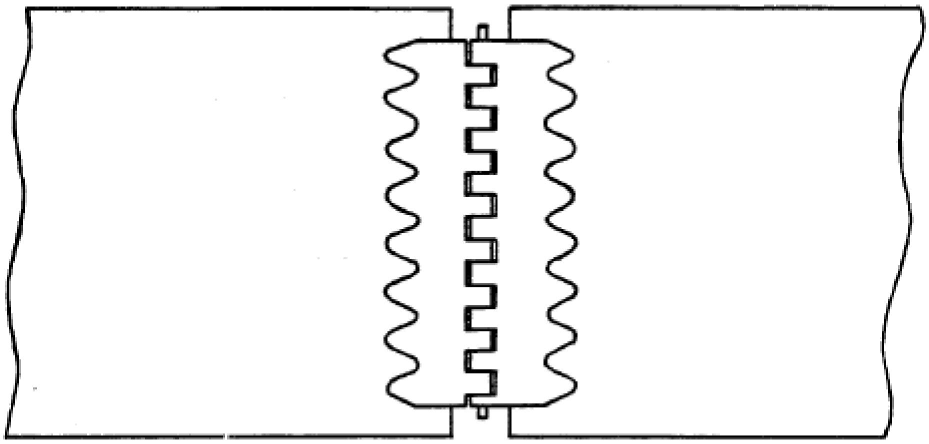


Figure 9.4 Alligator fastener

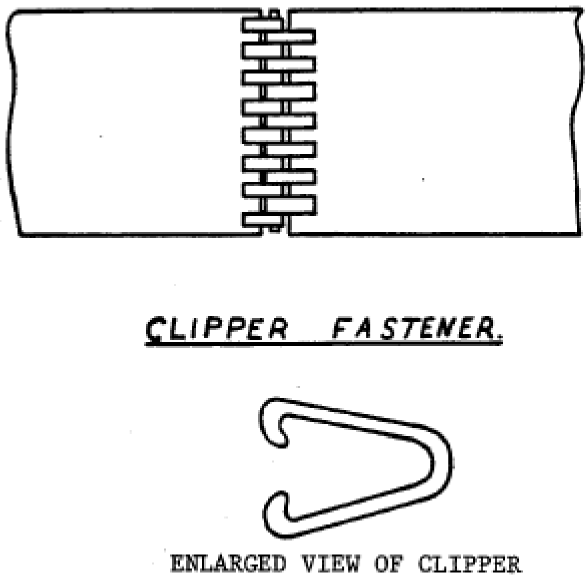


Figure 9.5

9.5.3 Belts transmitting heavy loads

Holes are punched into the two ends of the belt, the fasteners inserted as shown and then flattened down with a hammer.

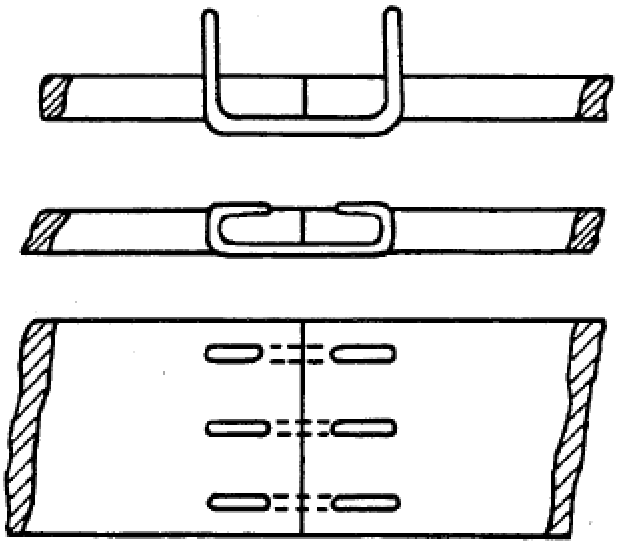


Figure 9.6 Butt joint with metal fasteners

9.6 Pulleys

They are made of wood, cast iron, steel or wrought iron. Pulleys constructed entirely from wood, have the advantage of lightness. They are generally built up in two halves and held together by bolts. They are mostly used on main shafts and other light drives.

Large pulleys that are run at high speed should be accurately balanced before being assembled.

9.6.1 Solid pulleys

These pulleys are cast in one piece.

A disadvantage in the use of these is that if replacement becomes necessary, the entire arrangement of shaft and pulleys has to be taken down.

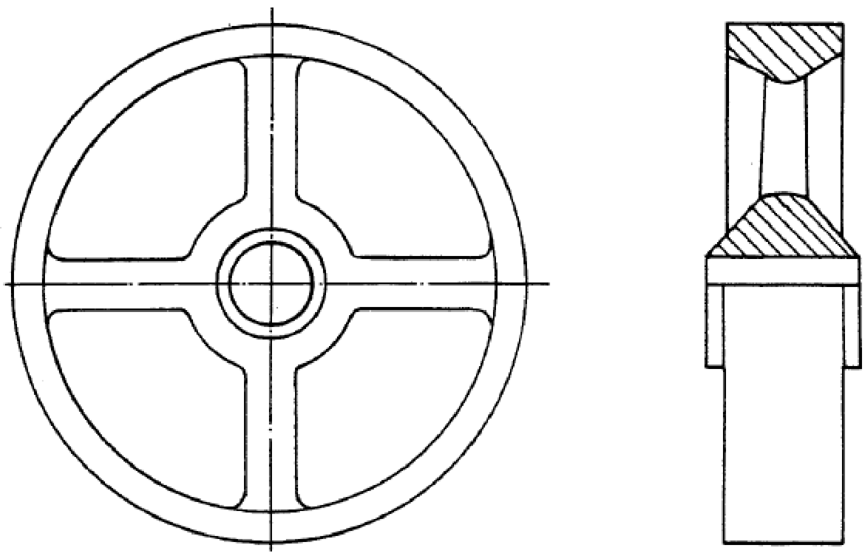


Figure 9.7 Straight-armed cast iron solid pulley

9.6.2 Split pulleys

These are used where it is necessary to mount a pulley some distance from the end of a shaft already carrying pulleys. It is obvious that if replacement becomes necessary, no trouble will be encountered.

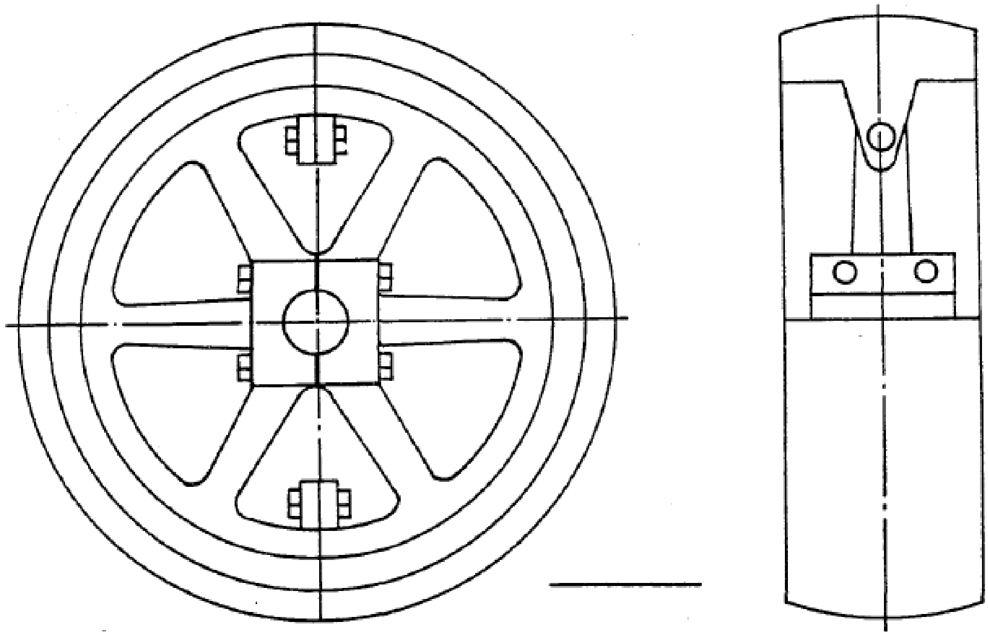


Figure 9.8 Cast-iron split pulley

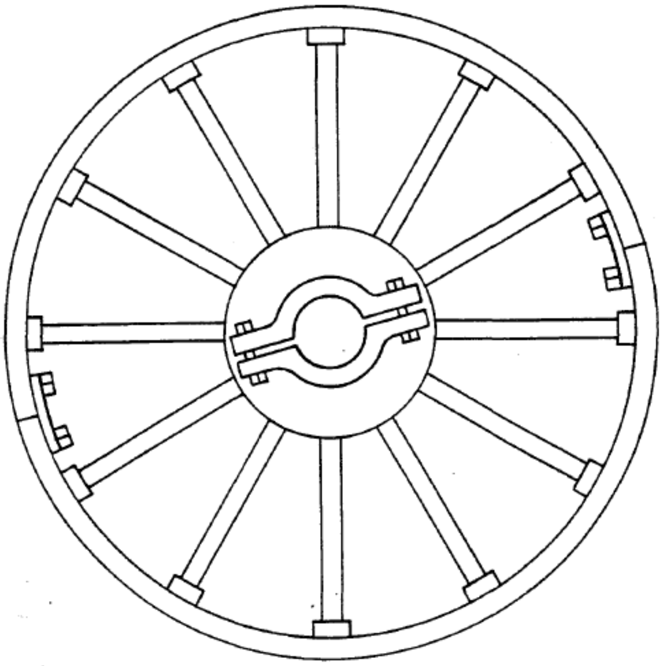


Figure 9.9 Built-up split pulley

9.6.3 Cone Pulleys

This type of pulley is very useful where it is desired to provide for a suitable range of spindle speeds, for example in a lathe where it is desired to obtain the correct cutting speeds for large and small diameters, different metals, etc.

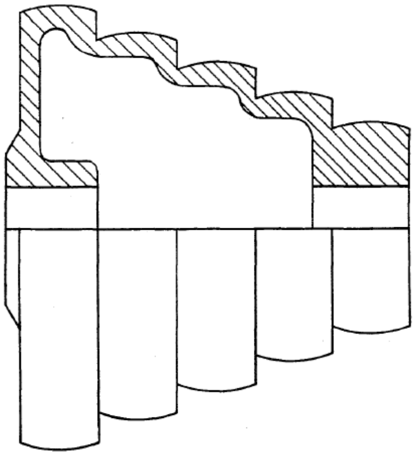


Figure 9.10 Five-step cone pulley

9.7 Shapes of pulley rims

The rims of flat belt pulleys can be formed to suit the particular type of drive and may be flat, crowned or flanged.

9.7.1 Flat-rim pulley

This type is used when it is required to move the belt from one pulley to another, for example fast and loose pulleys on lathe countershaft.

9.7.2 Crowned-rim pulley

Rounding or crowning is done to keep the belts on the pulleys. The belt takes the shape of the crowned rim and therefore, cannot slip off easily.

9.7.3 Flanged-rim pulley

They are used when it is desired to ensure that the belt does not come off the pulley.

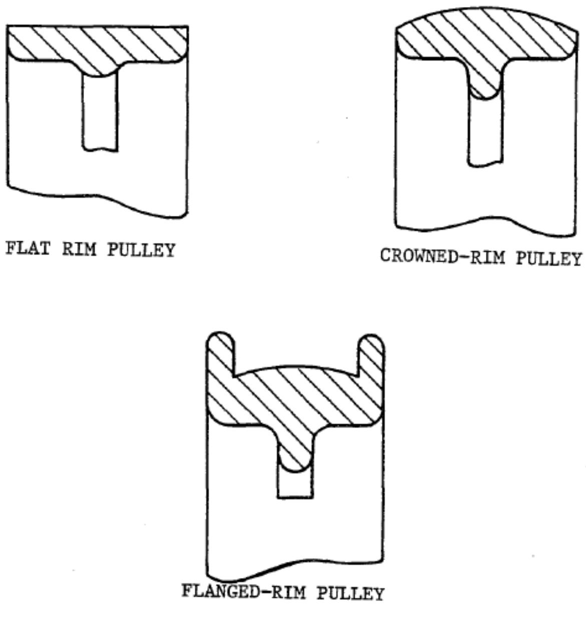


Figure 9.11

9.8 Belt calculations

If there is no slipping of the belt on the pulleys, then the speed of the pulleys is inversely proportional to the diameters, in other words small pulley, high speed; large pulley, slow speed.

$$Speed\ of\ driver = Speed\ of\ driven$$
$$\pi dn = \pi Dn$$

Where d = driver-pulley diameter D = driven-pulley diameter
n = driver-pulley r/min N = driven-pulley r/min

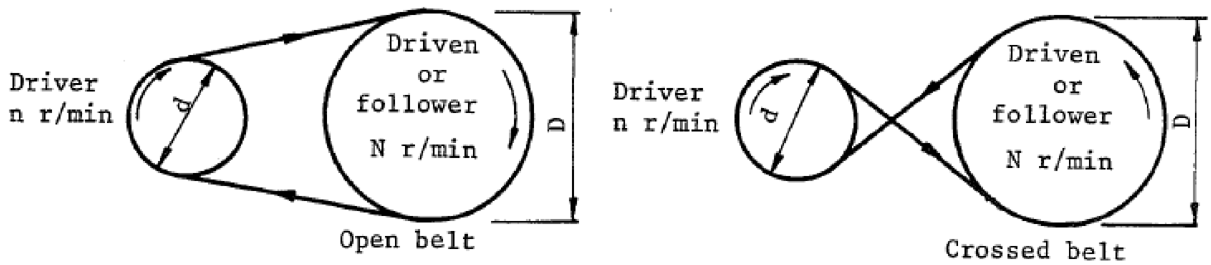


Figure 9.12

9.8.1 Speed of belt

If the belt is not slipping, the speed of the belt over both pulleys will be the same, therefore it must be moving at the same speed as the circumference of either of the pulleys.

$$Speed\ of\ belt\ in\ metres\ per\ minute = \pi Dn$$

where D = $Effective\ diameter\ of\ pulley\ in\ metres$
N = $r/min\ of\ pulley$

9.8.2 Effective diameter of pulley

If the belt thickness is appreciable in comparison with the pulley diameters, the effective diameter of the pulley is measured to the centre of the belt, but if the belt is thin or the pulley large, the belt thickness may be neglected.

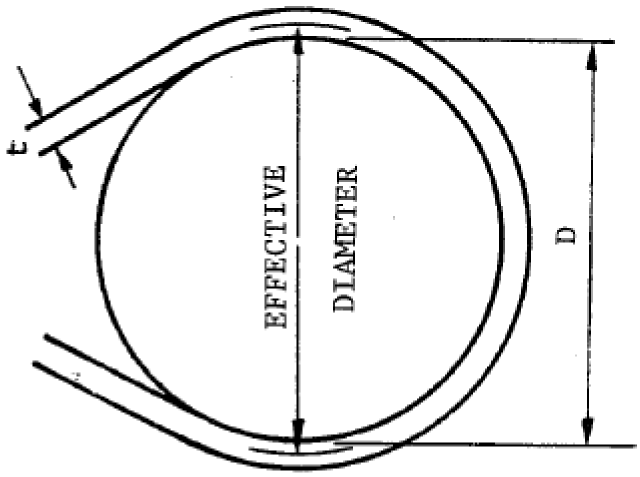


Figure 9.13

$$Effective\ diameter = D + \frac{1}{2}t + \frac{1}{2}t$$
$$= (D + t)\ metres$$

In practice, there is always a small amount of slip or creep, and the driven pulley runs at a slightly slower speed than it should in theory. The loss in speed should not be more that about 2%. If it is more than this, then either the belt is too loose, or it is too small for the job, or it is being run at too high a speed.

9.8.3 Power transmitted by a belt

When a belt drive is operating, one side is "tight" and the other side is "slack". This does not mean that the "slack" side is completely loose, since there must be some tension in it or the belt would slip around the pulley and not drive it.

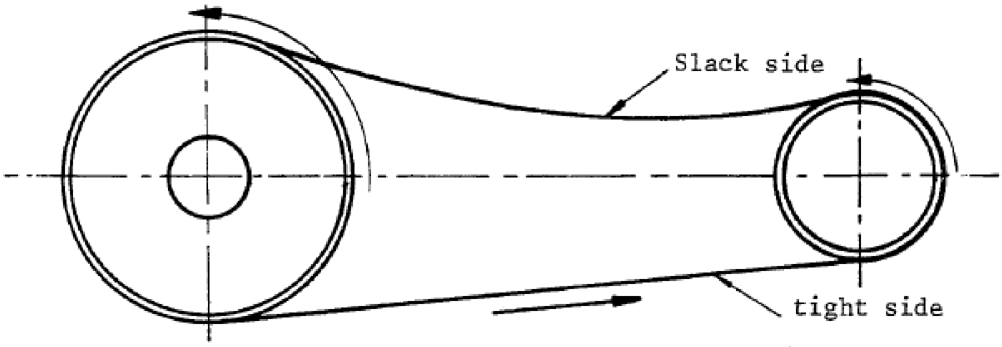


Figure 9.14 Open belt

If this is not clear to the student, he should imagine a piece of belting lying over a pulley on a line shaft as shown. If end A is pulled, the belt will just slip over the pulley and not turn it.

If a pull is now applied to end B, the belt will not slip, and it will be possible to make the pulley turn. Obviously, pull A must be greater than pull B, and the effective pull tending to turn the pulley, will be pull A minus pull B.

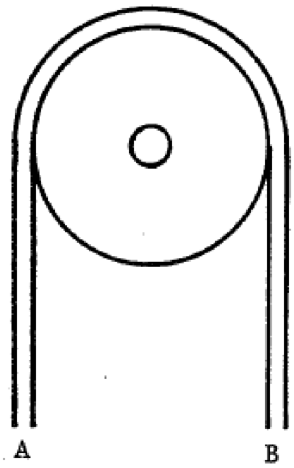


Figure 9.15

A belt pulley thus has two pulls acting on it - the force T_1 on the tight side and the force T2 on the slack side, and the effective force turning the pulley is $(T_1 - T_2)$.

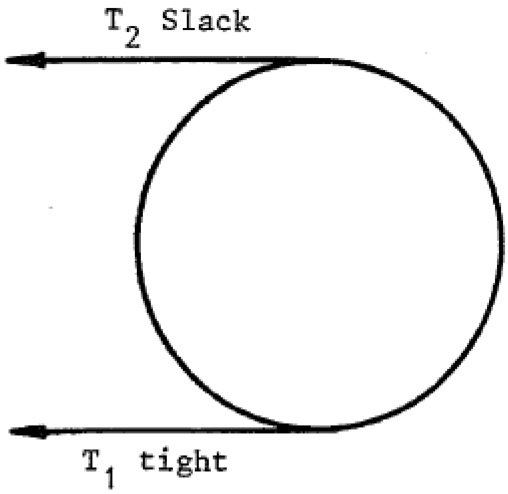


Figure 9.16

Now,

$$\begin{aligned} work\ done &= Force\ \times\ Distance \\ Work\ done\ per\ minute &= Force\ \times Distance\ moved\ per\ minute \\ &= (T_1 - T_2) \times Speed\ of\ belt\ in\ metres\ per\ minute \\ &= (T_1 - T_2) \times \pi DN \\ Power\ transmitted &= \frac{(T_1 - T_2) \times \pi DN}{60} \end{aligned}$$

or taking belt thickness into account

$$\begin{aligned} Work\ done\ per\ minute &= (T_1 - T_2) \times \pi(D + t)N \\ Power\ transmitted &= \frac{(T_1 - T_2) \times \pi(D + t)N}{60} \end{aligned}$$

Note:

T_1 = Force in belt on tight side
T_2 = Force in belt on slack side
π = 3,14
D = Diameter of pulley in metres
t = Thickness of belt in metres
N = revolutions per minute (r/min).

9.9 Angle of lap

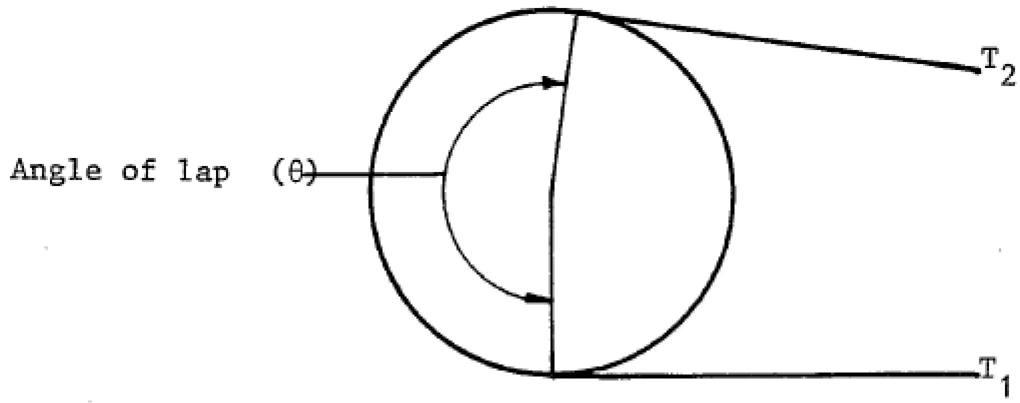


Figure 9.17

The angle of lap on the smaller pulley is always used, since the belt will slip on it before it does so on the larger one.

9.9.1 Open belt

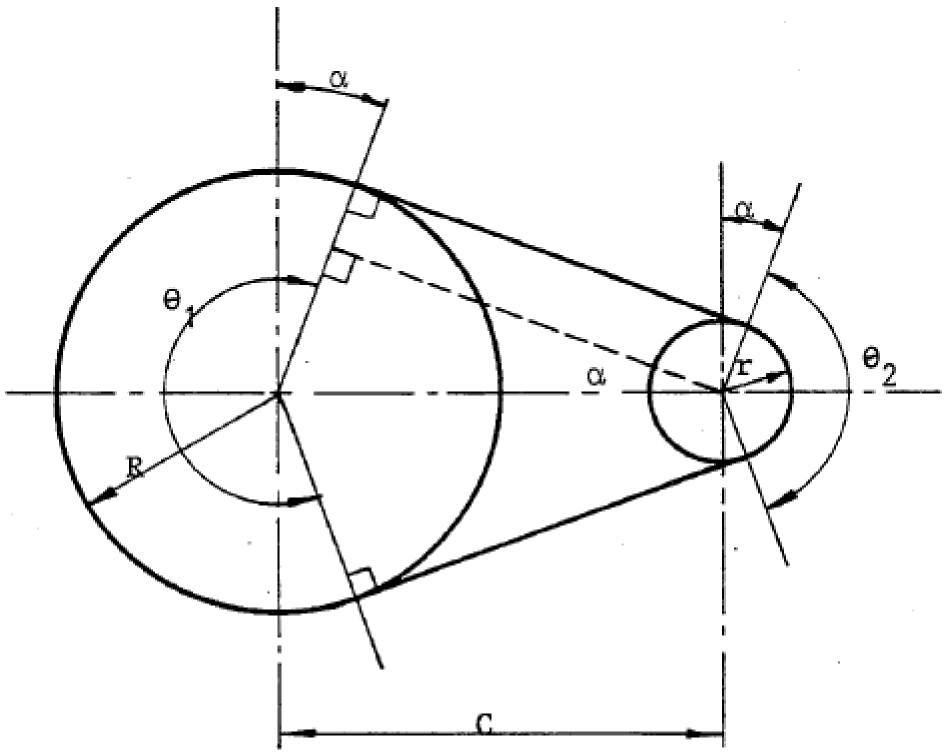


Figure 9.18

The angles of wrap θ_1 and θ_2 for an open belt may be determined by:

$$\sin\alpha = \frac{R-r}{c}$$

$$\theta_1 = 180° + 2\alpha$$

$$\theta_2 = 180° - 2\alpha$$

$$Length\ of\ belt = \frac{\pi}{2}(D+d) + \alpha(D+d) + 2 \times C \times \cos\alpha°$$

9.9.2 Crossed belt

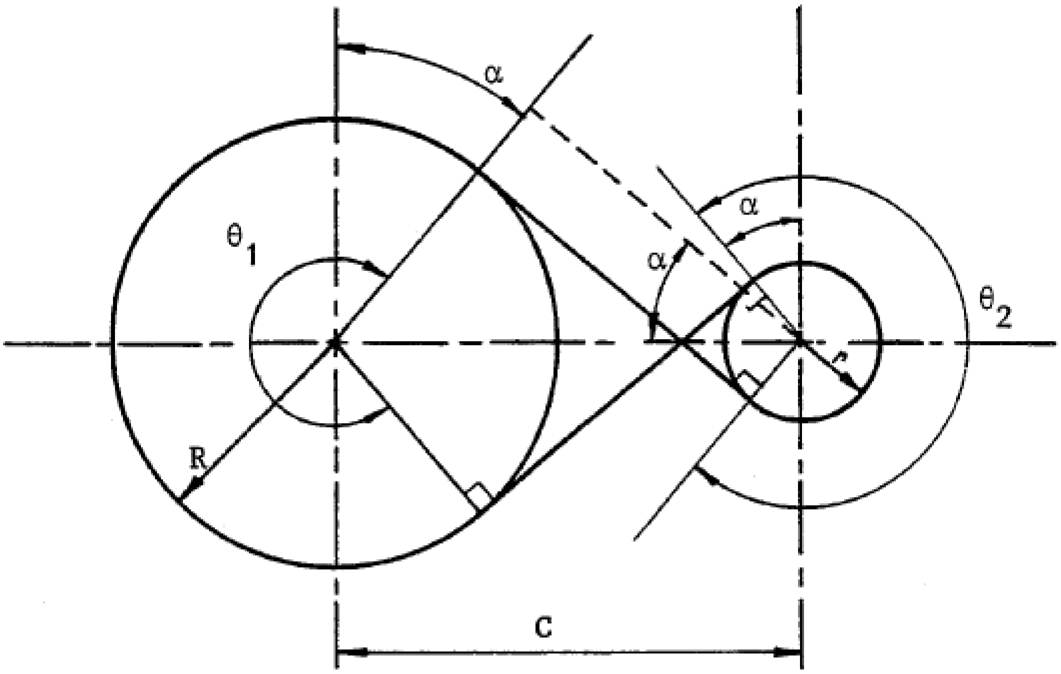


Figure 9.19

$$\sin\alpha = \frac{R-r}{c}$$
$$\theta_1 = \theta_2 = 180° + 2\alpha$$
$$Length\ of\ belt = \left(\frac{D}{2}\times\theta\right) + \left(\frac{d}{2}\times\theta\right) + 2\times C\times\cos\alpha°$$

Note:
$D = 2 \times R$ and $d = 2 \times r$

The ratio $\frac{T_1}{T_2}$ may be given in a question or it may be calculated. Its value depends on:

- The angle of lap of the belt on the pulley.
- Coefficient of friction between the belt and the pulley.

For flat belts $\frac{T_1}{T_2} = e^{u\theta}$

Where
$e = 2{,}718$
$u = coefficient\ of\ friction\ between\ belt\ and\ pulley$
$\theta = angle\ of\ lap\ of\ belt\ on\ pulley\ in\ radians$
$\emptyset = angle\ of\ vee$

Note:
$\pi \; radians = 180°$

Worked Example 9.1

An electric motor is fitted with a 300 mm diameter pulley from which a belt passes to a 1 metre diameter pulley on an intermediate shaft.

On the intermediate shaft, there is also a 230mm diameter pulley from which a belt passes to a 600mm diameter pulley on a machine shaft.

If the motor runs at 750 r/min, determine the speed of the machine shaft.

a) if there is no slip
b) b) if there is 2% slip in each drive

Solution:

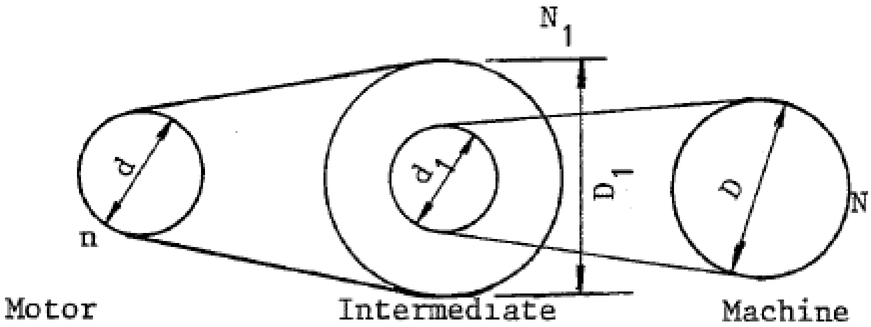


Figure 9.20

a)

$$Speed\ of\ driver = Speed\ of\ driven$$
$$Speed\ of\ motor = Speed\ of\ intermediate\ shaft$$
$$\pi DN = \pi D_1 N_1$$
$$N_1 = \frac{\pi dn}{\pi D_1} = \frac{dn}{D_1}$$
$$= \frac{0{,}3\ m \times 750\ r/min}{1\ m}$$
$$N_1 = 225\ r/min$$

$$Speed\ of\ intermediate\ shaft = Speed\ of\ machine$$
$$\pi d_1 N_1 = \pi DN$$
$$N = \frac{\pi d_1 N_1}{\pi D} = \frac{d_1 N_1}{D}$$
$$= \frac{0{,}23\ m \times 225\ r/min}{0{,}6\ m}$$
$$N = 86{,}25\ r/\min \rightarrow$$

b) $Speed\ of\ intermediate\ shaft = 225\ r/min \times \frac{98}{100}$

$$
\begin{aligned}
&= 220{,}5\ r/min \\
Speed\ of\ machine &= \frac{d_1 N_1}{D} \\
&= \frac{0{,}23\ m \times 220{,}5\ r/min}{0{,}6\ m} \times \frac{98}{100} \\
&= 82{,}84\ r/min
\end{aligned}
$$

Worked Example 9.2

A 230mm belt pulley is running at 75U r/min. Find the speed of the belt in metres per minute.

Solution:

$$
\begin{aligned}
Speed\ of\ belt &= \pi DN \\
&= \pi \times 0{,}23\ m \times 750\ r/min \\
&= 541\ {,}92\ metres\ per\ min \rightarrow
\end{aligned}
$$

Worked Example 9.3

A 200mm diameter belt pulley is turning at 90U r/min. The force on the tight side is 1 335 N and on the slack side 554 N. What power is being transmitted?

Solution:

$$
\begin{aligned}
Power\ transmitted &= \frac{(T_1 - T_2) \times \pi DN}{60} \\
&= \frac{(1335N - 554N) \times \pi \times 0{,}2\ m \times 900}{60} \\
&= 7360{,}75\ watts \\
&= 7360\ 75\ W \rightarrow
\end{aligned}
$$

Worked Example 9.4

What power can be transmitted by a leather belt 100mm wide by 10mm thick from a 250mm diameter pulley at 800 r/min if the allowable tensile stress in the belt is 1,38 MPa, and it is assumed that the force on the tight side of the belt is 2,2 times that on the slack side? (***Note***: $1\ MPa = 1 \times 10^6\ N/m^2$).

Solution:

$$
\begin{aligned}
Allowable\ tensile\ stress\ in\ the\ belt &= \frac{Allowable\ force\ in\ belt}{Cross{-}sectional\ area\ of\ belt} \\
Allowable\ force\ in\ belt &= Allowable\ tensile\ stress\ in\ the\ belt \\
&= 1{,}38 \times 10^6\ N/m^2 \times 0{,}1\ m \times 0{,}01\ m \\
&= 1\ 380\ N \rightarrow
\end{aligned}
$$

This is the maximum pull we are allowed to have in the belt, so it will be the pull in the tight side (T_1).

But we are told that $T_1 = 2{,}2\,T_2$

$\therefore T_2 = \frac{T_1}{2{,}2}$

$= \frac{1380}{2{,}2}$

$T_2 = 627{,}3\ N \rightarrow$

$Power\ transmitted = \frac{(T_1 - T_2) \times \pi(D+t)N}{60}$

$= \frac{(1380 - 627{,}3) \times \pi(0{,}25+0{,}01)800}{60}$

$= 8197{,}6\ watts$

$= 8{,}1976\ W \rightarrow$

Worked Example 9.5

What width belt is required to transmit 11 kW from a 280mm diameter pulley at 750 r/min if the allowable tensile force of the belt is 14 newtons per millimetre width of belt and it is assumed that the ratio of tension on the tight and slack sides is 2,3?

Solution:

$Power\ transmitted = \frac{(T_1 - T_2) \times \pi DN}{60}$

$(T_1 - T_2) = \frac{Power\ transmitted \times 60}{\pi DN}$

$= \frac{11 \times 10^3 \times 60}{\pi \times 0{,}28 \times 750}$

$T_1 - T_2 = 1000{,}4\ N \rightarrow$

But $ratio\ of\ force = \frac{T_1}{T_2} = 2{,}3$

$T_2 = \frac{T_1}{2{,}3}$

Substitute in $T_1 - T_2 = 1000{,}4\ N$

And we have $T_1 + \frac{T_1}{2{,}3} = 1000{,}4\ N$

$\frac{2{,}3T_1 - T_1}{2{,}3} = 1000{,}4\ N$

$\frac{1{,}3T_1 - T_1}{2{,}3} = 1000{,}4\ N$

$T_1 = \frac{1000{,}4 \times 2{,}3}{1{,}3} N$

$= 1770\ N$

But we are allowed 14 N/mm width of belt

$\therefore Width\ of\ belt = \frac{1770\ N}{14\ N/mm}$

$= 126{,}4\ mm \rightarrow$

Worked Example 9.6

The angle of lap of a belt on a pulley is 165° and the coefficient of friction between belt and pulley is 0,36. Find the ratio of tensions.

Solution:

$$180° = \pi\ radians$$
$$165° = \frac{\pi \times 165}{180}$$
$$= 2{,}88\ radians \rightarrow$$
$$\frac{T_1}{T_2} = e^{u\theta}$$
$$\frac{T_1}{T_2} = 2{,}718^{0{,}36 \times 2{,}88}$$
$$= 2{,}718^{1{,}037}$$
$$\frac{T_1}{T_2} = 2{,}82 \rightarrow$$

Worked Example 9.7

Determine the width of the flat belt required to transmit 9 kW from a 230mm diameter pulley running at 1 250 r/min.

The belt is in contact with the pulley rim for 160°, and the coefficient of friction is 0,24.

The maximum allowable pull in the belt is 16 newtons per millimetre of width.

Solution:

$$180° = \pi\ radians$$
$$165° = \frac{\pi \times 160}{180}$$
$$= 2{,}79\ radians \rightarrow$$
$$\frac{T_1}{T_2} = e^{u\theta}$$
$$\frac{T_1}{T_2} = 2{,}718^{0{,}24 \times 2{,}79}$$
$$= 2{,}718^{0{,}67}$$
$$\frac{T_1}{T_2} = 1{,}954 \rightarrow$$
$$T_1 = 1{,}954T_2$$
$$Power\ transmitted = \frac{(T_1 - T_2) \times \pi DN}{60}$$
$$(T_1 - T_2) = \frac{Power\ transmitted \times 60}{\pi DN}$$
$$= \frac{9 \times 10^3 \times 60}{\pi \times 0{,}23 \times 1250}$$
$$= 598\ N \rightarrow$$

Substitute for T_1 in equation 1

$$1{,}954\ T_2 - T_2 = 598\ N$$
$$0{,}954\ T_2 = 598\ N$$
$$T_2 = \frac{598}{0{,}954}$$
$$T_2 = 627\ N$$

Now $T_1 = 1{,}954\ T_2$

$= 1{,}954 \times 627\ N$

$= 1225\ N \rightarrow$

$Width\ of\ belt = \frac{T_1}{Maximum\ allowable\ pull\ in\ belt}$

$= \frac{1225\ N}{16\ N/mm}$

$= 76{,}56\ mm$

say $77\ mm \rightarrow$

Worked Example 9.8

Determine the maximum power that can be transmitted by a flat belt, given the following data:

- Width of belt 100 mm
- diameter of pulley 450 mm
- speed 500 r/min
- angle of contact of belt on pulley 190°
- coefficient of friction 0,32
- working tension of the belt not to exceed 13 newtons per millimetre width

Solution:

$180° = \pi\ radians$

$190° = \frac{190 \times \pi}{180}$

$= 3{,}316\ radians \rightarrow$

$\frac{T_1}{T_2} = e^{u\theta}$

$\frac{T_1}{T_2} = 2{,}718^{0{,}32 \times 3{,}316}$

$= 2{,}718^{1{,}061}$

$\frac{T_1}{T_2} = 2{,}89 \rightarrow$

Now $T_2 = \frac{T_1}{2{,}89} \rightarrow$

$T_1 = 13\ N/mm \times 100\ mm$

$= 1{,}300\ N \rightarrow$

$T_2 = \frac{1300N}{2{,}89} \rightarrow$

$T_2 = 450\ N \rightarrow$

$Power\ transmitted = \frac{(T_1 - T_2) \times \pi DN}{60}$

$= \frac{(1300 \times 450) \times \pi \times 0{,}45 \times 500}{60}$

$= 10{,}01\ kW \rightarrow$

Worked Example 9.9

Two pulleys, one 450 mm in diameter and the other 200 mm in diameter, are on parallel shafts 1,95 m apart, and are connected by a crossed belt.

Find the length of belt required and the angle of contact between the belt and each pulley.

What power can be transmitted by the belt when the larger pulley rotates at 200 r/min, if the maximum permissible tension in the belt is 1kN and the coefficient of friction between belt and pulley is 0,25?

Solution:

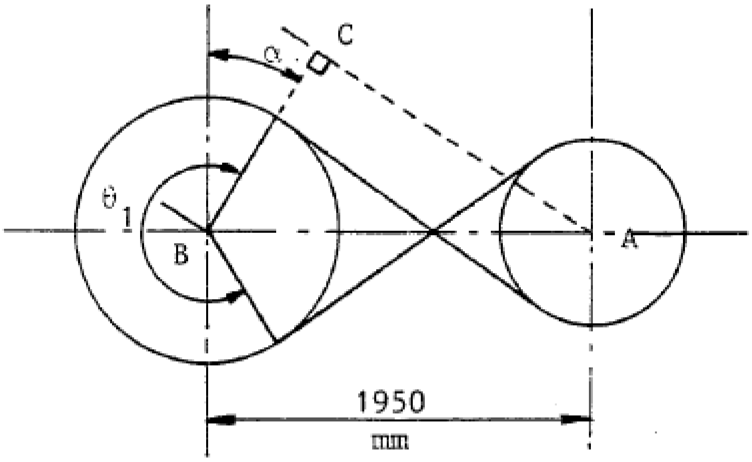


Figure 9.21

$$D = 2R$$

$$\therefore R = \frac{D}{2} = \frac{450\ mm}{2} = 225\ mm$$

$$\therefore r = \frac{d}{2} = \frac{200\ mm}{2} = 100\ mm$$

$$\sin\alpha = \frac{R + r}{c} = \frac{225 + 100}{1950} = 0{,}1667$$

$$\alpha = 9{,}59°$$

Angle of lap $\theta_1 = \theta_2 = 180° + 2\alpha = 180° + 2(9{,}59°) = 180° + 19{,}18° = 199{,}18°$

$$180° = \pi\ radians$$

$$\therefore 199{,}18° = \frac{\pi \times 199{,}18}{180} = 3{,}476\ radians$$

$$Length\ of\ belt = \left(\frac{D}{2} \times \theta\right) + \left(\frac{d}{2} \times \theta\right) + 2 \times C \times \sin\alpha$$

$$= \left(\frac{450}{2} \times 3{,}476\right) + \left(\frac{200}{2} \times 3{,}476\right) + 2 \times 1950 \times \cos 9{,}59°$$

$$= 782{,}1 + 347{,}6 + 3845{,}5$$

$$= 4975{,}2\ mm$$

$$= 4{,}9752\ m$$

$\frac{T_1}{T_2} = e^{u\theta}$

$\frac{T_1}{T_2} = 2{,}718^{0{,}25\times3{,}476}$

$= 2{,}718^{0{,}869}$

$\frac{T_1}{T_2} = 2{,}385 \;\rightarrow$

$T_1 = 1000\ N$ given

$T_2 = \frac{T_1}{2{,}385}$

$= \frac{1000\ N}{2{,}385}$

$= 419{,}3\ N$

$Power = \frac{T_1 - T_2 \times \pi DN}{60}$

$(T_1 - T_2) = \frac{(1000 - 419{,}3)\times\pi\times0{,}45\times200}{60}$

$= 2736{,}5\ watt$

$= 2{,}7365\ kW$

Activity 9.1

1. A pulley, 900 mm in diameter, is mounted on a line shaft running at 240 r/min. A machine which is to run at 600 r/min, and requires 22 kW is to be driven by a flat belt from this pulley. Determine:
   i. the diameter of the pulley that must be fitted to the machine
   ii. the width of belting that must be used, assuming that the angle of contact is 161° on the machine pulley, the coefficient of friction is 0,3 and the allowable tension of belting 15 newtons per millimetre width
2. Determine the maximum power that can be transmitted from a 380mm diameter pulley running at 720 r/min by a flat belt if the angle of contact of the belt on the pulley is 170°, the coefficient of friction is 0,3, and the tension of the belt is not to exceed 178 N.
3. An open flat belt drive is required to transmit 25 kW at 200 r/min. The diameters of the driving pulley and the driven pulley are 200mm and 150mm, respectively. The angle of contact of the driven pulley is 176° and the centre distance between the two parallel shafts is 1,5 metre. Coefficient of friction = 0,37.
   Calculate:
   i. the belt speed in metres per second
   ii. the ratio of tensions
   iii. the tension in the slack side
   iv. the tension in the tight side
   v. the angle of contact of the two pulleys if a crossed belt drive is used
   vi. the length of belt for an open drive
   vii. the lengths of belt for a crossed drive
   Sketch the following two views of the larger pulley:
   a) a full-sectional front view
   b) an outside side view

Self-Check		
I am able to:	**Yes**	**No**
• Describe belt drives		
• Describe belt materials and the choice of belting material		
• Describe belt joints		
• Describe pulleys		
• Describe the shapes of pulley rims		
• Perform belt calculation		
o Angle of lap		
o Ration of tensions		
• Describe the size of flat belt for transmitting a given power		
If you have answered ‘no’ to any of the outcomes listed above, then speak to your facilitator for guidance and further development.		

Module 10

Welded Joints

Learning Outcomes
On the completion of this module the student must be able to:

- Describe the types of welding
- Describe welding properties of materials
- Describe the types of welded joints
- Describe recommended proportions for weld joints
- Describe the design of welds
- Describe the strength of butt and fillet welded joints in simple cases of:
    - Bending
    - Tensions
    - Compression
    - Torsion
- Describe the eccentric load parallel with the weld group
- Describe the eccentric loading perpendicular to the plane of the weld group
- Describe the eccentric loading in the same plane as the weld group (twisting)

10.1 Introduction

The quality of the electrode determines the strength of a welding joint. When comparing the strength of a mild steel plate with an electrode, the breaking strengths of different electrodes can be impressive.

Definition: Welding
The art of joining metals by pressure, after heating to a plastic or semi-molten state, or of joining the metals by fusion alone.

10.2 Types of welding

Welding may be divided into five general classes, each of which has its particular field of use in manufacturing.

These classes are:
- forge welding

- thermit welding
- gas welding
- electric-resistance welding
- electric-arc welding

10.2.1 Forge welding
Forge welding consists of heating the parts in a forge or furnace until plastic and then hammering them together, a suitable flux being used to carry away the scale or oxide formed by contact of the heated metal with the air.

This process is practically obsolete, except in a modified form used in the manufacture of small pipes and tubes.

10.2.2 Thermit welding
In thermit welding, a suitable mould is built around the parts to be welded, and thermit, a mixture of finely powdered aluminium and iron oxide, is confined in a crucible above the mould.

When the thermit is ignited, the iron is released by the heat of combustion, and drops into the mould through an opening in the bottom of the crucible.

The weld metal is essentially cast steel fused into the parts welded. This process is used principally in the repair of heavy machine parts and in the building up of defective castings.

10.2.3 Gas welding
Gas welding utilises the heat produced by the combustion of either acetylene or hydrogen in a stream of pure oxygen. The flame is directed against the edges of the part to be welded, bringing the parent metal to the melting point; and extra metal required to fill the space between the parts is supplied by a welding rod of suitable material, melted in the gas flame.

Did you know?
Acetylene welding is the most common form of gas welding and is widely used for repair work, for welding thin plates, and for welding gas, steam and hydraulic pipe lines.

A torch that supplies streams of pure oxygen around the heating flame makes an excellent device for cutting heavy slabs up to 300 mm thick.

Flame cutting of irregular shapes by hand or by machine is becoming an important fabricating process.

10.2.4 Electric resistance welding
In the electric-resistance process, the parts are brought into contact, and a heavy current at low voltage is passed through the junction. Because of the high electrical resistance at the junction, the metal is rapidly brought up to the fusion

temperature. Pressure, applied mechanically, forces the parts together and forms the weld.

The resistance process is divided into classes such as spot welding, butt welding, flash welding, and seam welding. Butt and flash welding are economical where mass production justifies the special equipment required for each individual job.

Did you know?
This process is widely used in the assembly of the bodies of automobiles, refrigerators, and other pressed-steel parts.

The ordinary spot welder requires little special equipment, and is used extensively in the manufacture of such parts as gear housings, switch housings, lamp reflectors, and other similar parts built in small lots.

The field of resistance welding has been extended by the development of electronic controls such as the thyratron, so that many dissimilar metals can now be joined by welding, for instance, copper to aluminium, bronze to steel, and copper to steel.

10.2.5 Electric-arc welding

In the electric-arc process, the heat is supplied by a continuous arc drawn between two electrodes. In the original process, now practically obsolete, the arc is drawn between carbon electrodes, the heat being reflected onto the parts to be welded.

In the carbon-arc process, the work itself forms one electrode, a carbon rod being used for the second electrode. With the carbon electrode, it is difficult to make vertical and overhead welds, and excess carbon is Likely to be present in the weld metal.

Did you know?
The metallic-arc process is the most common electric welding process. The work forms one electrode, and the welding rod forms the second electrode.

Overhead welding is possible, since molten metal from the tip of the welding rod is carried by the arc to the weld. The electric-arc process is readily adapted to welding machines with automatic regulation of arc length, speed and other variables.

Semi-automatic machines are used when the paths of the seams are irregular and not easily followed by fully automatic machines.

Molten steel has an affinity for oxygen and nitrogen, which make up the air; hence the weld metal is likely to contain gas pockets and nitrides, which weaken the weld and reduce the corrosion resistance.

To prevent this, a shielded arc may be used. The welding rod is heavily coated with a material which, in the heat of the arc, gives off large quantities of inactive gas, thereby protecting the weld metal from contact with the air.

Think about it!
Welds made in this manner are about 20 per cent stronger than those made with bare welding rods.

The atomic-hydrogen are-welding process is a recent development used to prevent oxidation of the metal. A reducing atmosphere is created by forcing a jet of hydrogen through the arc drawn between two tungsten electrodes.

The heat of the arc separates the hydrogen into atoms, which later recombine, giving back the heat of disassociation.

The atomic-hydrogen atmosphere protects the weld metal.

10.3 Welding properties of materials

Most metals can be welded by some process, but some are more readily welded than others, and the properties of the weld depend upon many factors.

At the temperatures reached, structural changes in the metal that change the physical properties and the corrosion resistance may take place. Some elements in the base metal, such as zinc, may vaporise during the welding and cause porous weld metal.

Gaseous oxides may cause blow holes, soluble oxides in the molten metal reduce the strength and toughness of the weld, and insoluble oxides cause slag inclusions in the weld.

Metals of high thermal expansion and low thermal conductivity are subject to high cooling stresses in the weld. The elements present as impurities or as alloys, the kind of metal used in the welding rod, the material used for shielding the rod, the fluxing material, and the welding procedure all affect the weld characteristics.

All the plain carbon steels except spring steel and tool steel (with carbon contents from 0,75 to 1,50 per cent), can be satisfactorily welded, but the Lower-carbon steels are the most readily welded. Nickel, chromium and vanadium improve the welding qualities slightly.

Since the weld metal is essentially cast steel, it follows that cast-steel parts are easily welded by either the gas or electric processes.

Think about it!
The high strength and other desirable properties of alloy steels are chiefly due to their action on the carbon and to their response to heat treatment.

The cast weld material is normally weaker than the heat-treated alloys, so that the weld is weaker than the base metal unless special precautions are taken.

Special-composition welding rods, usually of the shielded type, producing weld material of nearly the same analysis as the base metal, should be used, and the parts should be heat-treated after welding.

Note:
Cast iron is difficult to weld by any process, and even under the most favourable conditions, the results are more or less unreliable.

Satisfactory welds can be made only by care in preheating, preparation, and welding procedure. What is more important welds should be made by experienced operators only.

Gas welding is, in general, superior to arc welding in strength, reliability and machinability. Gas welding, requires careful pre-heating to prevent warpage and shrinkage stresses, whereas the more localised heating with the arc reduces the seriousness of these effects.

Practically all the common metals, including stainless steel, copper, bronze, nickel, monel, and nickel silver can now be satisfactorily welded. Metals containing high percentages of lead, tin, zinc, aluminium, magnesium and molybdenum are somewhat difficult to weld.

This is due to the vaporizing of some of the ingredients, and to the fact that oxide forms and acts as an insulator interfering with the flow of current and heat.

10.4 Types of welded joints

10.4.1 Butt welds

A butt weld is a weld in which the fused metal lies substantially within the extension of the planes of the surfaces of the parts joined or within the extension of the planes of the smaller of two parts of differing size.

The basic forms of butt weld are shown in **Figure 10.1** but in practice many variations of these examples are permissible.

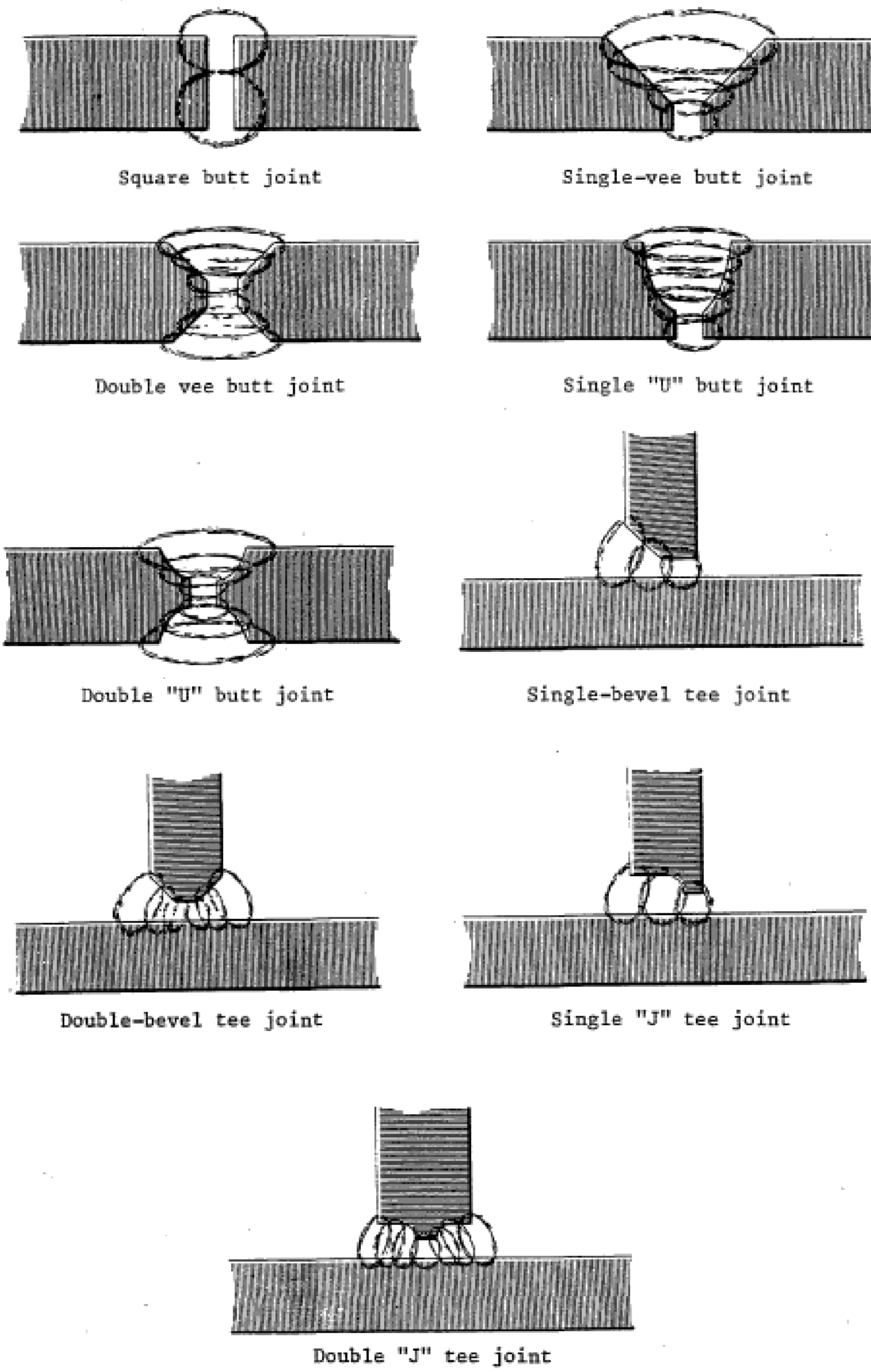


Figure 10.1

10.4.1.1 Terms applied to butt welds

Although a single vee joint is shown in **Figure 10.2**, the terms apply to other forms of preparation.

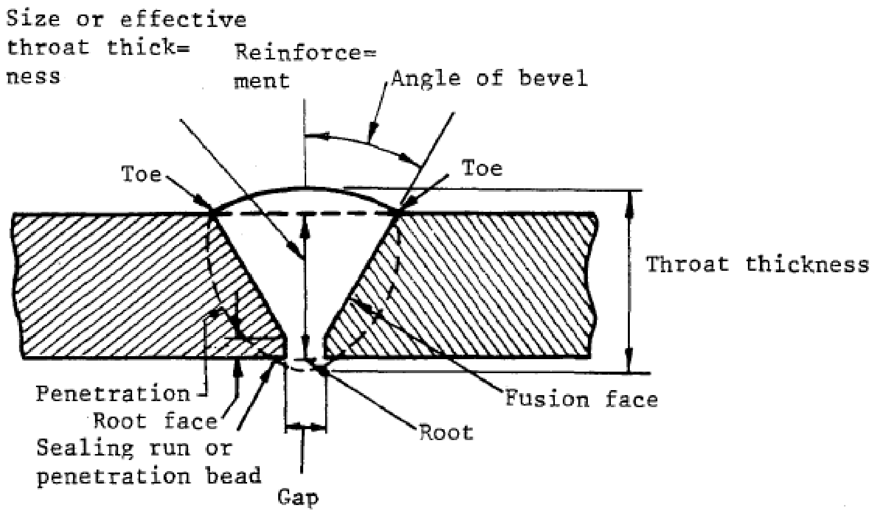


Figure 10.2

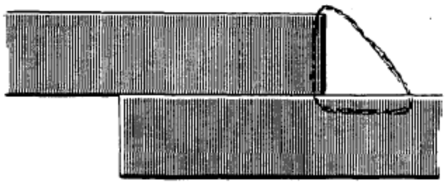
The throat thickness is the minimum thickness of weld metal measured:

- for a vee, U, J or bevel-butt weld, along a line passing through the root
- for a close square-butt weld, in the plane of the abutting faces; or
- for an open square-butt weld at the centre of the original gap in a plane parallel to the fusion faces.

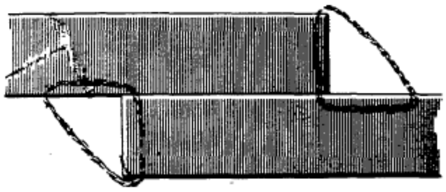
The effective throat thickness is the nominal throat thickness used for the purpose of design. (In a complete penetration butt weld, this is the thickness of the thinner part joined.)

10.4.2 Fillet welds

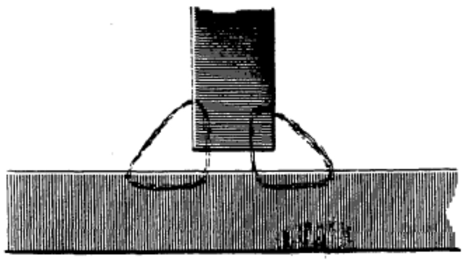
A fillet weld is any fusion weld approximately triangular in cross section, not a butt weld as previously defined, but including the weld in a corner joint.



Single-fillet lap joint



Double-fillet lap joint



Square tee joint

Figure 10.3

10.4.2.1 Terms applied to fillet welds

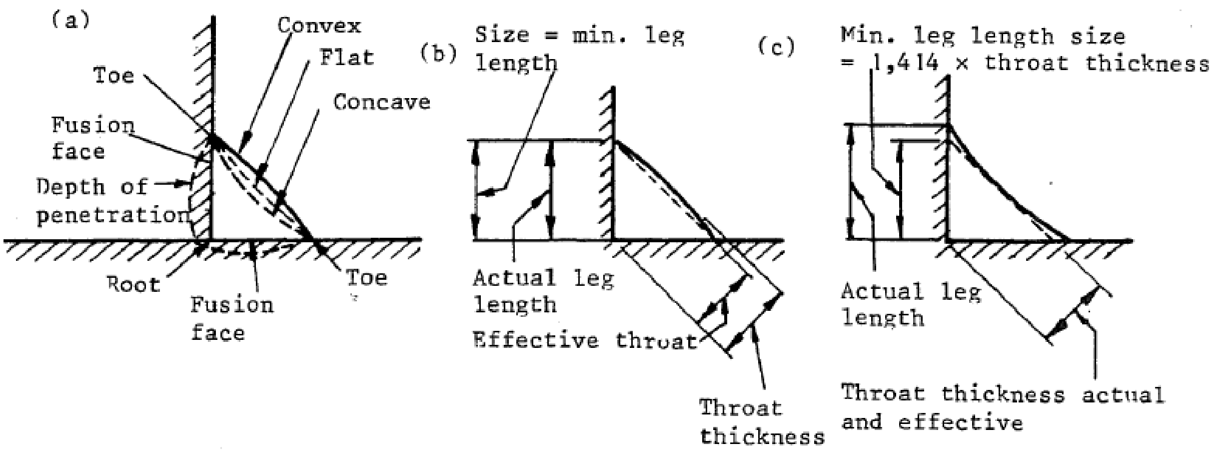


Figure 10.4

The throat thickness is the minimum thickness of weld metal measured along a line passing through the root.

Effective throat thickness is the throat thickness used for purposes of design. (This may be the actual throat thickness or a fraction of it, depending on the shape of the fillet.)

10.5 Recommended proportions for weld joints

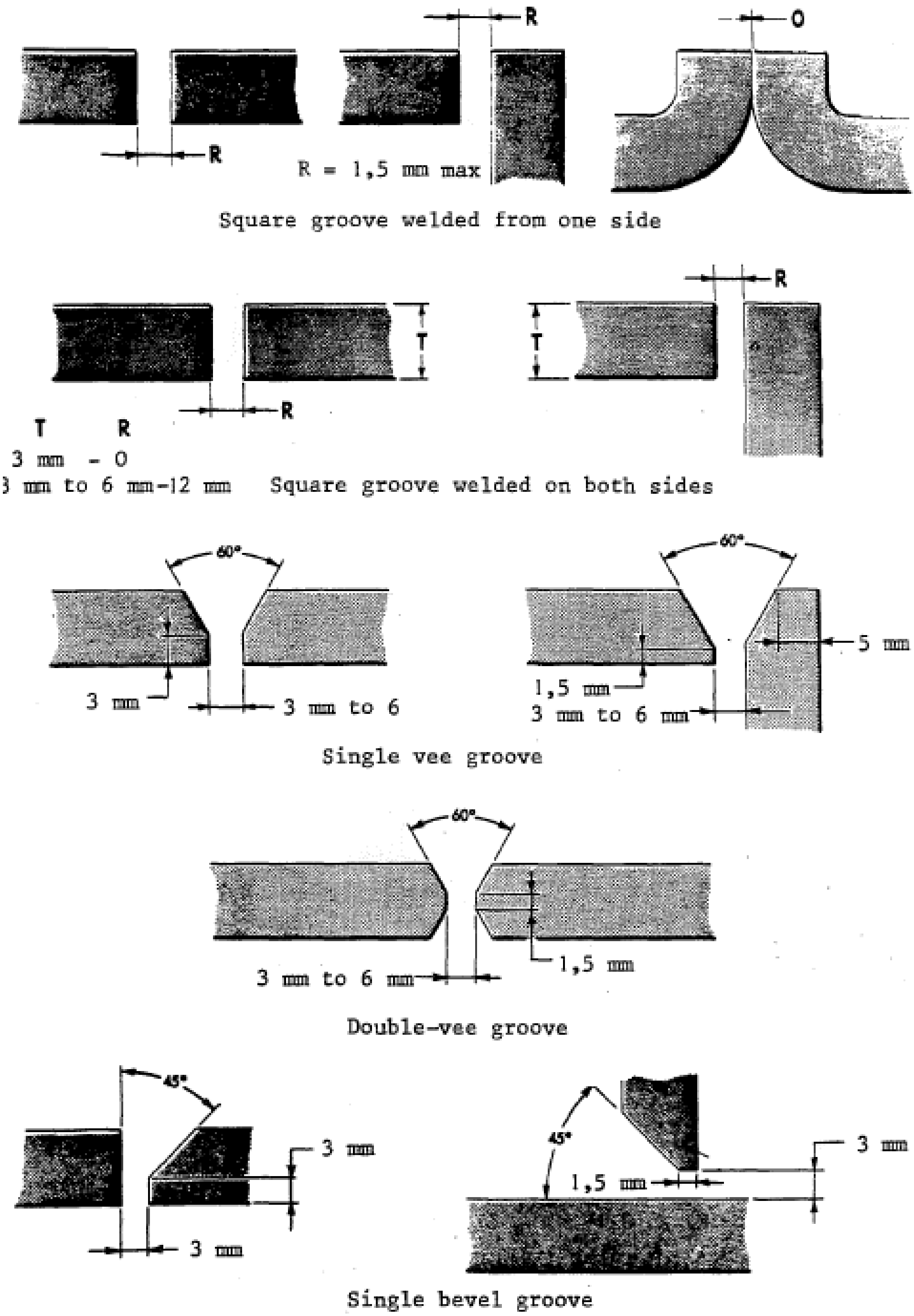


Figure 10.5a

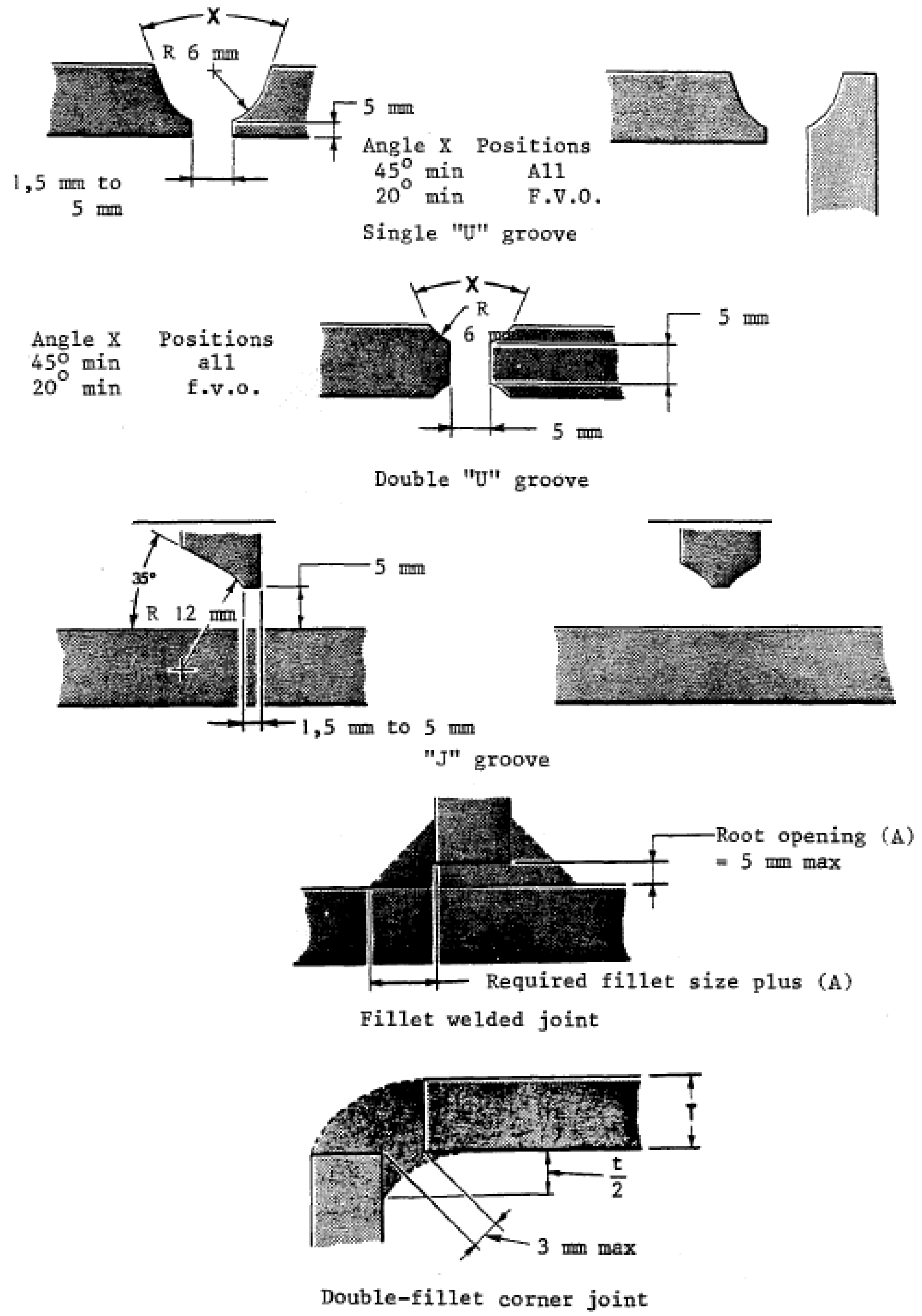


Figure 10.5b

10.6 Design of welds

One of the most important factors in the design of a weldment is determining the weld size. Weld size affects cost as well as strength and distort ion.

> **Note:**
> In a butt weld where full strength of the joint is required, the weld must have the same thickness as the connecting plates.

As a rule, the stress and weld size are not calculated, because a butt weld has strength equal or greater than the base metal.

The strength of a fillet weld is based on the effective throat thickness which is the shortest distance from the root to the face of the weld.

Fillet welds are classified according to the direction of the load.

- parallel load
- transverse load

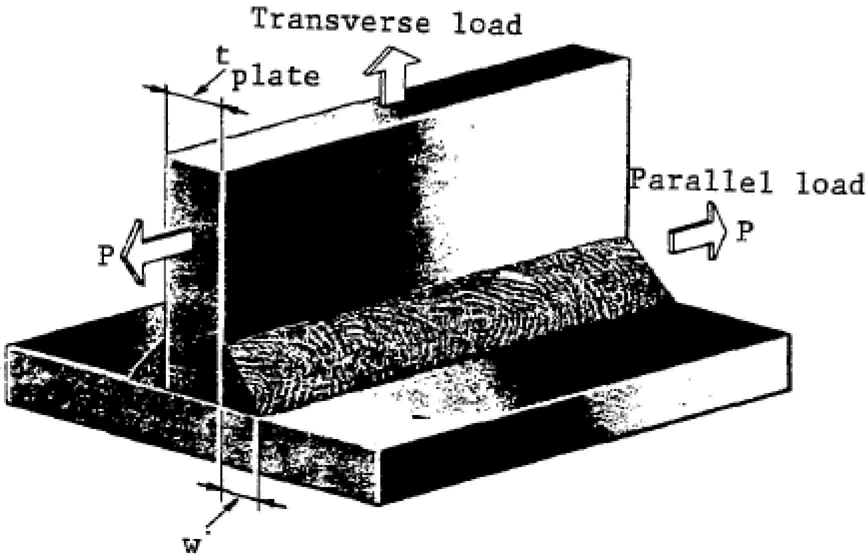


Figure 10.6

10.6.1 Parallel loading

The plane of maximum shear stress in the conventional 45° fillet weld is 45° throat as shown in **Figure 10.7**.

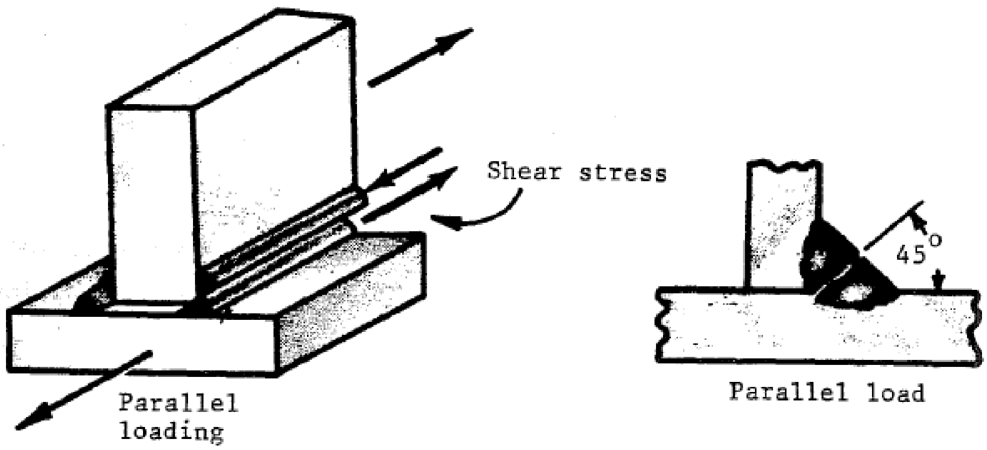


Figure 10.7

The size of a fillet weld is specified by the leg length of the largest inscribed isosceles right triangle or the Lengths of the largest inscribed right triangle.

The leg length of a fillet weld with equal legs is given by w and the leg lengths of a fillet weld with unequal legs are given by a and b.

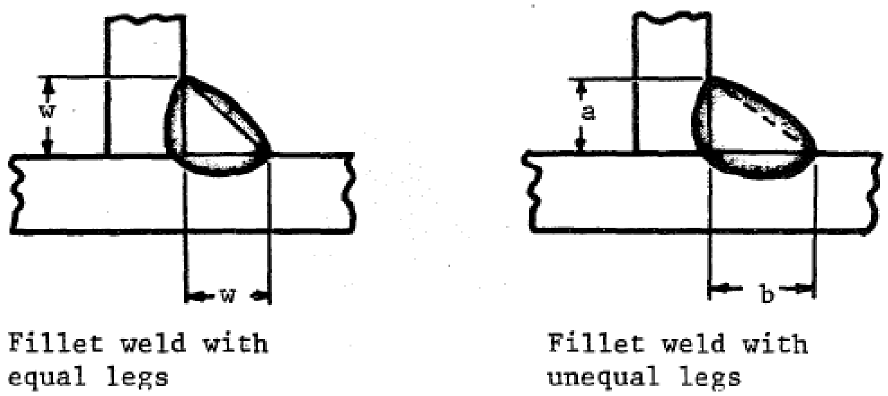


Figure 10.8

The strength of a fillet weld subjected to parallel loading is based on the effective throat thickness which is the shortest distance from the root to the face of the weld. For an equal leg (45°) fillet weld the throat is 0,707 (cos of 45°) times the leg size of the weld.

10.6.1.1 Throat size of fillet weld with equal legs

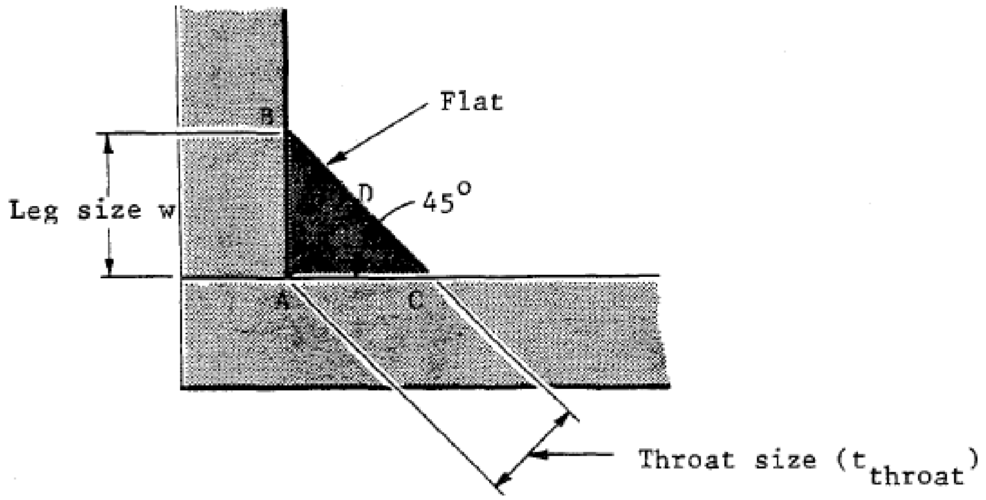


Figure 10.9

$$
\begin{aligned}
Throat\ size &= Leg\ size \times \cos 45° \\
AD &= AB \times \cos 45° \\
t_{throat} &= w \times 0{,}707 \\
\text{or } Leg\ size &= \frac{Throat\ size}{\cos 45°} \\
AB &= \frac{AD}{\cos 45°} \\
w &= \frac{t_{throat}}{0{,}707} \\
w &= 1{,}414 \times t_{throat} \text{ or } t_{throat} = 00{,}707\, w
\end{aligned}
$$

10.6.1.2 Throat size of fillet weld with unequal legs

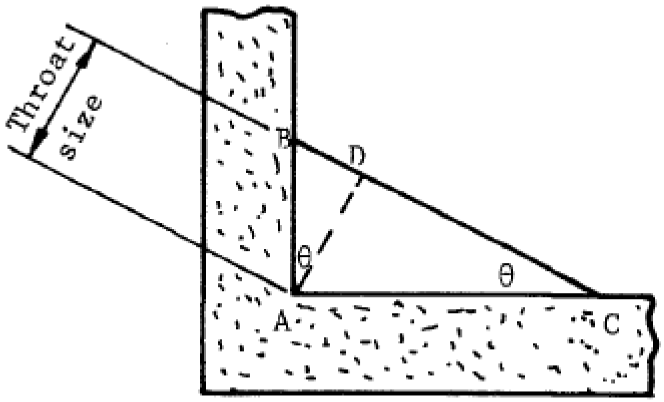


Figure 10.10

$$\tan\theta = \frac{AB}{AC}$$
$$Throat\ size\ AD = \cos\theta \times AB$$

10.6.2 Transverse loading

The plane of maximum shear stress in the conventional 45° fillet is the $67\frac{1}{2}$° throat as shown in **Figure 10. 11**.

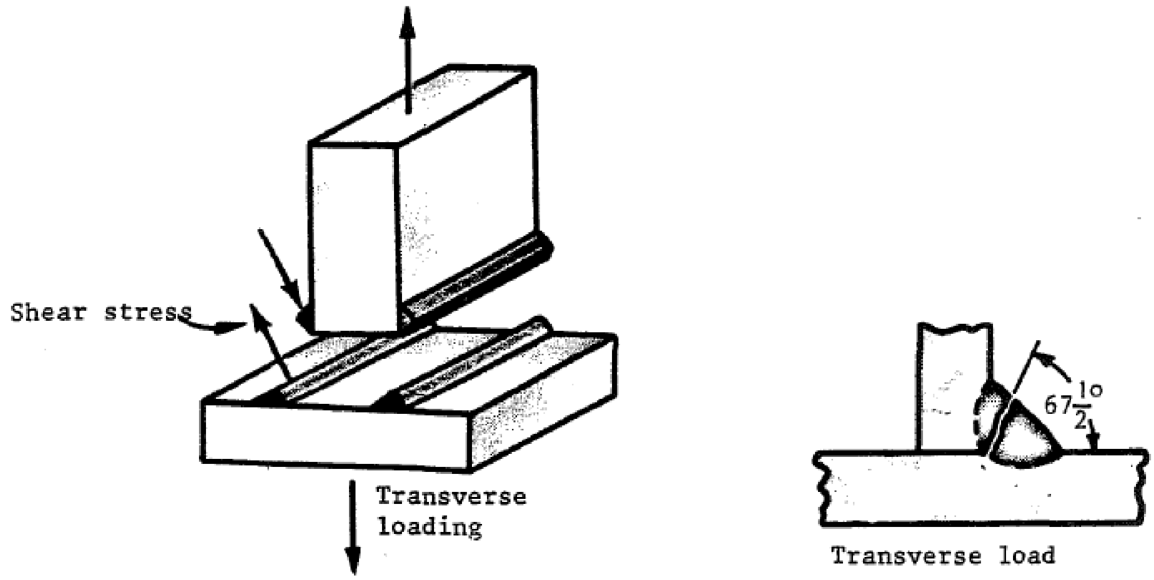


Figure 10.11

The strength of a fillet weld subjected to transverse Loading is based on the effective throat thickness, which is the distance from the root to the face of the weld at an angle of $67\frac{1}{2}$°.

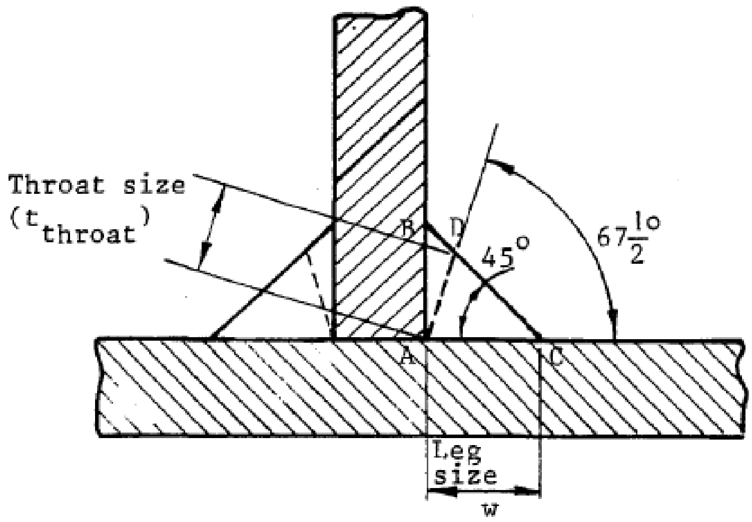


Figure 10.12

Throat size (Distance AD)

$$\frac{\sin 45°}{AD} = \frac{\sin 67\frac{1}{2}°}{leg\ size\ (AC)}$$

$$AD = \frac{\sin 45° \times AC}{\sin 67\frac{1}{2}°}$$

$$t_{throat} = 0{,}765\ w$$

or

$$Leg\ size = \frac{t_{throat}}{0{,}765}$$

$$w = 1{,}31\ t_{throat} \text{ or } c\ w$$

From the above it can be seen that fillet welds placed perpendicular to the direction of the applied load (transverse loading) are somewhat stronger than fillet welds in the direction of loading (parallel loading).

It is common practice to treat both of these types of fillet welds as being of equal strength, using the smaller of the two, in other words: $t_{throat} = 707\ w$.

10.7 Strength of welded joints

To properly design welded products, the strength of the welded joints must be analysed.

10.7.1 Butt welds

The stress on a butt joint is calculated by the formula:

$$Tensile\ stress = \frac{Tensile\ load}{Cross - sectional\ area}$$

10.7.1.1 Square butt joint

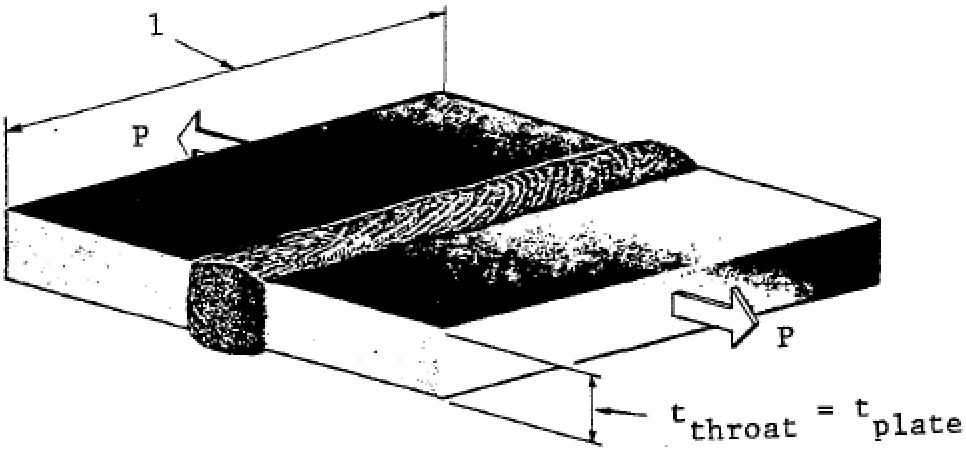


Figure 10.13 Determining strength of butt joints

$$\sigma_t = \frac{P}{t_{throat}} \times l$$

where σ_t = Tensile stress
P = Tensile load
l = Length of weld – width of plate
t_{throat} = Effective throat thickness equals thickness of plate

Allowable tensile stress of 138 MPa is recommended for butt joints.

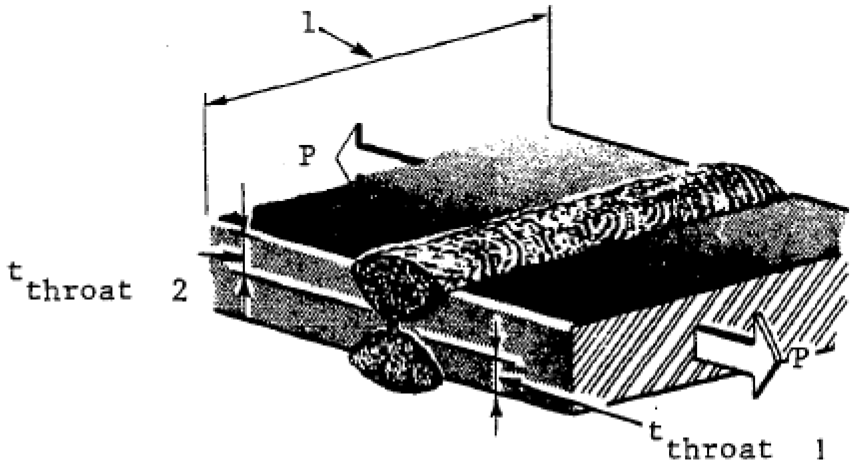


Figure 10.14

$$\sigma_t = \frac{P}{t_{throat}} \times l$$

10.7.2 Fillet welds

10.7.2.1 Lap joint with side and end weld

In considering a typical fillet weld such as shown in **Figure 10.15**, it is noted that the opposing forces tend to make the members slip and cause shearing stresses in the welds.

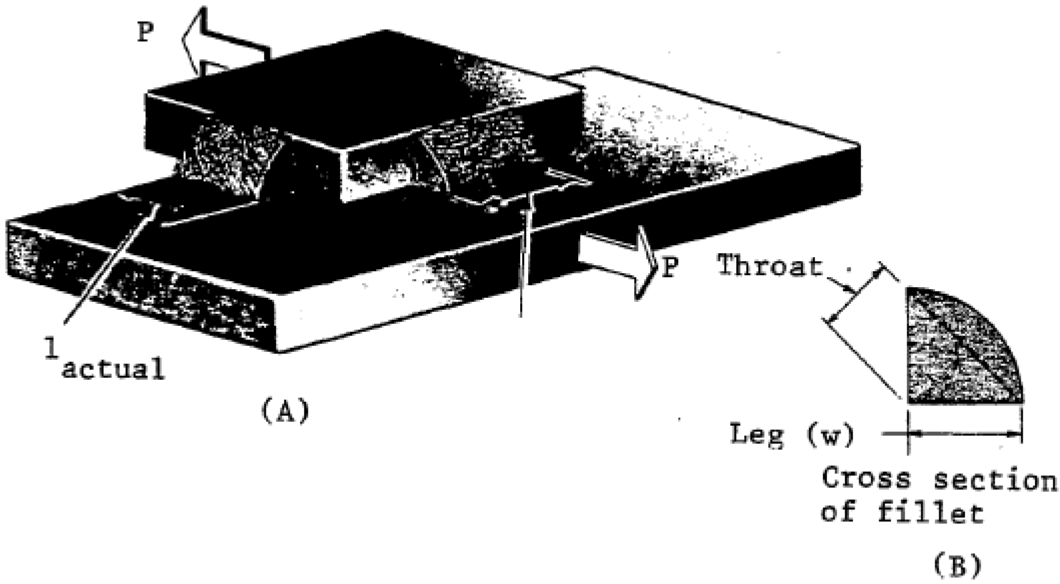


Figure 10.15

The stress is highest on the plane where the thickness of the weld is the smallest which occurs at the throat. Hence the shear stress on the weld is determined by the formula:

$$Shear\ stress = \frac{Shear\ load}{Shear\ area}$$

$$\tau = \frac{P}{t_{throat} \times l_{effective}}$$

$$= \frac{P}{0{,}707\ w \times l_{effective}}$$

The effective Length of a fillet weld must be taken as that length only which continues to have the specified weld size, and throat thickness corresponding.

It is common practice in the case of fillet welds with open ends, ie not specially end-treated, to obtain the effective length, for calculation purposes, by subtracting twice the weld size from the actual length, ie $1 \times Leg\ size\ for\ starting\ of\ weld + 1 \times Leg\ size\ for\ stopping\ of\ weld.$

$$Effective\ Length\ of\ weld = Actual\ ength\ of\ weld - 2 \times Leg\ size$$

$$l_{effective} = l_{actual} - 2w$$

$$l_{effective} = l_{actual} - 2\,w$$

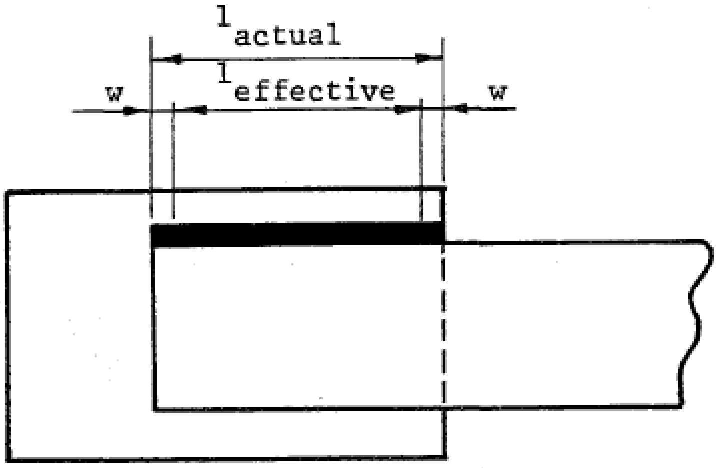


Figure 10.16a

$$l_{effective} = l_{actual} - 2w$$
$$Throat\ area\ of\ weld = 0{,}707 \times w \times l_{effective}$$

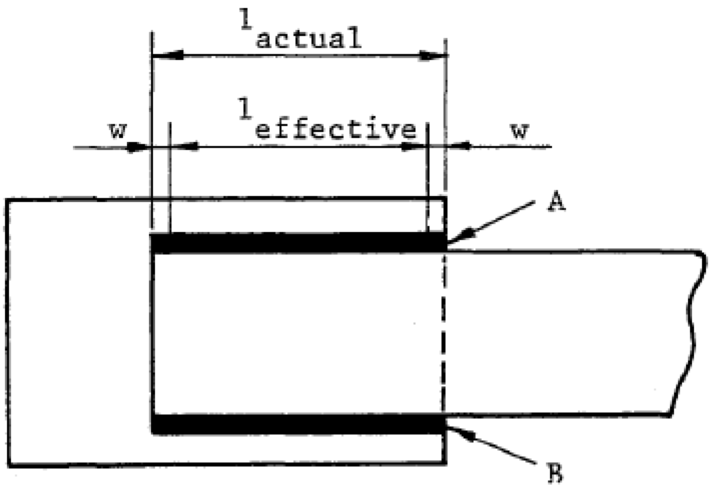


Figure 10.16b

$$Throat\ area\ of\ weld = Throat\ area\ of\ weld\ A - Throat\ area\ of\ weld\ B$$
$$= 2(0{,}702 \times w \times l_{effective})$$

Note:

$$Throat\ area\ of\ weld\ A\ =\ Throat\ area\ of\ weld\ B$$

Fillet welds terminating at the ends or sides of parts or members must, wherever practicable, be returned continuously around the corners for a distance not less than twice the size of the weld.

This provision applies, in particular, to side and end-fillet welds in tension which connect brackets, beam seatings and similar parts

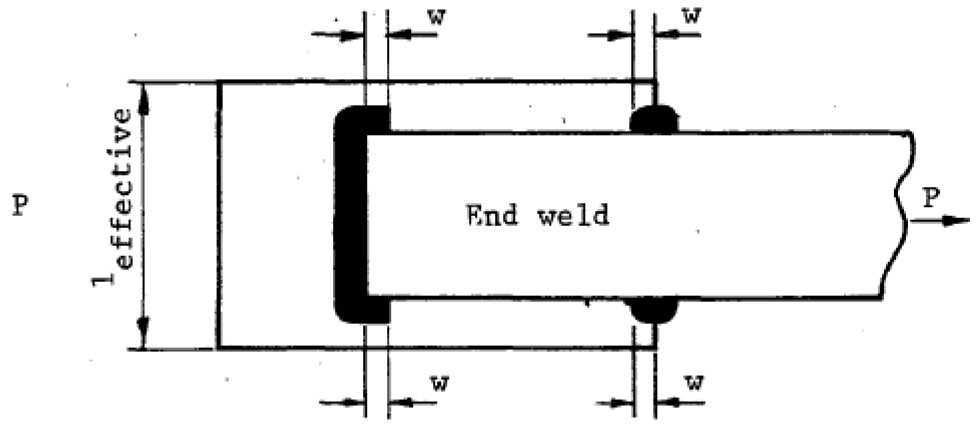


Figure 10.17a

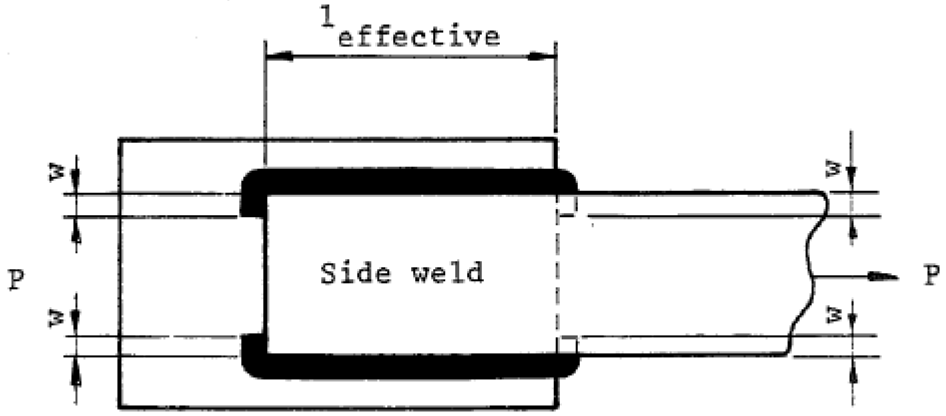


Figure 10.17b

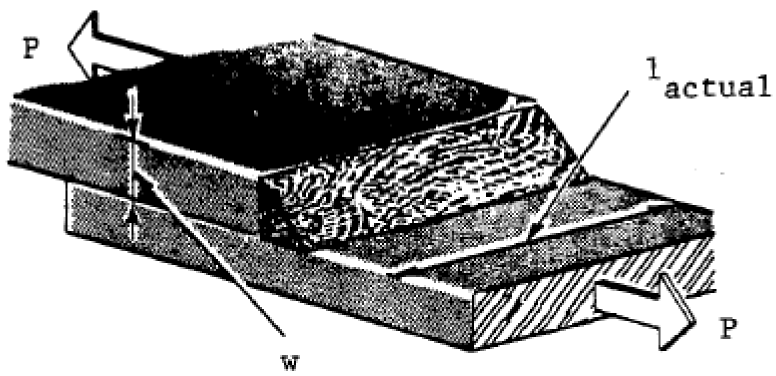


Figure 10.18

$$Normal\ stress = \frac{Normal\ load}{Troat\ area\ of\ weld}$$

$$\sigma = \frac{P}{0{,}707\ w \times l_{effective}}$$

$$\therefore \sigma = \frac{1{,}414\ P}{w \times l_{effective}}$$

10.7.3 Lap joint with double weld

10.7.3.1 When size of weld equals size of weld B

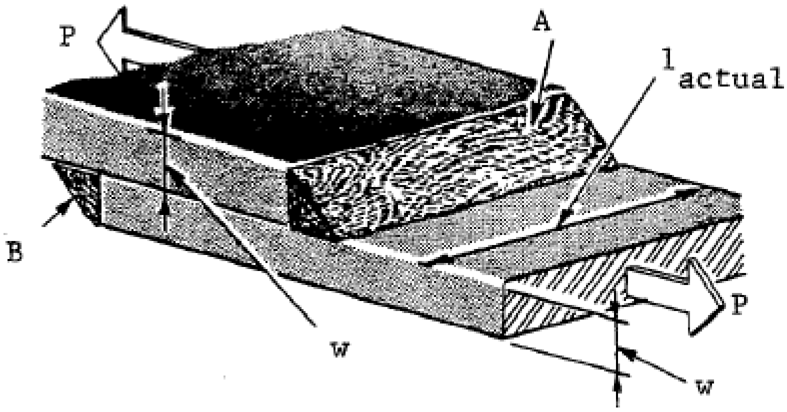


Figure 10.19

$$Normal\ stress = \frac{Normal\ load}{Troat\ area\ of\ weld\ A + Troat\ area\ of\ weld\ B}$$

$$\sigma = \frac{P}{(0{,}707\ w \times l_{effective}) + (0{,}707\ w \times l_{effective})}$$

$$\sigma = \frac{0{,}707\ P}{w \times l_{effective}}$$

10.7.3.2 When size of weld equals size of weld B

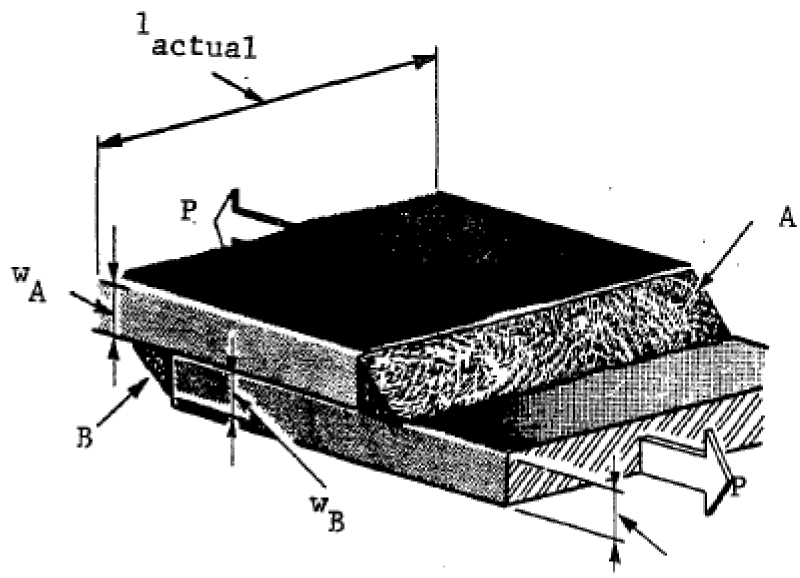


Figure 10.20

$$Stress\ in\ weld\ A = \frac{0{,}707\ P}{w_A \times l_{effective}}$$

$$Stress\ in\ weld\ B = \frac{0{,}707\ P}{w_B \times l_{effective}}$$

10.7.4 Perpendicular joint with top and bottom fillet welds

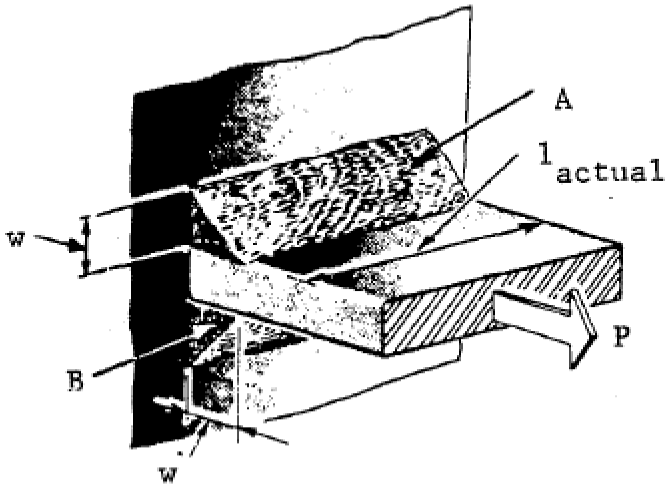


Figure 10.21

$$P = \sigma(t_{throat\ A} \times l_{A\ effective} + t_{throat\ B} \times l_{B\ effective})$$
$$= \sigma(0{,}707\ w_A \times l_{A\ effective} + 0{,}707\ w_B \times l_{B\ effective})$$
$$= 2 \times \sigma \times 0{,}707\ \times w_A \times l_{A\ effective}$$

Note:
$w_A = W_B$ and $l_{A\ effective} = \times\ l_{B\ effective}$

$$= \frac{P}{1{,}414\ w_A \times l_{A\ effective}}$$
$$\sigma = \frac{0{,}707\ P}{w_A \times l_{A\ effective}}$$

10.7.5 Perpendicular joint with circumferential fillet

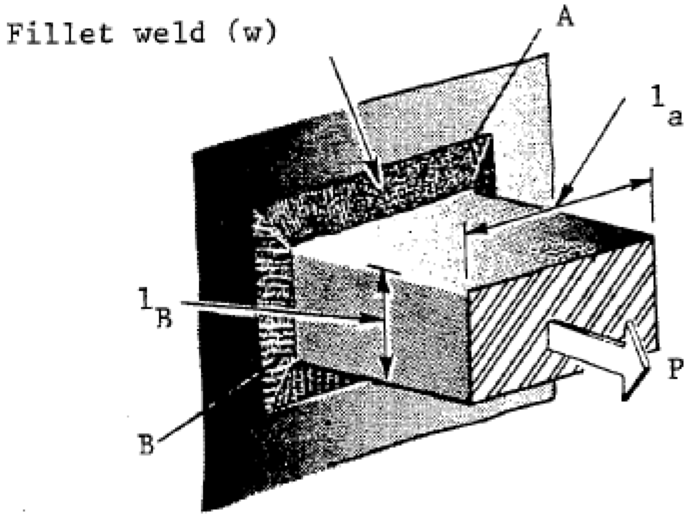


Figure 10.22

$$
\begin{aligned}
Normal\ force &= Normal\ stress \times (2 \times Throat\ area\ weld\ A + 2 \times Throat\ area\ weld\ B) \\
P &= \sigma \times (2\ t_{throat\ A} \times l_A\ + 2 \times\ t_{throat\ B} \times l_B) \\
&= \sigma(2 \times 0{,}707\ \times w_A \times l_A\ + 2 \times 0{,}707\ \times w_B \times l_B) \\
&= \sigma \times 2 \times 0{,}707 \times w_A(l_A\ + l_B) \\
&= 1{,}414 \times \sigma \times w_A(l_A\ + l_B) \\
\therefore \sigma &= \frac{P}{1{,}414 \times w_A(l_A\ + l_B)} \\
&\frac{0{,}707\ P}{w_A(l_A\ + l_B)}
\end{aligned}
$$

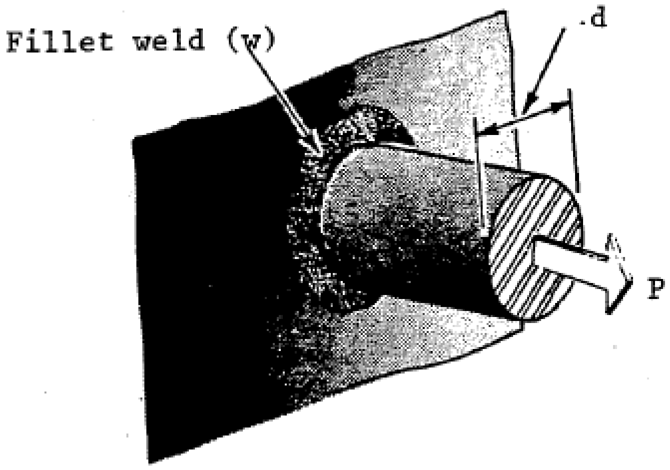


Figure 10.23

$$
\begin{aligned}
Normal\ force &= Normal\ stress \times Throat\ area\ weld \\
P &= \sigma \times\ t_{throat} \times \pi d \\
&= \sigma \times 0{,}707\ w \times \pi \times d \\
&= 2{,}22\ \times \sigma \times w \times d \\
\sigma &= \frac{P}{2{,}22 \times w \times d} \\
\sigma &= \frac{0{,}45\ P}{w \times d}
\end{aligned}
$$

Allowable shear stress of 94 MPa is recommended for fillet welds, and the normal stress to be taken as 138 MPa.

Worked Example 10.1

Calculate the throat thickness to be taken in the calculation of the weld strength in each of the following cases:

(a) 6 mm fillet weld
(b) a fillet weld having one Leg dimension 10 mm and the other 13 mm
(c) a butt weld joining two 12 mm plates
(d) a butt weld joining two plates, one being 12 mm thick and the other 14 mm

Solution:

(a)

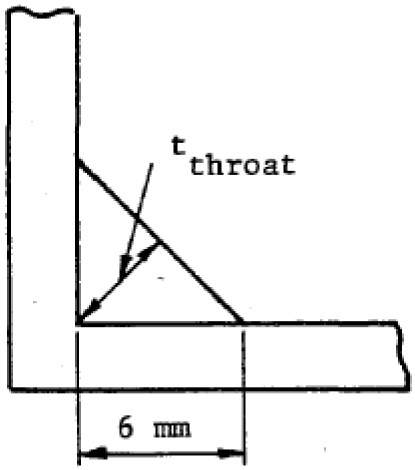


Figure 10.24

$$t_{throat} = 0{,}707 \times w$$
$$= 0{,}707 \times 6\ mm$$
$$= 4{,}242\ mm$$

(b)

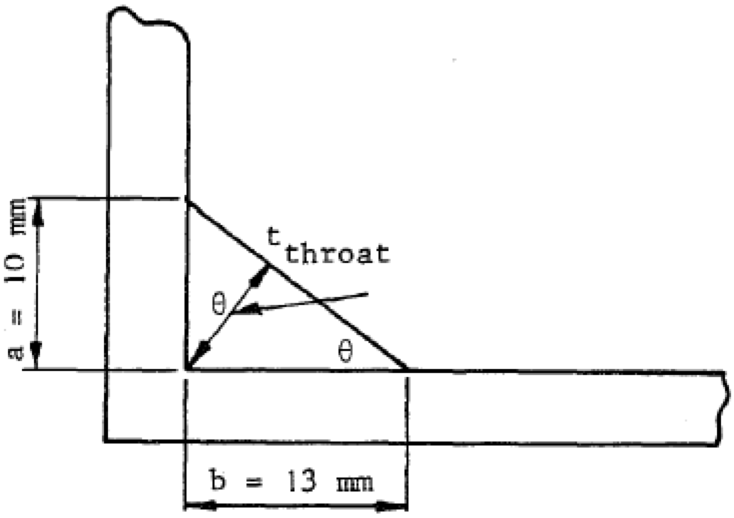


Figure 10.25

$$\tan\theta = \frac{a}{b}$$
$$= \frac{10\ mm}{13\ mm}$$
$$= 0{,}769$$
$$\theta = 37{,}57°$$
$$t_{throat} = \cos\theta \times a$$
$$= \cos 37{,}57° \times 10\ mm$$
$$= 0{,}793 \times 10\ mm$$
$$= 7{,}93\ mm$$

(c)

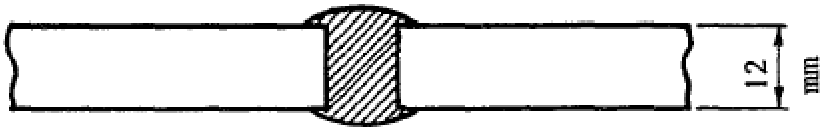


Figure 10.26

$$t_{throat} = plate\ thickness$$
$$= 12\ mm$$

(d)

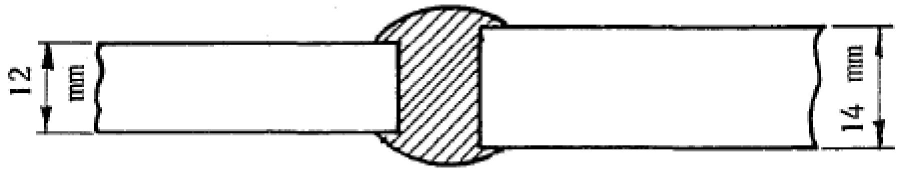


Figure 10.27

$$t_{throat} = thickness\ of\ thinner\ platte$$
$$= 12\ mm$$

Worked Example 10.2

Obtain the throat area to be taken in the case of a 10 mm end fillet of 150 mm overall Length,

(a) if no return side fillets are used
(b) if return fillets 25 mm Long are employed at both ends of the weld.

Solution:

(a)

$$Effective\ length\ of\ weld = Overall\ length\ of\ weld - 2\ x\ Leg\ size$$
$$l = 150\ mm - 2 \times 10\ mm$$
$$= 150\ mm - 20\ mm$$
$$l = 130\ mm$$
$$Throat\ size = 0{,}707 \times w$$
$$= 0{,}707 \times 10\ mm$$
$$= 7{,}07\ mm$$
$$Throat\ area = Throat\ size \times effective\ Length$$
$$= 7{,}07\ mm \times 130\ mm$$
$$= 919{,}1\ mm^2$$

$$Effective\ length\ of\ weld = The\ full\ 150\ mm\ Length\ is\ taken$$
$$Throat\ area = Throat\ size \times effective\ length$$
$$= 7{,}07\ mm \times 150\ mm$$
$$= 1\ 060{,}5\ mm^2$$

Worked Example 10.3

If two 6 mm thick plates are to be welded with a butt joint, determine the required length of weld if the joint is subjected to a tensile load of 44,5 kN. Take allowable tensile stress as 138 MPa.

Solution:

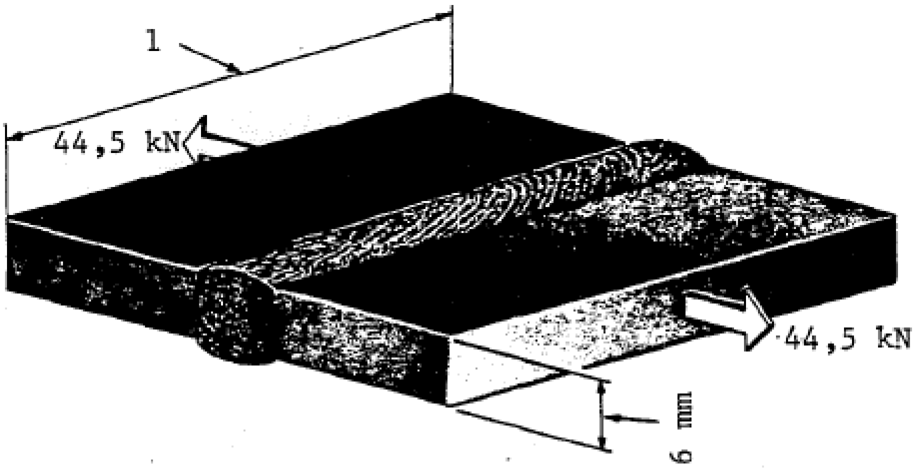


Figure 10.28

$$Tensile\ stress = \frac{Tensile\ load}{Cross\text{−}sectional\ area}$$

$$\sigma_t = \frac{P}{t_{throat} \times l}$$

$$l = \frac{P}{t_{throat} \times \sigma_t}$$

$$= \frac{44{,}5 \times 10^3 N}{0{,}006\ m \times 138 \times 10^6\ N/m^2}$$

$$= 0{,}0537\ m$$

say $l = 54\ mm$

Worked Example 10.4

If two 6 mm thick plates are to be joined by fillet welds, what is the required length of the weld if a load of 45 kN is applied as shown. Allowable shear stress equals 93 MPa.

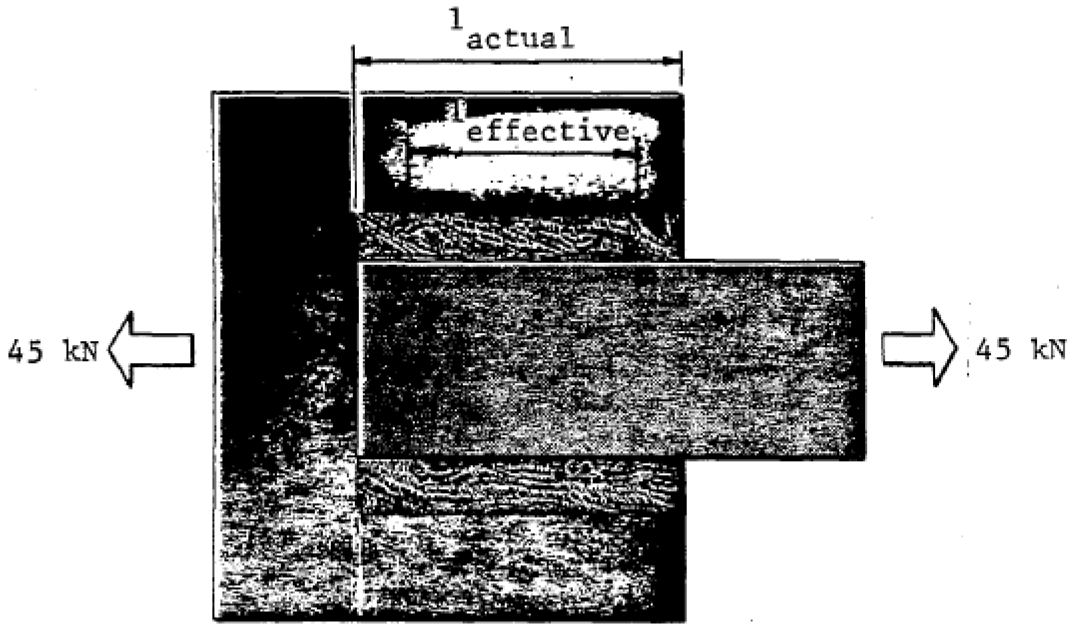


Figure 10.29

Solution:

$$Shear\ stress = \frac{Shear\ load}{Throat\ area\ of\ weld\ A + Throat\ area\ of\ weld\ B}$$

$$\tau = \frac{P}{2(0{,}707\ w \times l_{effective})}$$

$$l_{effective} = \frac{P}{2 \times 0{,}707 \times w \times \tau}$$

$$= \frac{45 \times 10^3 N}{2 \times 0{,}707 \times 0{,}006\ m \times 93 \times 10^6 N/m^2}$$

say $= 57{,}03\ mm$

$$Actual\ length\ of\ weld = Effective\ length + 2w$$

$= 57{,}03\ mm + 2 \times 6\ mm$

$= 57{,}03\ mm + 12\ mm$

$= 69{,}03\ mm$

say $l_{actual} = 70\ mm$

Worked Example 10.5

Determine the factor of safety for the joint shown in **Figure 10.30** below. The material is steel and the allowable shear stress for fillet welds is94 MPa. The leg size of the fillet weld is 5 mm.

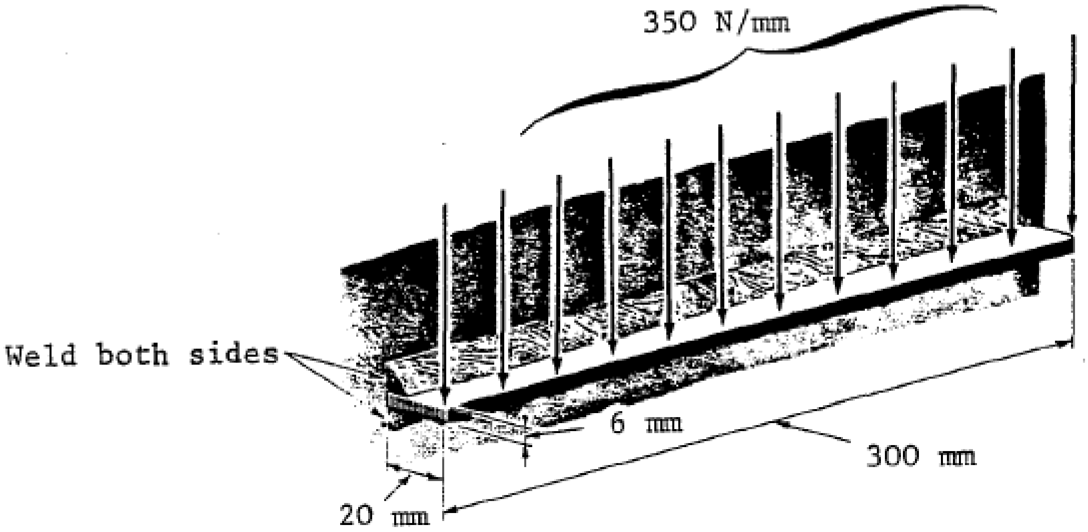


Figure 10.30

Solution:

The beam is so short (20 mml that the bending moment is neglected and the joint is studied for vertical shear only.

$$Total\ shear\ Load\ on\ beam = 350\ N/mm \times 300\ mm$$
$$= 105\ kN$$
$$Effective\ Length\ of\ weld = Actual\ length - 2 \times\ leg\ size$$
$$l_{effective} = 300\ mm - 2 \times 5\ mm$$
$$= 300\ mm - 10\ mm$$
$$= 290\ mm$$
$$Shear\ stress\ in\ weld = \frac{Shear\ load}{Throat\ area\ of\ weld\ A + Throat\ area\ of\ weld\ B}$$
$$\tau = \frac{P}{2(0{,}707 \times w \times l_{effective})}$$
$$= \frac{105 \times 10^3 N}{2(0{,}707 \times 0{,}005\ m \times 0{,}29\ m)}$$
$$= 51{,}21 \times 10^6\ N/m^2$$
$$Factor\ of\ safety = \frac{94 \times 10^6\ N/m^2}{51{,}21 \times 10^6\ N/m^2}$$
$$= 1{,}84$$

Worked Example 10.6

Determine the weld sizes required for the joint shown in **Figure 10.31**. Take the working stress for' the fillet weld as 80 MPa.

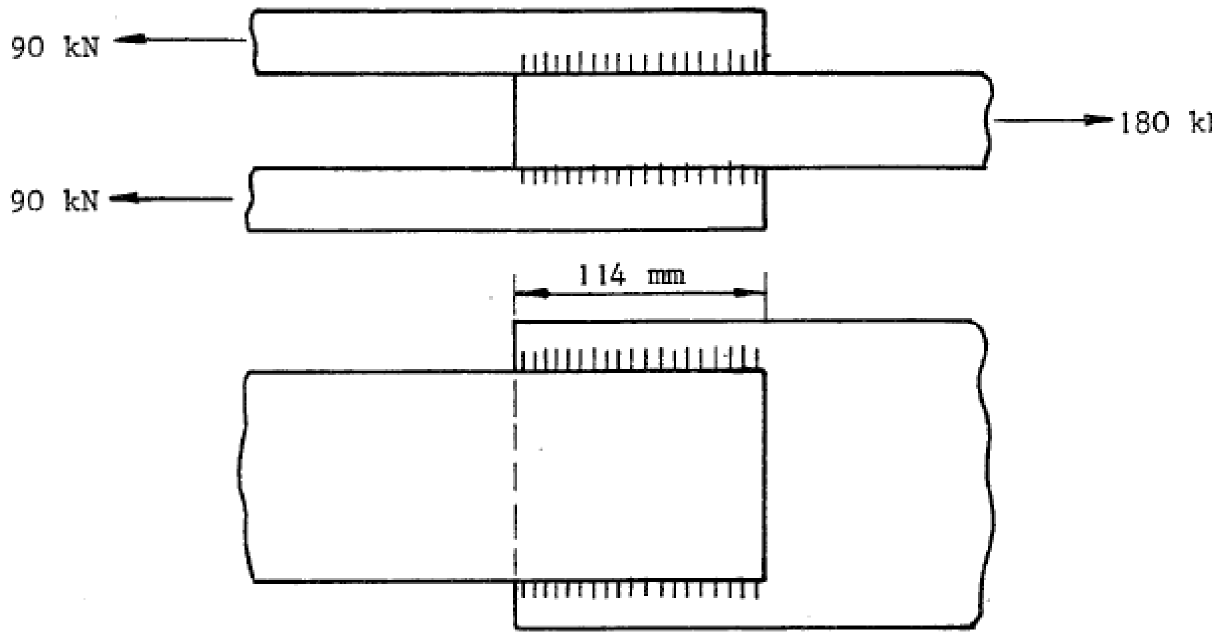


Figure 10.31

Solution:
The beam is so short (20 mml that the bending moment is neglected and the joint is studied for vertical shear only.

$$l_{effective} = l_{actual} - 2\,w$$
$$l_{effective} = 144\,mm - 2\,w$$
$$Shear\ stress = \frac{Shear\ load}{Total\ throat\ area\ of\ welds}$$
$$\tau = \frac{P}{4\times 0{,}707\times w\times l_{effective}}$$
$$80\times 10^{6}\,N/m^{2} = \frac{180\times 10^{3} N}{4\times 0{,}707\times w\times(0{,}114\,m-2\,w)}$$
$$80\times 10^{6}\,N/m^{2} = \frac{180\times 10^{3} N}{0{,}322\,w-5{,}656\,w^{2}}$$
$$0{,}322\,w-5{,}656\,w^{2} = \frac{180\times 10^{3} N}{80\times 10^{6}\,N/m^{2}}$$
$$= 2{,}25\times 10^{-3} m^{2}$$
$$w^{2}-0{,}057\,w + ,000398 = 0$$
$$w = \frac{-b\pm\sqrt{b^{2}-4ac}}{2a}$$
$$= \frac{0{,}057\pm\sqrt{0{,}00325-0{,}001592}}{2}$$
$$= \frac{0{,}057\pm 0{,}041}{2}$$
$$= \frac{0{,}098}{2} \text{ or } \frac{0{,}016}{2}$$
$$= 49\,mm \text{ or } 8\,mm$$

Use the practical size 8 mm.

Worked Example 10.7

It is required to calculate the side weld lengths for the joint shown using 10 mm fillet welds. Use a stress of 100 MPa for the end welds and 75 MPa for the side welds.

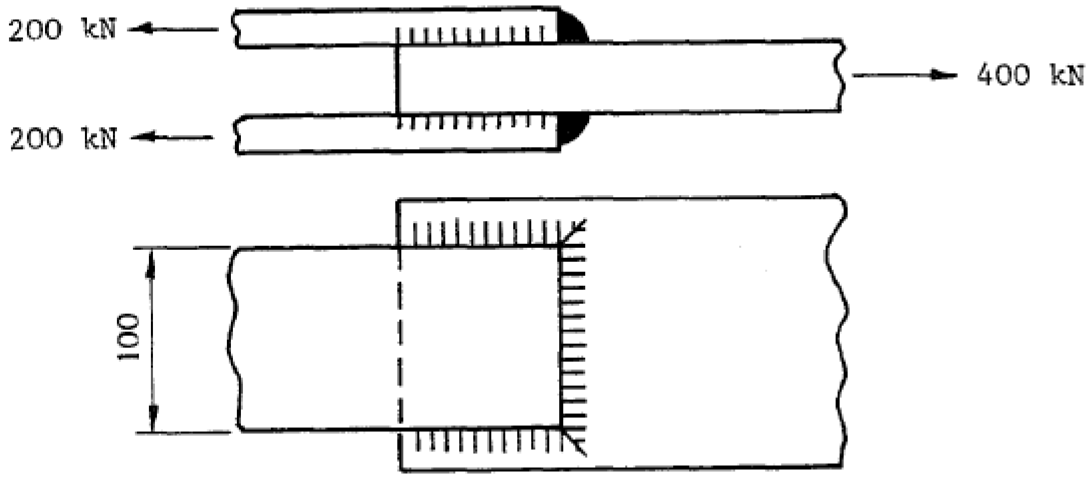


Figure 10.32

Solution:

$$Normal\ stress\ in\ end\ welds = \frac{Normal\ load\ carried\ by\ end\ welds}{Total\ throat\ area\ of\ endwelds}$$

$$\sigma = \frac{Normal\ load\ carried\ by\ end\ welds}{2(0{,}707 \times w \times l_{effective})}$$

$$Normal\ load\ carried\ by\ end\ welds = 100 \times 10^6\ N/m^2 \times 2 \times 0{,}707 \times 0{,}01m \times 0{,}1m$$
$$= 141{,}4\ kN$$

The remaining Load is to be carried by the side welds

$$Remaining\ load = 400 \times 10^3 N - 141{,}4 \times 10^3 N$$
$$= 258{,}6\ kN$$

$$Shear\ stress = \frac{Shear\ load}{Total\ throat\ area\ of\ side\ welds}$$

$$\tau = \frac{P}{4(0{,}707 \times w \times l_{effective})}$$

$$l_{effective} = \frac{258{,}6 \times 10^3 N}{75 \times 10^6\ N/m^2 \times 4 \times 0{,}707 \times 0{,}01\ m}$$
$$= 0{,}122\ m$$

$$l_{actual} = 122\ mm + 10mm$$
$$= 132\ mm$$

It is not necessary to add twice the weld size in this case, as one end of each weld is continuous with the end weld.

10.8 Eccentric load parallel with the welds

With axial loads on unsymmetrical sections such as angles welded on the flange edges, the throat area of the welds should be proportioned so that the sum of the resisting moments of the welds about the neutral axis of the angle is zero.

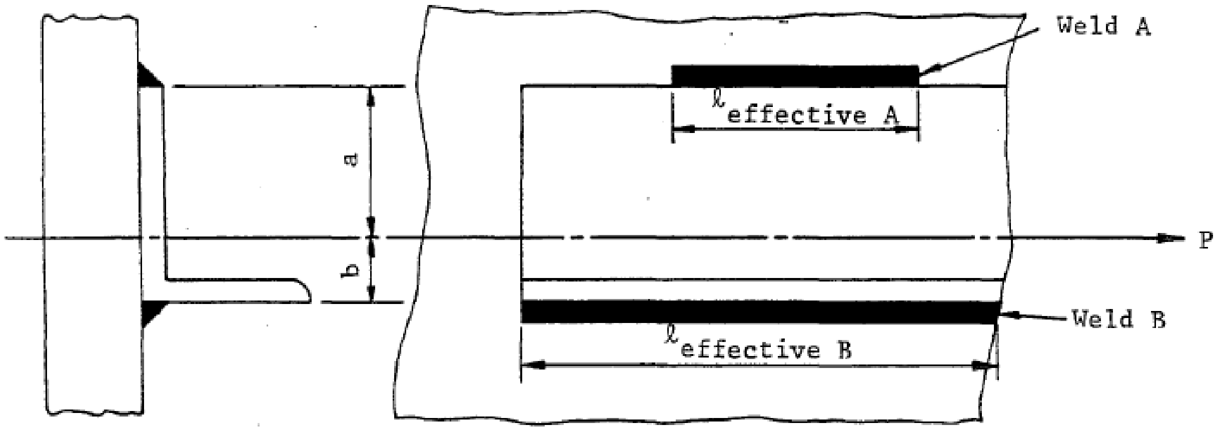


Figure 10.33

$$Resisting\ moment\ of\ weld\ A = Resisting\ moment\ of\ weld\ B$$
$$Throat\ area\ of\ weld\ A \times Shear\ stress \times a = Throat\ area\ of\ weld\ B \times Shear\ stress \times b$$
$$0{,}707 \times w \times l \times l_{effect} Ⓐ \times \tau \times a = 0{,}707 \times w \times l \times l_{effect} Ⓑ \times \tau \times b$$
$$l_{effect} Ⓐ \times a = l_{effect} Ⓑ \times b$$
$$l_{effect} Ⓐ = \frac{l_{effect} Ⓑ \times b}{a} \quad \text{.............................} ①$$

The required weld length to resist the load

$$Load = Shear\ stress \times Throat\ area\ of\ weld$$
$$P = \tau \times 0{,}707 \times w \times \left(l_{effect} Ⓐ + l_{effect} Ⓑ\right)$$
$$l_{effect} Ⓐ + l_{effect} Ⓑ = \frac{P}{\tau \times 0{,}707 \times w} \quad \text{.............................} ②$$

Substitute for l_{effect} Ⓐ in ②

$$\frac{l_{effect} Ⓑ \times b}{a} + l_{effect} Ⓑ = \frac{P}{\tau \times 0{,}707 \times w}$$

10.9 Eccentric loading perpendicular to the plane of the weld group

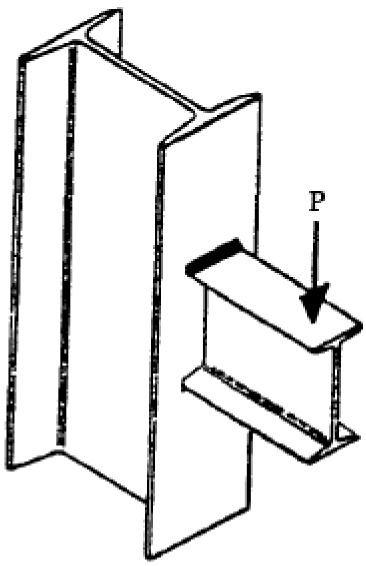


Figure 10.34

In the case of eccentric loading perpendicular to the group of welds, the welds are commonly designed to carry the vector sum of the direct shear stress and the maximum bending stress, based on a working throat stress not exceeding 77 MPa.

10.9.1 Design of welds in bending

A few typical welded connections subjected to eccentric loading perpendicular to the plane of the weld group are shown in the following figures.

10.9.1.1 Vertical welds

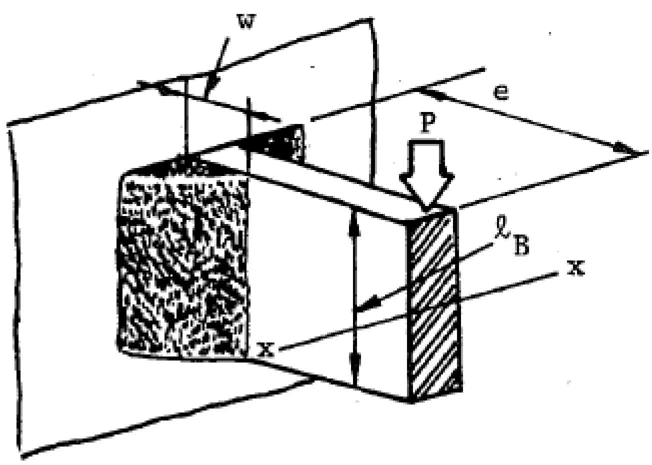


Figure 10.35

10.9.1.2 Horizontal welds

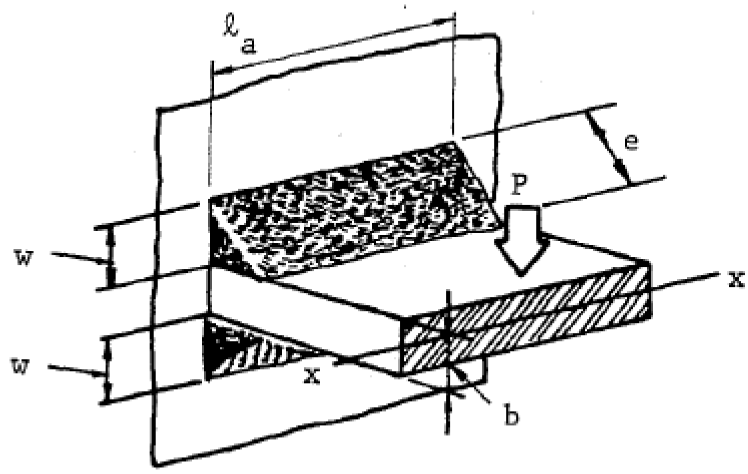


Figure 10.36

10.9.1.3 Circumferential welds

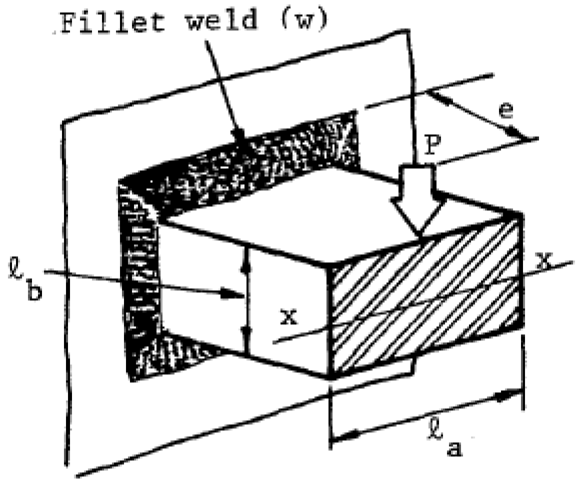


Figure 10.37

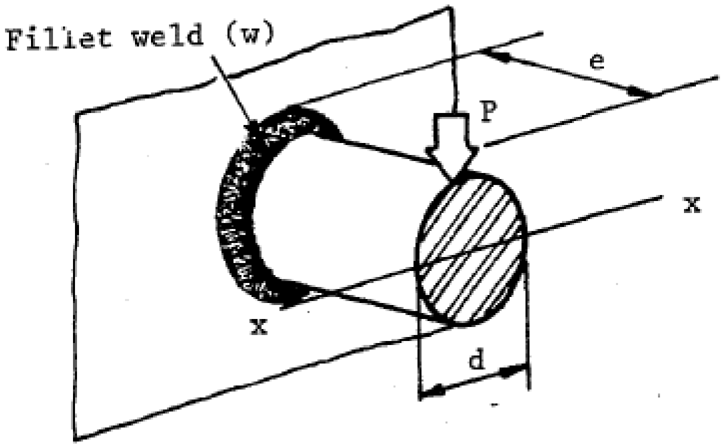


Figure 10.38

10.9.2 Design formulae

The design of a welded connection where the load is eccentrically applied perpendicularly to the plane of the weld group is carried out by using the following standard formulae.

(a) $$Direct\ shear\ stress = \frac{Shear\ load}{Total\ throat\ area\ of\ weld}$$

$$\tau_D = \frac{P}{0{,}707 \times w \times total\ effective\ length\ of\ weld}$$

(b) $$Bending\ stress = \frac{Bending\ moment}{Section\ modulus}$$

$$= \frac{M}{Z_{xx}}$$

$$\sigma_B = \frac{P \times e}{Z_{xx}}$$

where P = applied load

e = eccentricity of the applied load from the weld

$Z_{xx} = \frac{1_{xx}}{y}$

1_{xx} = Total moment of inertia of the welds about the x-x axis

y = distance from the neutral axis to the outer layers of the welded section along the y-y axis.

(c) $Resultant\ shear\ stress\ = \sqrt{\tau_D{}^2 + \sigma_B{}^2}$

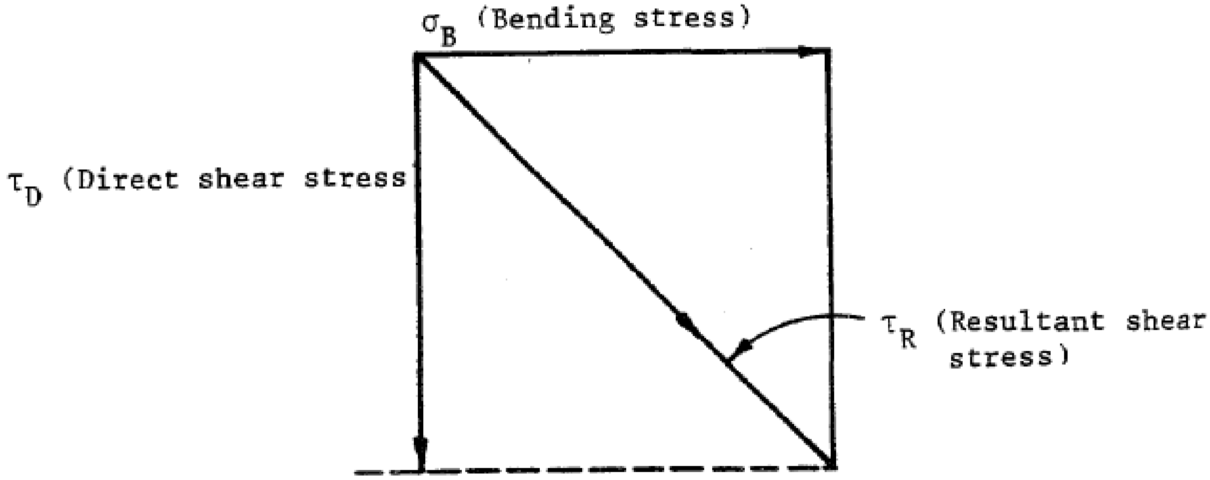


Figure 10.39

The corresponding approximate section modulus in bending of nine typical welded connections are shown in **Figure 10.40**.

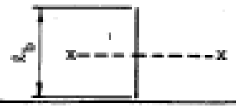
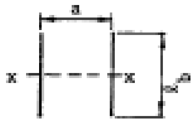
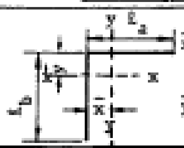
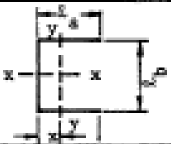
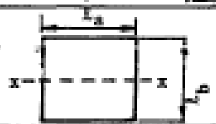
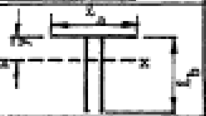
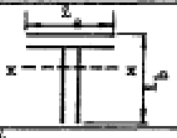
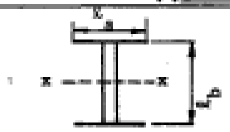
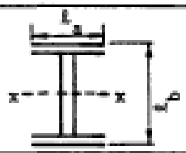
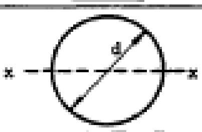
OUTLINE OF WELDED JOINT	BENDING (ABOUT HORIZONTAL AXIS x-x) Z_{xx}	
	$0{,}118\ w\ \ell_b^2$	
	$0{,}236\ w\ \ell_b^2$	
	$0{,}707\ bw\ \ell_a$	
 $\bar{x} = \frac{\ell_a^2}{2(\ell_a+\ell_b)}$ $\bar{y} = \frac{\ell_b^2}{2(\ell_a+\ell_b)}$	$0{,}118\ w\ \ell_b(4\ell_a + \ell_b)$ Top	$\frac{0{,}118\ w\ \ell_b^2(4\ell_a + \ell_b)}{2\ell_a + \ell_b}$ Bottom
$\bar{x} = \frac{\ell_a^2}{2\ell_a + \ell_b}$ 	$0{,}118\ w\ \ell_b(6\ell_a + \ell_b)$	
$\bar{y} = \frac{\ell_b^2}{2\ell_b+\ell_a}$ 	$0{,}236\ w\ \ell_b(2\ell_a + \ell_b)$ Top	$0{,}236\ w\ \ell_b^2\frac{(2\ell_a + \ell_b)}{\ell_a + \ell_b}$ Bottom
	$0{,}236\ w\ \ell_b(3\ell_a + \ell_b)$	
$\bar{y} = \frac{\ell_b^2}{\ell_a+2\ell_b}$ 	$0{,}236\ w\ \ell_b(2\ell_a + \ell_b)$ Top	$0{,}236\ w\ \ell_b^2\frac{(2\ell_a + \ell_b)}{\ell_a + \ell_b}$ Bottom
$\bar{y} = \frac{\ell_b^2}{2(\ell_a+\ell_b)}$ 	$0{,}236\ w\ \ell_b(4\ell_a + \ell_b)$ Top	$0{,}236\ w\ \ell_b^2\frac{(4\ell_a + \ell_b)}{(2\ell_a + \ell_b)}$ Bottom
	$0{,}236\ w\ \ell_b(3\ell_a + \ell_b)$	
	$0{,}236\ w\ \ell_b(6\ell_a + \ell_b)$	
	$0{,}555\ w\ d^2$	

Figure 10.40

10.10 Eccentric loading in the same plane as the weld group (twisting)

A general case of eccentric loading in the same plane as the weld group is shown in **Figure 10.41**.

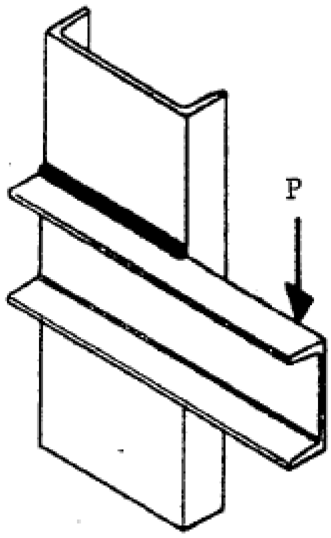


Figure 10.41

10.10.1 Design of welds in twisting

A few typical welded connections subjected to eccentric loading in the same plane as the weld group is shown in the following figures.

10.10.1.1 Vertical welds

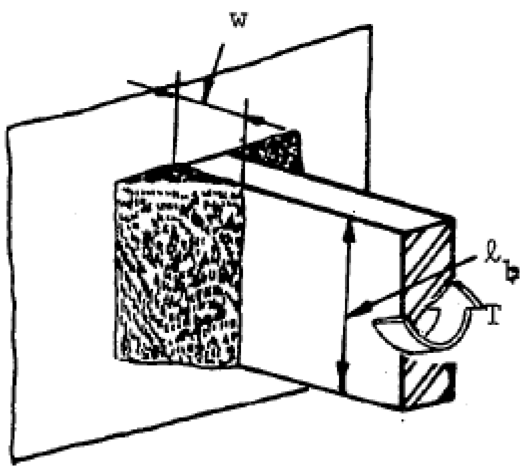


Figure 10.42

10.10.1.2 Horizontal welds

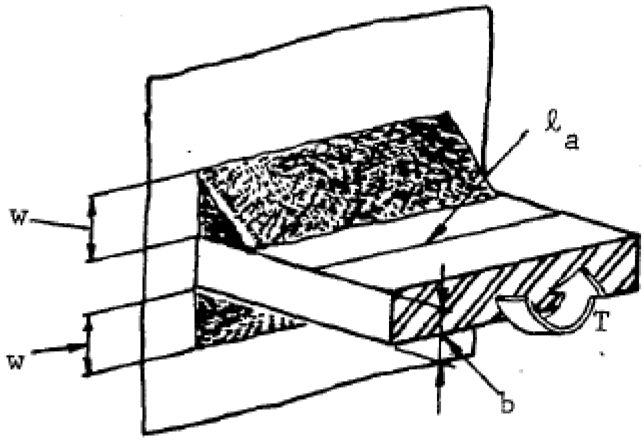


Figure 10.43

10.10.1.3 Circumferential welds

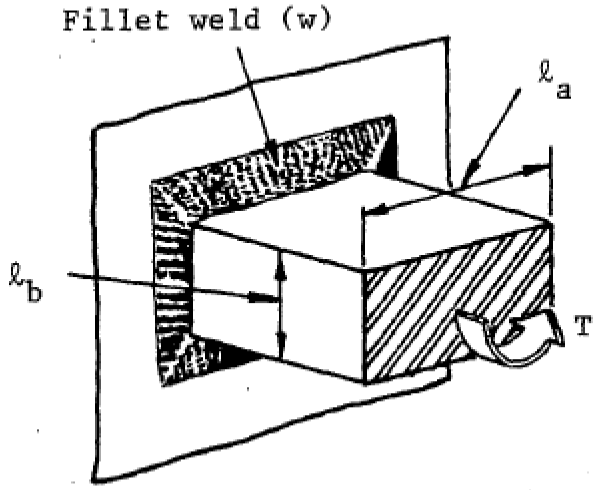


Figure 10.44

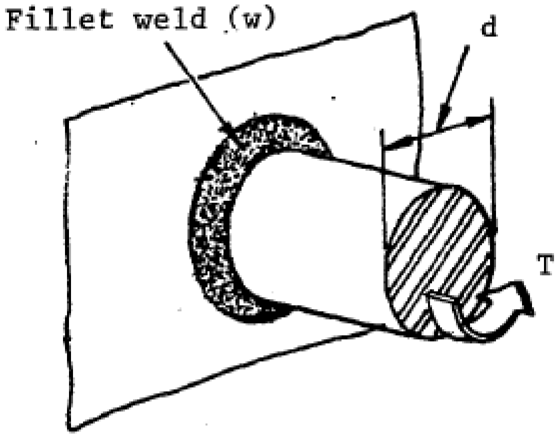


Figure 10.45

10.10.2 Design formulae

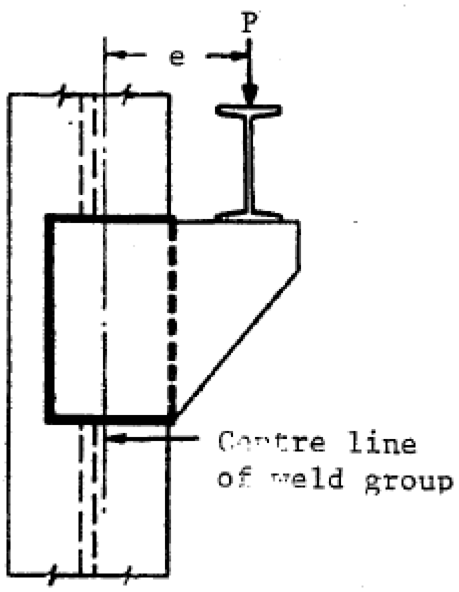


Figure 10.46

The fillet welds are subjected to the action of a load P acting at a distance e from the centre of gravity of the welds.

In **Figure 10.4**, let abed represent the fillet welds of the connection shown in **Figure 10. 46**.

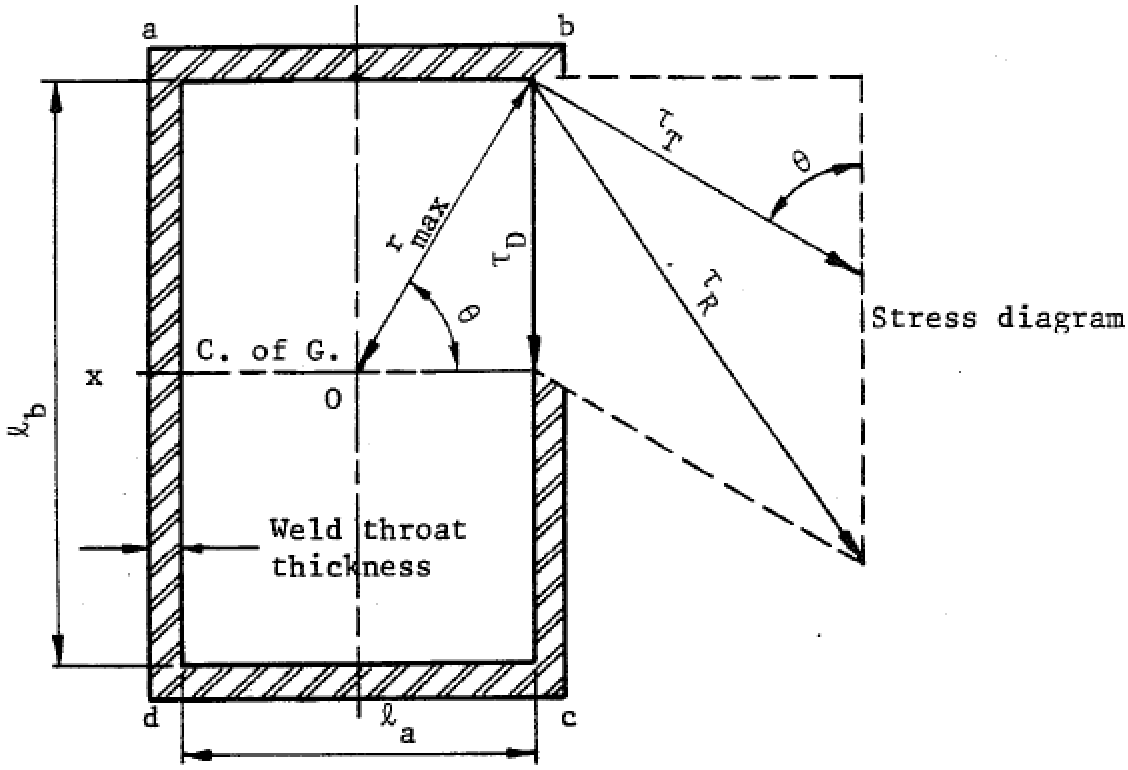


Figure 10.47

The welds are subjected to a direct shear (T_D) and to a turning shear τ_T, which is proportional to the distance of the weld section from the centre of gravity and is a maximum at the corners of the weld.

In the stress diagram in **Figure 10.47**, τ_T is to represent its maximum value. The maximum total (resultant) shear (T_R) is then the vector sum of T_D and τ_T.

(a) $$Direct\ shear\ stress = \frac{Shear\ load}{Total\ throat\ area\ of\ weld}$$

$$\tau_D = \frac{P}{0{,}707 \times w \times total\ effective\ length\ of\ weld}$$

(b) $$Turning\ shear = \frac{Turning\ moment \times r_{max}}{Polar\ moment\ of\ inertia} = \frac{P \times e \times r_{max}}{l_p}$$

where P = applied load

e = eccentricity of the applied load from the centre of gravity of the weld group

$1_p = 1_{xx} + 1_{yy}$

$r_{max} = \frac{1}{2}\sqrt{{l_a}^2 + {l_b}^2}$

(c) $$Maximum\ resultant\ shear\ stress\ (T_R) = \sqrt{{\tau_D}^2 + {\tau_T}^2 + 2\tau_T\tau_D \times \cos\theta}$$

and $$\cos\theta = \frac{l_a}{\sqrt{{l_a}^2 + {l_b}^2}}$$

The foregoing is the fundamental theory underlying the calculation of the welds for brackets eccentrically loaded in the plane of the welding.

Formulae for the polar moment of inertia of various combinations of top and bottom end side fillet welds are given in **Figure 10.48**.

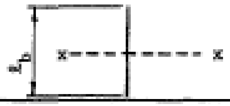
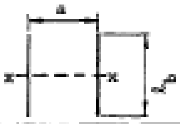
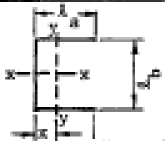
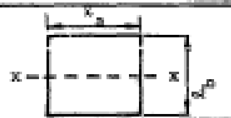
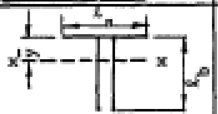
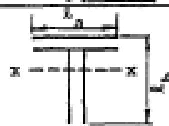
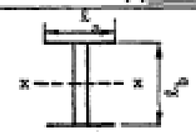
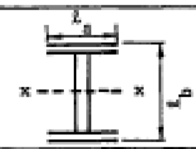
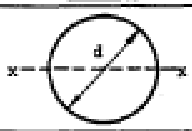
OUTLINE OF WELDED JOINT	TWISTING I_p	
	$0{,}059\ w\ \ell_b^3$	
	$0{,}118\ w\ \ell_b(\ell_b^2 + 3a^2)$	
	$0{,}118\ w\ \ell_a(3b^2 + \ell_a^2)$	
 $\bar{x} = \frac{\ell_a^2}{2(\ell_a+\ell_b)}$ $\bar{y} = \frac{\ell_b^2}{2(\ell_a+\ell_b)}$	$0{,}059\ w\left[\frac{(\ell_a + \ell_b)^4 - 6\ell_a^2\ell_b^2}{(\ell_a + \ell_b)}\right]$	
$\bar{x} = \frac{\ell_a^2}{2\ell_a + \ell_b}$ 	$0{,}059\ w(2\ell_a + \ell_b)^3 - \frac{0{,}707\ w\ \ell_a^2(\ell_a+\ell_b)^2}{2\ell_a + \ell_b}$	
$\bar{y} = \frac{\ell_b^2}{2\ell_b+\ell_a}$ 	$0{,}059\ w(\ell_a + 2\ell_b)^3 - \frac{0{,}707\ w\ \ell_b^2(\ell_a+\ell_b)^2}{\ell_a + 2\ell_b}$	
	$0{,}118\ w(\ell_b + \ell_a)^3$	
$\bar{y} = \frac{\ell_b^2}{\ell_a+2\ell_b}$ 	$0{,}059\ w(\ell_a + 2\ell_b)^3 - \frac{0{,}707\ w\ \ell_b^2(\ell_a+\ell_b)^2}{(\ell_a + 2\ell_b)}$	
$\bar{y} = \frac{\ell_b^2}{2(\ell_a+\ell_b)}$ 	$0{,}118\ w\ \ell_b^3\ \frac{(4\ell_a + \ell_b)}{(\ell_a + \ell_b)} + 0{,}118\ w\ \ell_a^3$	
	$0{,}118\ w(\ell_a^3 + 3\ell_a\ell_b^2 + \ell_b^3)$	
	$0{,}118\ w(2\ell_a^3 + 6\ell_a\ell_b^2 + \ell_b^3)$	
	$0{,}555\ w\ d^3$	

Figure 10.48

Worked Example 10.8

An angle measuring 150 mm x 90 mm x 10 mm is to be welded to a steel plate by fillet welds along the edges of the 150 mm leg. The angle is subjected to a tension of 220 kN. Determine the weld lengths required if placed as shown in **Figure 10.49**. (Allowable shear stress not to exceed 63,5 MPa).

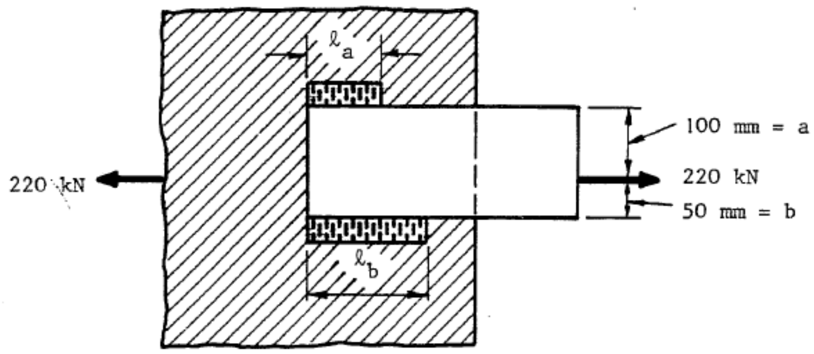


Figure 10.49

<u>Solution:</u>

$$\frac{l_{effect}\text{Ⓑ} \times b}{a} + l_{effect}\text{Ⓑ} = \frac{P}{\tau \times 0{,}707 \times w}$$

The angle thickness is 10 mm, therefore the leg size of the weld is taken as 10 mm.

$$\frac{l_{effect}\text{Ⓑ} \times 0{,}05\ m}{0{,}1\ m} + l_{effect}\text{Ⓑ} = \frac{220 \times 10^3\ N}{63{,}5 \times 10^6\ N/m^2\ \ 0{,}707 \times 0{,}01\ m}$$

$$0{,}5 l_{effect}\text{Ⓑ} + l_{effect}\text{Ⓑ} = 0{,}49\ m$$

$$1{,}5 l_{effect}\text{Ⓑ} = 0{,}49\ m$$

$$l_{effective}\text{Ⓑ} = \frac{0{,}49\ m}{1{,}5}$$

$$= 0{,}327\ m$$

$$Actual\ length\ of\ weld\text{Ⓑ} = Effective\ Length\ weld\ \text{Ⓑ} +\ 2\ x\ weld\ size\ for\ starting\ and\ stopping\ of\ weld$$

$$= 327\ mm + 2 \times 10\ mm$$

$$= 347\ mm$$

$$l_{effect}\text{Ⓐ} = \frac{l_{effective}\text{Ⓑ} \times b}{a}$$

$$= \frac{0{,}327\ m \times 0{,}05\ m}{0{,}1\ m}$$

$$= 0{,}164\ m$$

$$Actual\ length\ of\ weld\text{Ⓐ} = Effective\ Length\ weld\ \text{Ⓐ} +\ 2\ x\ weld\ size\ for\ starting\ and\ stopping\ of\ weld$$

$$= 164\ mm + 2 \times 10\ mm$$

$$= 184\ mm$$

Worked Example 10.9

A bracket carrying a Load of 150 kN at an arm of 190 mm from the plane of the welds is connected to the face of the main member by a top and bottom weld 127 mm Long and 305 mm apart, and by 2 side welds with an effective Length of 305 mm, as shown in Figure 25.17.

Calculate the size of fillet weld to be used, if the allowable throat stress of the weld is not to exceed 77 MPa .

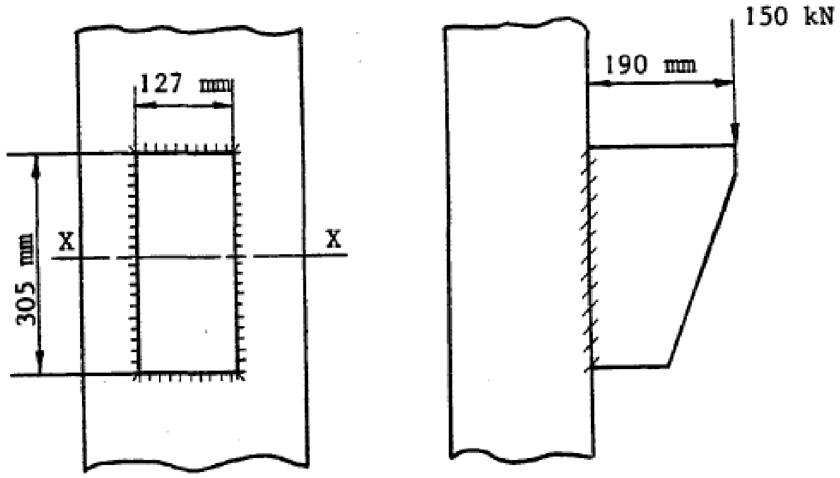


Figure 10.50

Solution:

$$Direct\ shear\ stress = \frac{Shear\ load}{Total\ throat\ area\ of\ weld}$$

$$\tau_D = \frac{150 \times 10^3\ N}{2 \times 0{,}707 \times w(0{,}127\ m + 0{,}305\ m)}$$

$$\tau_D = \frac{245{,}56 \times 10^3}{w} N/m$$

$$Bending\ stress = \frac{Bending\ moment}{Section\ modulus}$$

$$\sigma_B = \frac{P \times e}{Z_{xx}}$$ See **Figure 10.40** for Z_{xx}

$$= \frac{150 \times 10^3\ N \times 0{,}19\ m}{0{,}236 \times w \times 0{,}305\ m(3 \times 0{,}127\ m + 0{,}305\ m)}$$

$$= \frac{28\ 500\ Nm}{0{,}0494\ w\ m^2}$$

$$\sigma_B = \frac{577{,}22 \times 10^3}{w}\ Nm$$

$$Resultant\ shear\ stress = \sqrt{\tau_D{}^2 + \sigma_B{}^2}$$

$$\tau_R = \sqrt{\left[\frac{245{,}6 \times 10^3}{w}\right]^2 + \left[\frac{577{,}2 \times 10^3}{w}\right]^2}$$

$$= \sqrt{\frac{6{,}03 \times 10^{10} + 33{,}32 \times 10^{10}}{w^2}}$$

$$= \sqrt{\frac{39{,}35 \times 10^{10}}{w^2}}$$

$$= \frac{627{,}263 \times 10^3}{w} N/m$$

$$Working\ throat\ stress = Resultant\ shear\ stress$$

$$77 \times 10^6\ N/m^2 = \frac{627{,}263 \times 10^3 N/m}{w}$$

$$w = \frac{627{,}263 \times 10^3 N/m}{77 \times 10^6\ N/m^2}$$

$$w = 0{,}00815\ m$$

$$= 8{,}15\ mm$$

Use 9 mm fillet welds.

Worked Example 10.10

A circular bar is welded to a steel plate as shown in **Figure 10.51**. The bar diameter is 50 mm, and it carries a load of 9 000 N at the free end.

The size of fillet weld used is 20 mm, and the allowable shear stress in the welded joint should not exceed 66 x 10^6 Pa.

Calculate the maximum length of bar to be used if the mass of the shaft is neglected.

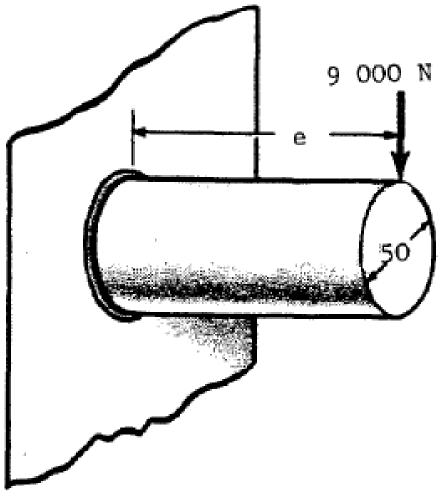


Figure 10.51

Solution:

$$Direct\ shear\ stress = \frac{Shear\ load}{Total\ throat\ area\ of\ weld}$$

$$\tau_D = \frac{P}{0{,}707 \times w \times circumference\ of\ weld}$$

$$\tau_D = \frac{9000\ N}{0{,}707 \times\ 0{,}02\ m \times \pi \times 0{,}05\ m}$$

$$\tau_D = 4052\ \times\ 10^3 N/m^2$$

$$Bending\ stress = \frac{Bending\ moment}{Section\ modulus}$$

$$\sigma_B = \frac{P \times e}{Z_{xx}}$$

$$\sigma_B = \frac{9000\,N \times e}{0{,}555 \times 0{,}02\,m \times 0{,}05\,m \times 0{,}05\,m}$$

$$= 324{,}32 \times 10^6\,N/m^2$$

$$Resultant\ shear\ stress = \sqrt{{\tau_D}^2 + {\sigma_B}^2}$$

$$\tau_R = \sqrt{(4052 \times 10^3)^2 + (324{,}32 \times 10^6 \times e)^2}$$

$$66 \times 10^6\,N/m^2 = \sqrt{1{,}64 \times 10^{13}} + \sqrt{10\,518{,}35 \times 10^{13} \times e}$$

$$(10\,518{,}35 \times 10^{13} \times e^2 \times N^2/m^6)^{\frac{1}{2}} = 66 \times 10^6\,N/m^2 - 4{,}052 \times 10^6\,N/m^2$$

$$= 61{,}948\,10^6\,N/m^2$$

$$10\,518{,}35 \times 10^{13} \times e^2 = (61{,}948 \times 10^6 N/m^2)^2$$

$$e^2 = \frac{3{,}838 \times 10^{15}\,N^2/m^4}{105{,}1835 \times 10^{15}\,N^2/m^6}$$

$$e^2 = 0{,}0365\,m^2$$

$$e = 0{,}191\,m$$

$$e = 191\,mm$$

Worked Example 10.11

A bracket carrying a load of 15 kN at an arm of 200 mm from the plane of the welds is connected to the face of the main member by a top weld 130 mm long and by two side welds, each having an effective length of 280 mm, as shown in **Figure 10.52**.

What size of fillet weld should be used if the allowable throat stress of the welds is not to exceed 77 MPa?

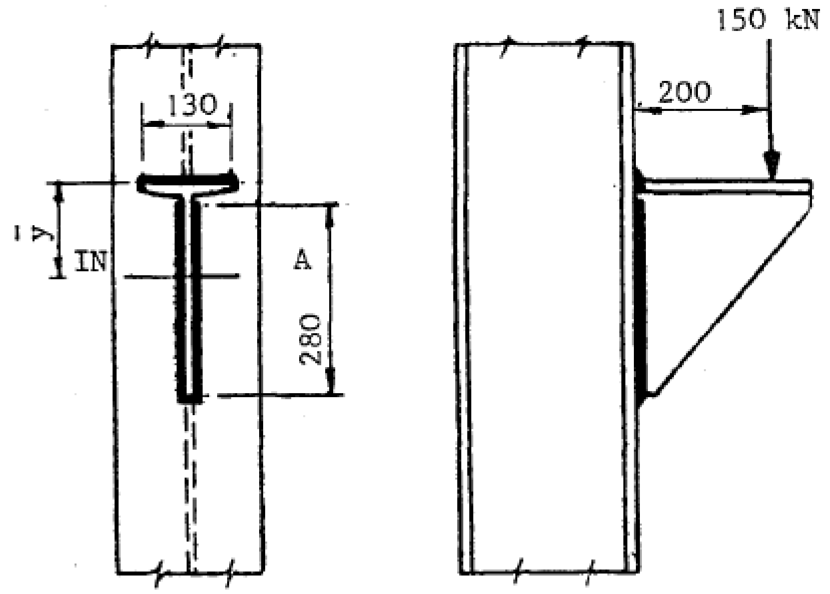


Figure 10.52

Solution:

$$Direct\ shear\ stress = \frac{Shear\ load}{Total\ throat\ area\ of\ weld}$$

$$\tau_D = \frac{150 \times 10^3\ N}{0{,}707 \times w \times (2 \times 0{,}28\ m + 0{,}13\ m)}$$

$$\tau_D = \frac{307{,}48 \times 10^3}{w}\ N/m$$

$$Bending\ stress = \frac{Bending\ moment}{Section\ modulus}$$

There are two section moduli in this case, a top one and a bottom one, and the least has to be used.

$$Z_{xx}\ top = 0{,}236\ w\ l_b(2b_a + l_b)$$
$$= 0{,}236\ \times w\ \times 0{,}28\ m(2\ \times 0{,}13\ m + 0{,}28\ m)$$
$$= 0{,}036\ \times w\ m^2$$

$$Z_{xx}\ bottom = 0{,}236\ w\ {l_b}^2 \frac{(2l_a + l_b)}{l_a + l_b}$$
$$= 0{,}236\ \times w\ (0{,}28\ m)^2 \frac{(2 \times 0{,}13\ m + 0{,}28\ m)}{0{,}13\ m + 0{,}28\ m}$$
$$= 0{,}0244\ w\ m^2 \text{ (least, therefore to be used)}$$

$$\sigma_B = \frac{150 \times 10^3\ N\ \times 0{,}2\ m}{0{,}0244\ w\ m^2}$$
$$= \frac{1229{,}51 \times 10^3\ N/m}{w}$$

$$Resultant\ shear\ stress = \sqrt{{\tau_D}^2 + {\sigma_B}^2}$$

$$\tau_R = \sqrt{\left[\frac{307{,}48 \times 10^3}{w}\right]^2 + \left[\frac{1229{,}51 \times 10^3}{w}\right]^2}$$
$$= \sqrt{\frac{9{,}45 \times 10^{10} + 151{,}17 \times 10^{10}}{w^2}}$$

$$\tau_R = \frac{1267{,}37 \times 10^3}{w}\ N/m$$

$$Allowable\ throat\ stress = Resultant\ shear\ stress$$

$$77 \times\ 10^6\ N/m^2 = \frac{1267{,}37 \times 10^3}{w}\ N/m$$

$$w = \frac{1267{,}37 \times 10^3\ N/m}{77 \times\ 10^6\ N/m^2}$$

$$w = 0{,}0165\ m$$

Use17 mm fillet welds.

Worked Example 10.12

Determine the size of fillet weld required, for the weld group shown in **Figure 10.53**, if the allowable stress is 77 MPa.

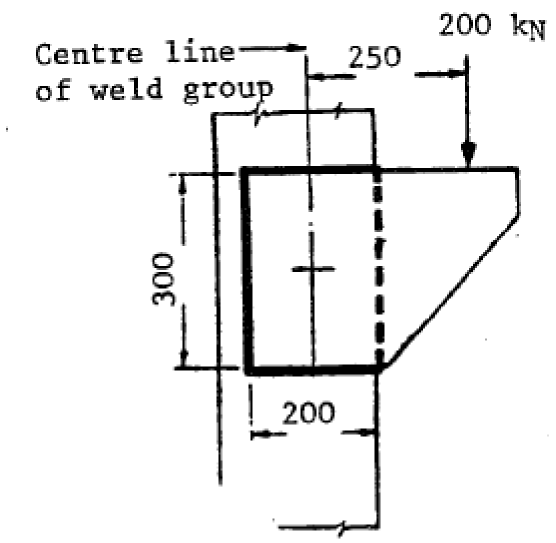


Figure 10.53

Solution:

$$Direct\ shear\ stress = \frac{Shear\ load}{Total\ throat\ area\ of\ weld}$$

$$\tau_D = \frac{200 \times 10^3\ N}{0{,}707 \times w \times 2(0{,}3\ m + 0{,}2\ m)}$$

$$\tau_D = \frac{282{,}89 \times 10^3}{w} N/m$$

$$Turning\ moment = P \times e$$
$$= 200 \times 10^3\ N \times 0{,}25\ m$$
$$= 50 \times 10^3\ Nm$$
$$r_{max} = \frac{1}{2}\sqrt{l_a + l_b}$$
$$= \frac{1}{2}\sqrt{(0{,}2\ m)^2 + (0{,}3\ m)^2}$$
$$r_{max} = 0{,}1803\ m$$

$$Polar\ moment\ of\ inertia = 0{,}118\ w(l_b + l_a)^3$$
$$= 0{,}118\ w(0{,}3\ m + 0{,}2\ m)^3$$
$$= 0{,}01475\ w\ m^3$$

$$Turning\ shear = \frac{Turning\ moment \times r_{max}}{Polar\ moment\ of\ inertia}$$
$$= \frac{50 \times 10^3\ Nm \times 0{,}1803\ m}{0{,}01475\ w\ m^3}$$
$$= \frac{611{,}186 \times 10^3}{w} N/m$$
$$\cos\theta = \frac{l_a}{\sqrt{l_a + l_b}}$$
$$= \frac{0{,}2\ m}{\sqrt{(0{,}2\ m)^2 + (0{,}3)^2}}$$
$$= 0{,}555$$

$$Resultant\ shear\ stress = \sqrt{{\tau_D}^2 + {\tau_T}^2 - 2\tau_T\tau_D \times \cos\theta}$$

$$\tau_R = \sqrt{\left[\frac{282{,}89\times10^3}{w}\right]^2 + \left[\frac{611{,}186\times10^3}{w}\right] + 2\times\frac{611{,}186\times10^3}{w}\times\frac{282{,}89\times10^3}{w}\times 0{,}55}$$

$$\tau_R = \sqrt{\frac{6{,}455\times10^{11}}{w^2}}$$

$$\tau_R = \frac{803{,}425\times10^3}{w} N/m$$

$$Allowable\ throat\ stress = Resultant\ shear\ stress$$

$$77\times\ 10^6\ N/m^2 = \frac{803{,}425\times10^3}{w} N/m$$

$$w = \frac{803{,}425\times10^3 N/m}{77\times10^6\ N/m^2}$$

$$w = 0{,}0104\ m$$

Say 11 mm fillet weld.

Worked Example 10.13

Determine the size of the required fillet weld for the bracket shown in **Figure 10.54** to carry a load r-F 200 kN. (Allowable stress not to exceed 66 MPa.)

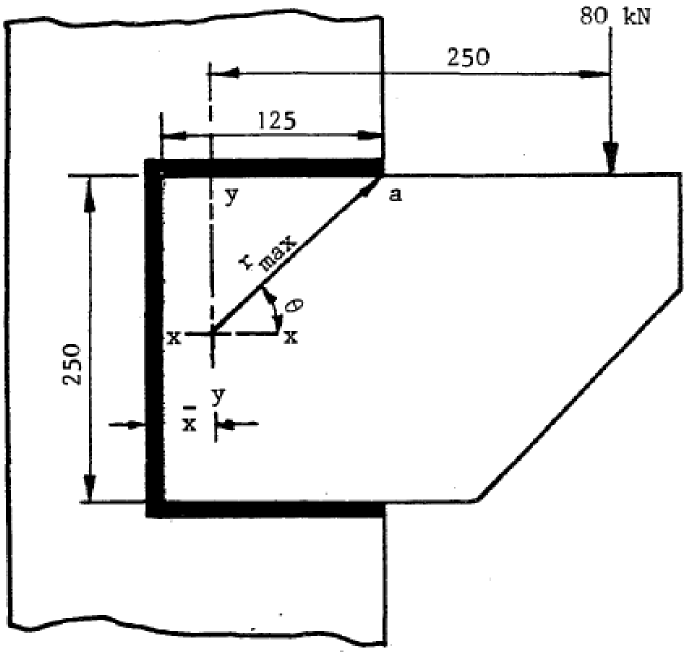


Figure 10.54

Solution:

$$x = \frac{{l_a}^2}{2l_a + l_b}$$

$$= \frac{(0{,}125\ m)^2}{2\times 0{,}125\ m + 0{,}25\ m}$$

$$= 0{,}03125\ m$$

$$= 31{,}25\ mm$$

$$Direct\ shear\ stress = \frac{Shear\ load}{Total\ throat\ area\ of\ weld}$$

$$\tau_D = \frac{80 \times 10^3\ N}{0{,}707 \times w \times (2l_a + l_b)}$$

$$= \frac{80 \times 10^3\ N}{0{,}707 \times w \times (2 \times 0{,}125\ m + 0{,}25\ m)}$$

$$= \frac{226308}{w}\ N/m$$

$$r_{max} = \frac{1}{2}\sqrt{{l_a}^2 + {l_b}^2}$$

$$= \frac{1}{2}\sqrt{(0{,}125\ \ m)^2 + (0{,}25\ m)^2}$$

$$r_{max} = 0{,}1398\ m$$

$$\cos\theta = \frac{l_a}{\sqrt{{l_a}^2 + {l_b}^2}}$$

$$= \frac{0{,}125\ m}{\sqrt{(0{,}125\ m)^2 + (0{,}25)^2}}$$

$$= 0{,}4472$$

$$l_p = 0{,}059\ w(2l_a + l_b)^3 - \frac{0{,}707 w {l_a}^2 (l_a + l_b)^2}{2l_a + l_b}$$

$$= 0{,}059\ w(2 \times 0{,}125 + 0{,}25)^3 - \frac{0{,}707 w \times 0{,}125^2 (0{,}125 + 0{,}25)^2}{2 \times 0{,}125 + 0{,}25}$$

$$= 7{,}375\ \times 10^{-3} w - 3{,}107 \times 10^{-3} w$$

$$= 4{,}268\ \times 10^{-3} w\ m^{-3}$$

$$Turning\ shear = \frac{Turning\ moment \times r_{max}}{Polar\ moment\ of\ inertia}$$

$$= \frac{80 \times 10^3\ N \times 0{,}025\ m \times 0{,}1398\ m}{4{,}268 \times 10^{-3}\ w\ m^3}$$

$$= \frac{665098}{w} N/m$$

$$Maximum\ resultant\ shear\ stress = \sqrt{{\tau_D}^2 + {\tau_T}^2 - 2\tau_T\tau_D \times \cos\theta}$$

$$\tau_R = \sqrt{\left[\frac{226308}{w}\right]^2 + \left[\frac{66598}{w}\right]^2 + 2\left[\frac{665098}{w}\right]\left[\frac{226308}{w}\right] 0{,}4472}$$

$$\tau_R = \frac{712062}{w} N/m$$

$$Allowable\ throat\ stress = Resultant\ shear\ stress$$

$$66 \times\ 10^6\ N/m^2 = \frac{712062}{w} N/m$$

$$w = \frac{712062\ N/m}{66 \times 10^6\ N/m^2}$$

$$w = 0{,}0108\ m$$

Use, say 11 mm fillet welds.

Worked Example 10.14

A shaft of 50 mm in diameter is joined to a plate by a 6 mm fillet weld. The working stress in shear may not exceed 77 MPa.

Determine the maximum torque the welded joint can sustain.

Solution:

$$Maximum\ \ shearing\ stress\ in\ the\ fillet\ weld = \frac{Torque \times \frac{d}{2}}{Polar\ moment\ of\ inertia}$$
$$= \frac{T \times \frac{0{,}05\ m}{2}}{0{,}555\ w\ d^2}$$
$$= \frac{T \times \frac{0{,}05\ m}{2}}{0{,}555\ w\ d^2}$$
$$= \frac{T \times 0{,}025\ m}{0{,}555 \times 0{,}006\ m \times (0{,}05\ m)^3}$$
$$= 60{,}06\ \times\ 10^3\ T\ m^{-3}$$

$$Working\ stress = Maximum\ shearing\ stress$$
$$77 \times\ 10^6\ N/m^2 = 60{,}06\ \times\ 10^3\ T\ m^{-3}$$
$$T = \frac{77 \times 10^6\ N/m^2}{60{,}06 \times 10^3\ \ m^{-3}}$$
$$= 1\ 2820{,}5\ Nm$$

Worked Example 10.15

A shaft having a diameter of 50 mm is welded to a pulley by means of fillet welding. If the allowable shear stress of both the shaft and the welded joint is 69 x 106 N/m2, calculate the size of weld to be used.

Solution:

$$Torque = \frac{\pi d^3}{16} \times \tau$$
$$= \frac{\pi \times \frac{(0{,}05\ m)^3}{16} \times 69 \times 10^6\ N/m^2}{0{,}555\ w\ d^2}$$
$$= 1693{,}5\ Nm$$

$$Maximum\ \ shear\ stress = \frac{T \times \frac{d}{2}}{l_p}$$
$$= \frac{1693{,}5\ Nm \times \frac{0{,}05\ m}{2}}{0{,}555 \times w \times (0{,}05\ m)^3}$$
$$= \frac{610\ 270}{w} N/m$$

$$Allowable\ shear\ stress = Maximum\ shear\ stress$$
$$69 \times\ 10^6\ N/m^2 = \frac{610\ 270}{w}\ N/m$$
$$w = \frac{610\ 270\ N/m}{69 \times 10^6\ N/m^2}$$
$$w = 0{,}00884\ Nm$$

Say 9 mm fillet weld.

Worked Example 10.16

Ascertain the size of fillet welds for the weld groups shown below under the loading indicated:

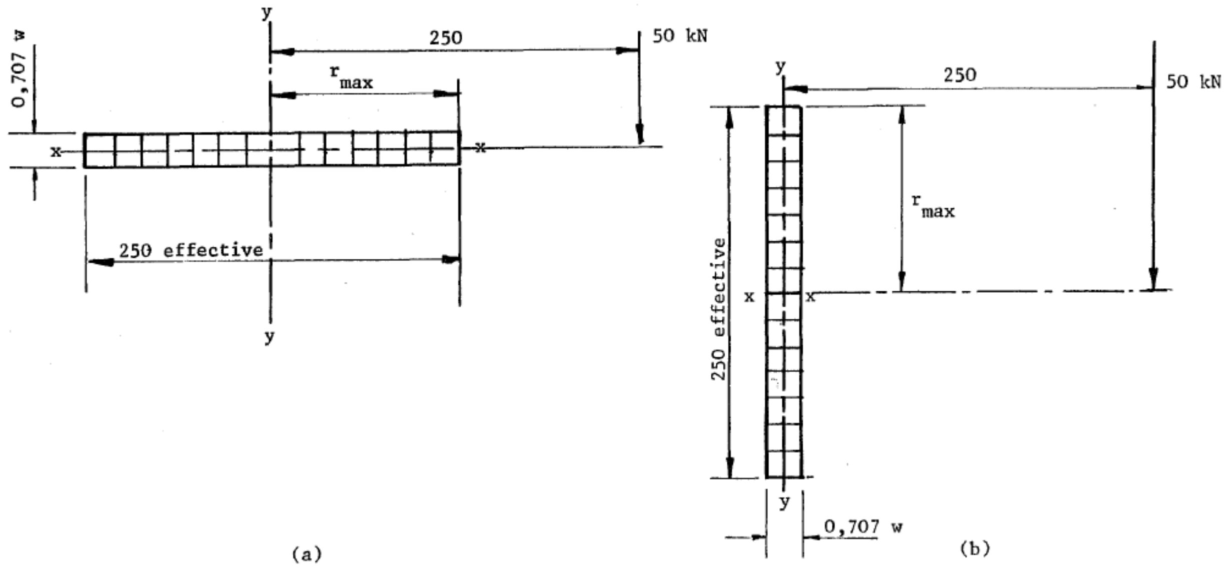


Figure 10.55

Solution:

(a)

$$l_{xx} = \frac{l_a \times t^3}{12} \text{ (neglect very small)}$$

where t = $throat\ size = 0{,}707\ w$

$$l_{yy} = \frac{t \times {l_a}^3}{12}$$

$$\begin{aligned} l_p &= l_{xx} + l_{yy} \\ &= l_{yy} \\ &= \frac{t \times {l_a}^3}{12} \\ &= \frac{0{,}707 \times w \times (0{,}25\ m)^3}{12} \\ &= 9{,}206\ \times\ 10^{-4}\ w\ m^3 \end{aligned}$$

$$Direct\ shear\ stress = \frac{Shear\ load}{Total\ throat\ area\ of\ weld}$$

$$\begin{aligned} \tau_D &= \frac{P}{0{,}707 \times w \times total\ effective\ length\ of\ \ weld} \\ &= \frac{50 \times 10^3\ N}{0{,}707 \times w \times 0{,}25\ m} \\ &= \frac{282\ 885}{w} N/m \end{aligned}$$

$$\begin{aligned} r_{max} &= \frac{1}{2}\sqrt{{l_a}^2 + {l_b}^2} \\ &= \frac{1}{2}\sqrt{(0{,}25\ \ m)^2 + 0} \\ &= 0{,}125\ m \end{aligned}$$

$$\cos\theta = \frac{l_a}{\sqrt{{l_a}^2+{l_b}^2}}$$
$$= \frac{0{,}25\ m}{\sqrt{(0{,}25\ m)^2+0}}$$
$$= 1$$

$$Turning\ shear = \frac{Turning\ moment \times r_{max}}{l_p}$$
$$= \frac{p \times e \times r_{max}}{Polar\ moment\ of\ inertia}$$
$$\tau_T = \frac{50 \times 10^3\ N \times 0{,}25\ m \times 0{,}125\ m}{9{,}206 \times 10^{-4}\ w\ m^3}$$
$$= \frac{1697313}{w} N/m$$

$$Resultant\ shear\ stress = \sqrt{{\tau_D}^2 + {\tau_T}^2 - 2\tau_T\tau_D \times \cos\theta}$$
$$= \sqrt{\left[\frac{282885}{w}\right]^2 + \left[\frac{1697313}{w}\right]^2 + 2\left[\frac{697313}{w}\right]\left[\frac{282885}{w}\right] \times 1}$$
$$= \sqrt{\frac{3{,}921 \times 10^{12}}{w^2}}$$
$$\tau_R = \frac{1980198}{w} N/m$$

$$Allowable\ throat\ stress = Resultant\ shear\ stress$$
$$77 \times 10^6\ N/m^2 = \frac{1980\ 198}{w} N/m$$
$$w = \frac{1980\ 198\ N/m}{77 \times 10^6\ N/m^2}$$
$$w = 0{,}0272\ m$$

Use, say 26 mm fillet welds.

(b)

$$l_{xx} = \frac{t \times {l_b}^3}{12}$$
$$l_{yy} = \frac{l_b \times t^3}{12}$$ (neglect very small)
$$l_p = l_{xx} + l_{yy}$$
$$= \frac{t \times {l_b}^3}{12}$$
$$= \frac{0{,}707 \times w \times (0{,}25\ m)^3}{12}$$
$$= 9{,}206 \times 10^{-4}\ w\ m^3$$

$$Direct\ shear\ stress = \frac{Shear\ load}{Total\ throat\ area\ of\ weld}$$
$$\tau_D = \frac{P}{0{,}707 \times w \times total\ effective\ length\ of\ weld}$$
$$= \frac{50 \times 10^3\ N}{0{,}707 \times w \times 0{,}25\ m}$$
$$= \frac{282\ 885}{w} N/m$$
$$r_{max} = \frac{1}{2}\sqrt{{l_a}^2 + {l_b}^2}$$
$$= \frac{1}{2}\sqrt{(0{,}25\ m)^2 + 0}$$

$$= 0{,}125\ m$$

$$\cos\theta = \frac{l_a}{\sqrt{{l_a}^2 + {l_b}^2}}$$
$$= 0$$

$$Turning\ shear = \frac{Turning\ moment \times r_{max}}{l_p}$$
$$= \frac{p \times e \times r_{max}}{Polar\ moment\ of\ inertia}$$
$$= \frac{50 \times 10^3\ N \times 0{,}25\ m \times 0{,}125\ m}{9{,}206 \times 10^{-4}\ w\ m^3}$$
$$= \frac{1697263}{w} N/m$$

$$Resultant\ shear\ stress = \sqrt{{\tau_D}^2 + {\tau_T}^2 + 2\tau_T\tau_D\cos\theta}$$
$$= \sqrt{\left[\frac{282885}{w}\right]^2 + \left[\frac{1697263}{w}\right]^2 + 0}$$
$$= \sqrt{\frac{2{,}961 \times 10^{12}}{w^2}}$$
$$= \frac{1\ 720\ 676}{w} N/m$$

$$Allowable\ throat\ stress = Resultant\ shear\ stress$$
$$77 \times 10^6\ N/m^2 = \frac{1720\ 676}{w} N/m$$
$$w = \frac{1720\ 676\ N/m}{77 \times 10^6\ N/m^2}$$
$$w = 0{,}0224\ m$$

say 23 mm fillet weld.

Activity 10.1

1. Calculate the safe load for the butt-welded mild steel tie-bars shown in the figure. Each bar is 100 mm wide x 12 mm thick. Allowable tensile stress = 90 MPa.

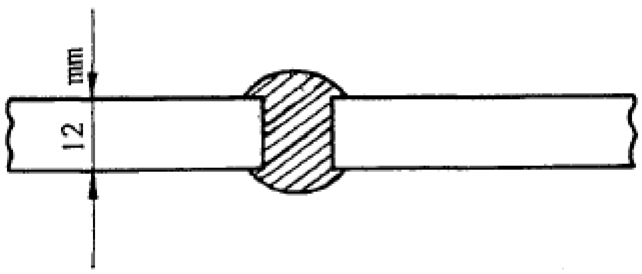


Figure 10.56

2. Find the safe value of P, in the example shown in the figure below, from the point of view of the side fillet welds. Allowable stress= 77 MPa.

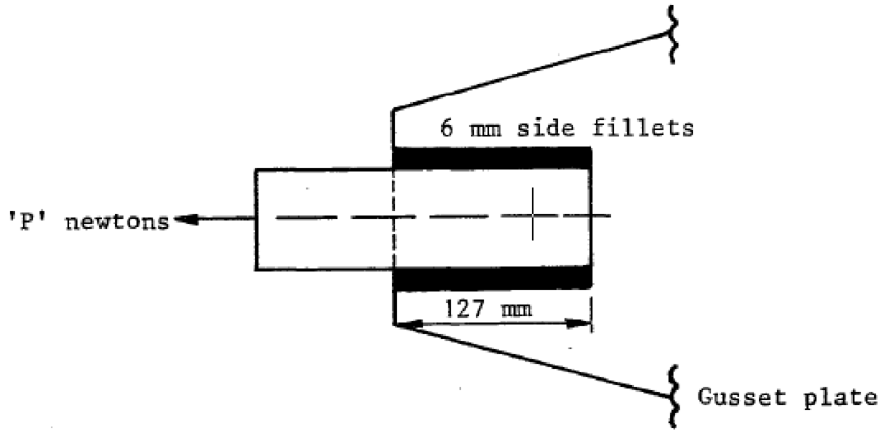


Figure 10.57

3. The figure shows a connection with two end welds both having 25 mm return side fillets. Calculate the strength of the welded joint, when the allowable stress is equal to 110 MPa.

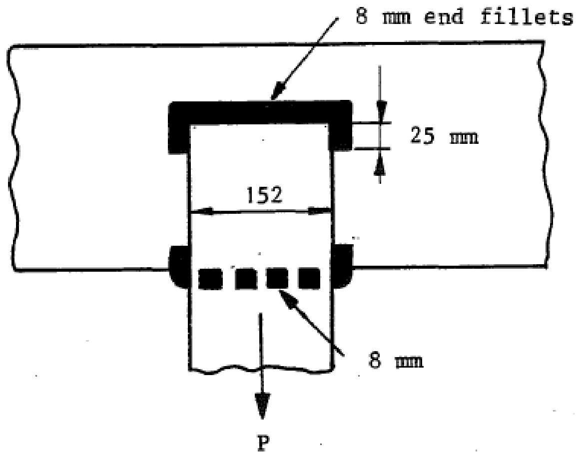


Figure 10.58

4. It is required to calculate the side weld length for the joint shown having fillet welds of 6 mm. Assume allowable stress for end welds to be 108 MPa and for side welds to be 77 MPa.

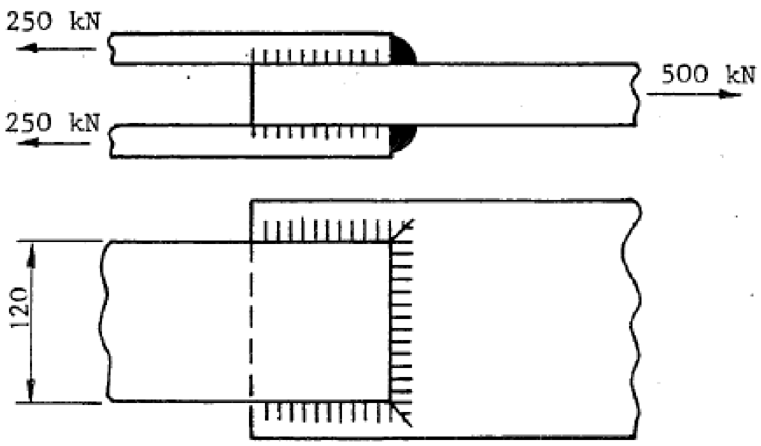


Figure 10.59

Activity 10.2

1. Determine the size of fillet weld required for the flat plate loaded as shown. Allowable shear stress equals 66 MPa.

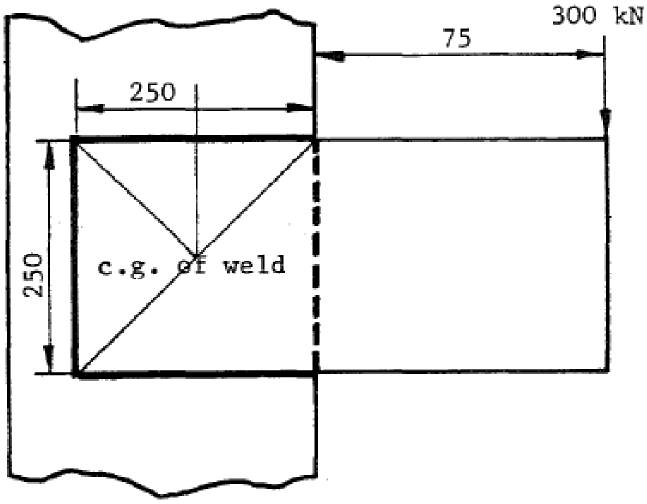


Figure 10.60

2. A channel is welded to a support. Determine the size of weld required for a steady load of 22 kN. Allowable shear stress equals 66 MPa.
3.

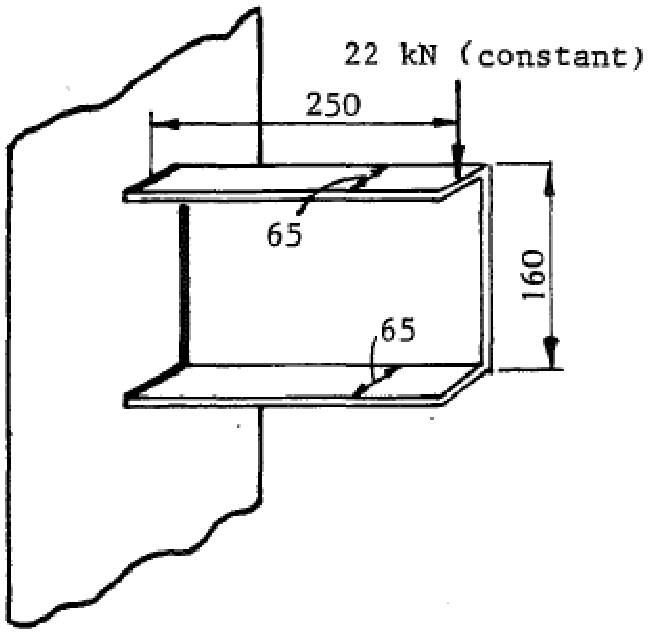


Figure 10.61

4. When welding an angle to an upright as shown, it was found that the total weld length was 140 mm. Determine the effective weld lengths of A and B.

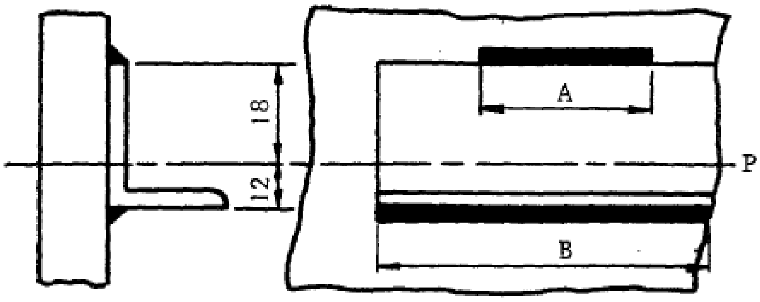


Figure 10.62

Self-Check		
I am able to:	**Yes**	**No**
• Describe the types of welding		
• Describe welding properties of materials		
• Describe the types of welded joints		
• Describe recommended proportions for weld joints		
• Describe the design of welds		
• Describe the strength of butt and fillet welded joints in simple cases of:		
o Bending		
o Tensions		
o Compression		
o Torsion		
• Describe the eccentric load parallel with the weld group		
• Describe the eccentric loading perpendicular to the plane of the weld group		
• Describe the eccentric loading in the same plane as the weld group (twisting)		
If you have answered 'no' to any of the outcomes listed above, then speak to your facilitator for guidance and further development.		

Past Examination Papers

higher education & training

Department: Higher Education and Training
REPUBLIC OF SOUTH AFRICA

August 2014

NATIONAL CERTIFICATE

MECHANICAL DRAWING AND DESIGN N5

(8090075)

4 August (Y-Paper)
13:00 – 17:00

Candidates may use text books and personal notes.

No swopping of books, notes may take place during the exam.

No communication between students may take place during the exam

Calculators and Mathematical tables may be used.

This question paper consists of 6 pages.

TIME: 4 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the information carefully.
3. Questions may be answered in any order, but subsections of questions must be kept together.
4. Number the answers according to the numbering system used in this question paper.
5. Show ALL the intermediate steps.
6. Sketches must be made FREEHAND and in good proportion in the ANSWER BOOK.
7. Where possible, standard size items must be used.
8. All the answers must be approximated accurately to THREE decimal places.
9. Write neatly and legibly.

QUESTION 1

DATA:

A lathe requires 30kW to operate at full capacity when the rotational frequency of the 80 mm diameter shaft is 200 revolutions per minute. One end of the shaft is reduced to 70 mm to provide for a spline connection with 8 splines each 6mm in depth.

The ultimate shear and crushing stress in the shaft material is 640 Mpa and 38 MPa respectively. A safety factor of 4 is used. The shear area of the splines is 45 percent greater than the crushing area of the splines.

The 80mm diameter shaft driving the lathe is driven by an open belt flat drive. Assume that no losses occur between the belt drive and the lathe. The 8 mm thick flat belt is 145 mm wide and has a maximum allowable safe stress of 160 N/cm of belt width.
The tension ratio between the belts is 2,56 :1. The power loss due to friction between the driving pulley and the driven pulley is 25W and the centre distance between the two pulleys is 1 ,2 m. The driven pulley diameter is 23o mm.

The 520 mm driving pulley is supported midway between two journal bearings on the solid shaft. The compressive stress between the shaft, and the bearings is 806 kPa.
The 600mm long shaft is driven by a flange coupling keyed to the shaft. There are no losses due to friction between the driving pulley and the flange coupling. The maximum torque on the shaft exceeds the mean 16,5 percent. The shear stress in the shaft may not exceed 127 MPa and the maximum modulus of rigidity is 82 GPa .

The flange coupling uses three M10 bolts. The maximum allowable shear stress for the bolts is 47,5 MPa. The shear stress and the compressive stress in the key is 76MPa and 192 MPa respectively.

The flange coupling is driven by a crank arm mounted on the same shaft. The crank is rotated by the connecting rod of a single cylinder steam engine which produces a constant steam pressure of 1 425 kPa. The cylinder with a diameter of 320 mm and a wall thickness of 20 mm requires15 studs to seal the cylinder. The safe allowable tensile stress for the studs is 25 MPa. Assume the core area equates to 70 per cent of the nominal area. The connecting rod is 2,37 m long and the stroke length of the crank is 127 m.

The 18 mm valve mechanism rods are connected by means of a knuckle joint. All the parts are made of the same material. The allowable tensile stress in the joint is 104,5 MPa.

Calculate the following on the spline shaft:

1.1 The torque transmitted by the spline connection (3)

1.2 The total compressive area (4)

1.3 The total shear area (2)

1.4 The length of the splines (2)

1.5 The width of the splines (3)

[14]

QUESTION 2

Calculate the following on the belt drive:

2.1 The tension in the tight side of the belt. (2)

2.2 The tension in the slack side of the belt. (2)

2.3 The power transmitted by the driving pulley. (3)

2.4 The velocity of the belt drive. (4)

2.5 The rotational frequency of the driven pulley (3)

2.6 The length of the belt. (3)

2.7 The rotational frequency of the driving pulley. (3)

[20]

QUESTION 3

Calculate the following on the shaft and the coupling:

3.1 The maximum torque in the shaft. (5)

3.2 The diameter of the shaft. (2)

3.3 The maximum angle of twist in radians (3)

[10]

QUESTION 4

Calculate the following on the bearing:

4.1 The forces acting on each bearing (3)

4.2 The required width of the bearings (3)

[6]

QUESTION 5

Calculate the following on the flange coupling:

5.1	The pitch circle diameter of the flange coupling.	(3)
5.2	The outside diameter of the boss	(1)
5.3	The length of the boss	(1)
5.4	The outside diameter of the flange	(1)
5.5	The flange thickness	(1)
5.6	The length of the key required to join the flange to the shaft	(7)

[14]

QUESTION 6

Calculate the following on the steam cover:

6.1	The force acting on the cover	(3)
6.2	The diameter of each stud	(3)
6.3	The pitch circle diameter	(3)
6.4	The circular pitch of the stud. Check for steam tightness.	(5)

[14]

QUESTION 7

Calculate when the crank arm is at right angles to the connecting rod centre line:

7.1	The magnitude of the angle the crank arm makes with the piston rod centre line	(3)
7.2	The magnitude of the angle between the connecting rod and the piston rod centre line	(1)
7.3	The magnitude of the force on the crank arm	(3)
7.4	The magnitude of the force on the cross head guides	(3)
7.5	The diameter of the piston rod if the safe compressive stress for the piston rod steel is 95 MPa	(3)

[13]

QUESTION 8

Calculate the following on the knuckle joint:

8.1 The tensile load on the knuckle joint (3)

8.2 The thickness of the eye required for tension (3)

8.3 The thickness of the fork ends (3)

[9]

TOTAL: 100

Marking Guidelines

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

AUGUST 2014

NATIONAL CERTIFICATE

MECHANICAL DRAWING AND DESIGN N5

(8090075)

This marking guideline consists of 8 pages.

QUESTION 1

Data

$P = 30kW;\ D_s = 80mm;\ N = 200r/min;\ D_{spline} = 70mm;\ n = 8;\ h_{spline} = 6mm;$
$\tau = \frac{640}{4} = 160MPa;\ \sigma_c = \frac{38}{4} = 9{,}5MPa;\ A_s = 1{,}45A_c$

1.1 For torque (3)

$$P = \frac{2\pi N T_{mean}}{60}$$

$$\therefore T_{mean} = \frac{60P}{2\pi N} = \frac{60(30\times10^3)✓}{2\pi(200)✓} = 1432{,}395Nm✓$$

1.2 For total compressive area $(T_{max} = T_{mean})$ (4)

$$T_{max} = \sigma_c A_c R_{mean}$$

where $d = D - 2h = [0{,}07 - 2(0{,}006)] = 0{,}058mm✓$

and $R_{mean} = \frac{1}{4}(D + d) = \frac{1}{4}(0{,}07 + 0{,}058) = 0{,}032mm✓$

$$\therefore A_c = \frac{T_{max}}{\sigma_c R_{mean}} = \frac{(1432{,}395)✓}{(9{,}5\times10^6)(0{,}032)} = 4{,}712\times10^{-3}m^2✓$$

1.3 For total shear area (2)

$$A_s = 1{,}45A_c = 1{,}45(4{,}712\times10^{-3})✓ = 6{,}832\times10^{-3}m^2✓$$

1.4 For length of spline (2)

$$A_c = nhL$$

$$\therefore L = \frac{A_c}{nh} = \frac{(4{,}712\times10^{-3})✓}{(8)(0{,}006)} = 0{,}098\dot{1}\dot{6}m = 99mm✓$$

1.5 For width of spline (3)

$$A_s = nwL$$

$$\therefore w = \frac{A_s}{nL} = \frac{(6{,}832\times10^{-3})✓}{(8)(0{,}099)✓} = 8{,}626\times10^{-3}m = 9mm✓$$

[14]

QUESTION 2

DATA

$d_s = 80mm;\ \ t = 8mm; w = 145mm;\ T/cm = 160N/cm\ ;\ T_1 = 2{,}56T_2; P_{loss} = 25W;$
$C = 1{,}2m; d = 23mm; D = 520mm$

2.1 For tension in tight side of belt (2)

$$T_1 = w \times {}^{T}/_{cm} = (14{,}5)\checkmark(160) \quad = 2320N\ \checkmark$$

2.2 For tension in slack side (2)

$$T_1 = 2{,}56T_2$$
$$\therefore T_2 = \frac{T_1}{2{,}56} = \frac{2320\checkmark}{2{,}56} = 906{,}25N\ \checkmark$$

2.3 The power transmitted by the driving pulley (3)

$$P_1 = P_2 + P_{loss} = (30 \times 10^3)\checkmark + (25)\checkmark = 30{,}025kW\ \checkmark$$

2.4 The velocity of the belt drive (4)

$$P_1 = (T_1 - T_2)v$$
$$\therefore v = \frac{P_1}{(T_1 - T_2)} = \frac{(30025)\checkmark}{(2320\checkmark - 906{,}25\checkmark)} = 21{,}238m/s\ \checkmark$$

2.5 The rotational frequency of the driven pulley (3)

$$v = \frac{\pi(d+t)n}{60}$$
$$\therefore n = \frac{60v}{\pi(d+t)} = \frac{60(21{,}238)\checkmark}{\pi(0{,}23+0{,}008)\checkmark} = 1704{,}269r/min\ \checkmark$$

2.6 The length of the belt (3)

$$L = \frac{\pi}{2}(D + d + 2t) + \frac{(D-d)^2}{4(C)} + 2(C)$$
$$L = \frac{\pi}{2}[0{,}52 + 0{,}23 + 2(0{,}008)\checkmark] + \frac{(0{,}52-0{,}23)^2\checkmark}{4(1{,}2)} + 2(1{,}2)$$
$$L = 3{,}621m\ \checkmark$$

2.7 The rotational frequency of the driving pulley (3)

$$v = \frac{\pi(D+t)N}{60}$$

$$\therefore N = \frac{60v}{\pi(D+t)} = \frac{60(21{,}238)✓}{\pi(0{,}52+0{,}008)✓} = 768{,}212r/min ✓$$

[20]

QUESTION 3

DATA

$L = 600mm;\ T_{max} = 1{,}165T_{mean};\ \tau_s = 127MPa; G = 82GPa$

3.1 For maximum torque on shaft (5)

$$P = \frac{2\pi N T_{mean}}{60}$$

$$\therefore T_{mean} = \frac{60P}{2\pi N} = \frac{60(30{,}025\times10^3)✓}{2\pi(768{,}212)✓} = 373{,}227Nm ✓$$

$$\therefore T_{max} = 1,xT_{mean} = (1{,}165)(373{,}227)✓ = 434{,}809Nm ✓$$

3.2 For shaft diameter (2)

$$T_{max} = \frac{\pi}{16}D^3\tau$$

$$\therefore D = \sqrt[3]{\frac{16T_{max}}{\pi\tau}} = \sqrt[3]{\frac{16(434{,}809)✓}{\pi(127\times10^6)}} = 0{,}025931m = 26mm ✓(std\ size)$$

3.3 For maximum angle of twist (3)

$$\theta = \frac{10{,}2T_{max}L}{GD^4} = \frac{(10.2)(434{,}809)✓(0{,}6)}{(82\times10^9)(0{,}026)^4✓} = 0{,}071\ radians ✓$$

[10]

QUESTION 4

DATA

$d_s = 26mm;\ \sigma_c = 806MPa$

4.1 For forces acting on each bearing (3)

$$F_c = T_1 + T_2 = 2320 + 906{,}25 = 3226{,}25N ✓$$
$$F_{bearing} = \frac{1}{2}F_c = \frac{1}{2}(3226{,}25) ✓ = 1613{,}125N ✓$$

4.2 For width of each bearing (3)

$$F = wd_s\sigma_c$$
$$\therefore w = \frac{F}{d_s\sigma_c} = \frac{(1613{,}125) ✓}{(0{,}026) ✓ (806\times10^3)} = 0{,}0769767m = 77mm ✓$$

[6]

QUESTION 5

DATA

$n = 3;\ d = M10;\ \tau_{bolts} = 47{,}5MPa;\ \tau_{key} = 76MPa;\ \sigma_{c_{key}} = 192MPa$

5.1 For pitch circle diameter of flange coupling (3)

$$T_{max} = T_{bolts} = \frac{n\pi}{8}d^2 PCD\tau_{bolt}$$
$$\therefore PCD = \frac{8T_{max}}{n\pi d^2\tau_b} = \frac{8(434{,}809) ✓}{(3\pi)(0{,}01)^2(47{,}5\times10^6) ✓} = 0{,}0777004m = 78mm ✓$$

5.2 For outside diameter of boss (1)

$$D_1 = 2D = 2(26) = 52mm ✓$$

5.3 For length of boss (1)

$$L_1 = 1{,}3D + 2mm = 1{,}3(26) + 2mm = 35{,}8mm = 36mm ✓$$

5.4 For outside diameter of flange (1)

$$D_2 = 4{,}5D = 4{,}5(26) = 117mm ✓$$

OR

$$D_2 = PCD + 3d = 78 + 3(10) = 108mm$$

5.5 For flange thickness (1)

$$t_1 = \frac{1}{4}D = \frac{1}{4}(26) = 6{,}5mm = 7mm \checkmark$$

5.6 For length of key required (7)

$$w \times t = 8 \times 7mm\ (std\ key\ size\ according\ to\ shaft\ size)$$

Consider compression

$$T_{max} = T_{key} = \frac{1}{4}tLD\sigma_c$$

$$L = \frac{4T_{max}}{tD\sigma_c} = \frac{4(434{,}809)\checkmark}{(0{,}007)\checkmark(0{,}026)(192\times10^6)} = 0{,}049772m = 50mm \checkmark$$

Consider shearing

$$T_{max} = T_{key} = \frac{1}{2}wLD\tau$$

$$L = \frac{2T_{max}}{WD\tau} = \frac{2(434{,}809)\checkmark}{(0{,}008)\checkmark(0{,}026)(76\times10^6)} = 0{,}055011m = 55mm \checkmark$$

$$Use\ key\ w \times t \times L = 8 \times 7 \times 55mm \checkmark$$

[14]

QUESTION 6

DATA

$\wp = 1425kPa; D = 320m;\ t = 20mm; n = 15;\ \sigma_{t\ studs} = 25MPa; A_c = 0{,}7A_n$

6.1 For force acting on cover (3)

$$F = \frac{\pi}{4}D^2\wp = \frac{\pi}{4}(0{,}32)^2\checkmark(1425\times10^3)\checkmark = 114605{,}3N = 114{,}605kN \checkmark$$

6.2 For diameter of each stud (3)

$$F = F_s = 0{,}7n\left(\frac{\pi}{4}d^2\right)\sigma_t$$

$$\therefore d = \sqrt[2]{\frac{4F}{0{,}7n\pi\sigma_t}} = \sqrt[2]{\frac{4(114{,}605\times10^3)\checkmark}{(0{,}7)\checkmark(15\pi)(25\times10^6)}} = 0{,}023577m = 24mm\checkmark$$

6.3 For pitch circle diameter (3)

$$PCD = D + 2t + d = 320 + 2(20)\checkmark + 24\checkmark = 384mm \checkmark$$

6.4 For circular pitch. Check for steam tightness. (5)

$CP = \frac{\pi PCD}{n} = \frac{\pi(384)}{(15)} = 80{,}425mm$ ✓

$\wp = 1425kPa\ falls\ above\ 1215kPa\ \therefore 3d < CP < 5d$

$3d = 3(24) = 72mm$ ✓
$5d = 5(24) = 120mm$ ✓

$\therefore Cp\ falls\ between\ 3d\ \&\ 5d.\ Yes, steam\ tight!$ ✓

[14]

QUESTION 7

DATA

$L = 2{,}37m; r = \frac{1{,}27}{2} = 0{,}635m;\ \sigma_{c\ piston} = 95MPa$

7.1 For magnitude of angle between crank arm and piston rod centre line (3)

$tan\alpha = \frac{L}{r}$

$\therefore \alpha = \tan^{-1}\frac{L}{r} = \tan^{-1}\left(\frac{2{,}37}{0{,}635}\right) = 75°$ ✓

7.2 For magnitude of angle between connecting rod and piston rod centre line (1)

$\beta = 90° - \alpha = 90° - 75° = 15°$ ✓

7.3 For force on crank pin (3)

$cos\theta = \frac{F}{k}$

$\therefore k = \frac{F}{cos\theta} = \frac{(114{,}605 \times 10^3)}{(\cos 15°)} = 118{,}648kN$ ✓

7.4 For force on cross head guide (3)

$tan\theta = \frac{R}{F}$

$\therefore R = F\ tan\theta = (114{,}605 \times 10^3)(tan15°) = 30{,}708kN$ ✓

7.5 For diameter of piston rod (3)

$$F = \frac{\pi}{4} d^2 \sigma_c$$

$$\therefore d = \sqrt[2]{\frac{4F}{\pi \sigma_c}} = \sqrt[2]{\frac{4(114{,}605 \times 10^3)✓}{\pi(95 \times 10^6)✓}} = 0{,}039192m = 40mm ✓ (std\ size)$$

[13]

QUESTION 8

DATA

$d = 18mm;\ \sigma_t = 104{,}5MPa$

8.1 For tensile load on the knuckle joint (3)

$$F_t = \frac{\pi}{4} d^2 \sigma_t = \frac{\pi}{4}(0{,}018^2)✓(104{,}5 \times 10^6)✓ = 26{,}592kN ✓$$

8.2 For thickness of eye required for tension (3)

$$F_t = (d_2 - d_1)t\sigma_t$$

$$where\ d_2 = 2d_1 = 2(18) = 36mm ✓$$

$$\therefore t = \frac{F}{(d_2 - d_1)\sigma_t} = \frac{(26{,}592 \times 10^3)}{[(0{,}036) - (0{,}018)✓](104{,}5 \times 10^6)} = 0{,}0141376m = 14\ or\ 15mm ✓$$

8.3 For thickness of the fork ends (3)

$$F_t = 2(d_2 - d_1)t_1\sigma_t$$

$$\therefore t_1 = \frac{F_t}{2(d_2 - d_1)\sigma_t} = \frac{(26{,}592 \times 10^3)}{2[(0{,}036)✓ - (0{,}018)✓](104{,}5 \times 10^6)} = 7{,}06858 \times 10^{-3}m = 7mm ✓$$

[9]

TOTAL: 100

Past Examination Papers

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

April 2014

NATIONAL CERTIFICATE

MECHANICAL DRAWING AND DESIGN N5

(8090075)

7 April (Y-Paper)
13:00 – 17:00

No exchange of books or notes, or communication between candidates, will be permitted during the examination .

Calculators and mathematical tables may be used.

This question paper consists of 6 pages.

TIME: 4 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the information and the questions carefully.
3. Questions may be answered in any order, but subsections of questions must be kept together.
4. Number the answers according to the numbering system used in this question paper.
5. Show ALL the intermediate steps.
6. Sketches must be made FREEHAND and in good proportion in the ANSWER BOOK.
7. Where possible, standard size items must be used.
8. All the answers must be approximated accurately to THREE decimal places.
9. Write neatly and legibly.

THE FOLLOWING DATA IS APPLICABLE TO ALL THE QUESTIONS.

Two rods of a valve mechanism in a steam engine are connected by means of a knuckle joint. The rods have the same diameter and all the parts are made of the same material. The joint has to withstand a load of 55 kN.

The following allowable stresses apply to the knuckle joint.

Tensile stress	112 MPa
Shear stress	63 MPa
Compressive stress	135 MPa

The engine has a cylinder diameter of 385 mm and produces a maximum steam pressure of 1 367 kPa. The crank arm is making an angle of 90° with the piston rod centre line and the piston rod is in compression. The stroke length is 466 mm and the connecting rod length is 960 mm.

The crank arm on the steam engine drives a shaft on which a flange coupling is mounted. The flange coupling is designed to transmit 480 kW at 294 revolutions per minute. The pitch circle diameter is equivalent to three times the shaft diameter.

The following ultimate stresses are used on the coupling:

Shear stress in shaft	465 MPa
Shear stress in bolts	486 MPa
Shear stress in key	381 MPa
Compressive stress in key	882 MPa

A safety factor of three must be used for the flange coupling.

A belt pulley, 360 mm in diameter is mounted on and driven by a 0,8 m long hollow shaft from the flange coupling. There is a 5 percent power loss between the flange coupling and the driving pulley.

The driving pulley drives a 280 mm pulley by means of a crossed flat belt. The driving pulley revolves at 281 revolutions per minute and the coefficient of friction is 0,4. The pulley centres are 1,2 m apart. The 15 mm thick belt can carry a tensile load of 105 kN and the tensile stress in the belt material is 26 MPa.

The driven pulley is mounted on a 65 mm diameter shaft which is supported by two bearings 1 m apart.

The driven pulley is mounted two thirds of the distance from the right hand bearing. The compressive stress between the shaft and the bearing is 19,3 MPa.

The driven pulley drives a compressor which delivers air at a pressure of 1 420 kPa into a storage tank. The storage tank has a longitudinal double riveted butt joint with

one cover plate. The stress in the joint may not exceed 60 MPa. The wall thickness is 18 mm. The longitudinal efficiency of the joint is 26,3 per cent The pitch of the rivets is48 mm.
The ultimate stresses for the rivets and plate are as follows:

Tensile stress	400 MPa
Shear stress	320 MPa

Use a safety factor of four for these stresses.

QUESTION 1

Calculate the following on the knuckle joint

1.1 The diameter of the two round valve mechanism rods (3)

1.2 The diameter of the pin (3)

1.3 The thickness of the eye (6)

1.4 The thickness of the fork ends (5)

1.5 The thickness of the curved section of the fork (2)

1.6 The diameter of the pin head and collar (1)

[20]

QUESTION 2

Calculate the following on the steam engines

2.1 The force on the piston rod (3)

2.2 The force on the connecting rod (6)

2.3 The force on the cross head guide (2)

2.4 The force on the crank pin (1)

[12]

QUESTION 3

Calculate the following on the flange coupling.

3.1 The shaft diameter (4)

3.2 The length of the key (7)

3.3 The number and the diameter of the bolts required (3)

[14]

QUESTION 4

The shaft driving the coupling must be replaced by a hollow shaft which has the same outside diameter as the solid shaft. The allowable shear stress in the hollow shaft exceeds that of the solid shaft by 15 per cent.

Calculate the following on the shaft:

4.1 The internal diameter of the hollow shaft (3)

4.2 The percentage saving in weight if the hollow shaft is used (2)

4.3 The power transmitted by the hollow shaft (2)

4.4 The maximum angle of twist in degrees for the hollow shaft between the coupling and the pulley, if the modulus of rigidity is 85 GPa and the torsion in the hollow is equal to that of the solid shaft. (3)

[10]

QUESTION 5

Calculate the following on the belt drive:

5.1 The contact angle on the driving pulley (4)

5.2 The width of the belt (3)

5.3 The length of the belt (3)

5.4 The tension in the slack side of the belt (4)

5.5 The velocity of the belt (2)

5.6 The power transmitted by the belt (3)

[19]

QUESTION 6

Calculate the following on the bearings:

6.1 The force acting on each bearing due to the forces in the belt (5)

6.2 The width of the bearing (3)

[8]

QUESTION 7

Calculate the following on the storage tank:

7.1	The diameter of the storage tank	(3)
7.2	The diameter of the rivets (Equate □□ = □□)	(4)
7.3	The tearing strength of the plate between the rivets	(2)
7.4	The strength of the solid plate	(2)
7.5	The shear strength of the rivets	(2)
7.6	The tearing efficiency of the joint	(3)
7.7	The thickness of the cover plate	(1)

[17]

TOTAL: 100

Marking Guidelines

higher education & training
Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

APRIL 2014

NATIONAL CERTIFICATE

MECHANICAL DRAWING AND DESIGN N5

(8090075)

This marking guideline consists of 10 pages.

QUESTION 1

Data:

$F = 55kN;\ \sigma_t = 112MPa;\ \tau = 63MPa;\ \sigma_c = 135MPa$

1.1 For diameter of the valve mechanism rods (3)

$$F_t = \frac{1}{4}\pi d^2 \sigma_t$$

$$\therefore d = \sqrt[2]{\frac{4F_t}{\pi\sigma_t}} = \sqrt[2]{\frac{4(55\times10^3)✓}{\pi(112\times10^6)✓}} = 0{,}025005m = 25mm✓\ (std\ size)$$

1.2 For diameter of pin (3)

$$F = 2\left(\frac{\pi}{4}d_1^2\right)\tau$$

$$\therefore d_1 = \sqrt[2]{\frac{4F}{2\pi\tau}} = \sqrt[2]{\frac{4(55\times10^3)✓}{2\pi(63\times10^6)✓}} = 0{,}02357496m = 24mm$$

$$but\ d = d_1 = 25mm✓$$

1.3 For thickness of the eye (6)

Consider tearing

$$d_2 = 2d_1 = 2(25) = 50mm$$

$$F = (d_2 - d_1)t\sigma_t$$

$$\therefore t = \frac{F}{(d_2 - d_1)\sigma_t} = \frac{(55\times10^3)}{(0.05✓ - 0.025✓)(112\times10^6)} = 0{,}0196m = 20mm✓$$

Consider crushing

$$F = d_1 t\sigma_c$$

$$\therefore t = \frac{F}{d_1\sigma_c} = \frac{(55\times10^3)}{(0{,}025)✓(135\times10^6)} = 0{,}01629m = 17mm✓$$

$$Use\ t = 20mm✓$$

1.4 For thickness of the fork ends (5)

Consider tearing

$F = 2(d_2 - d_1)t_1\sigma_t$

$\therefore t_1 = \frac{F}{2(d_2-d_1)\sigma_t} = \frac{(55\times10^3)}{2(0,05-0,025✓)(112\times10^6)} = 9,821 \times 10^{-3}m = 10mm$✓

Consider crushing

$F = 2d_1t_1\sigma_c$

$\therefore t_1 = \frac{F}{2d_1\sigma_c} = \frac{(55\times10^3)}{2(0,025)✓(135\times10^6)} = 8,148148 \times 10^{-3}m = 8mm \text{ or } 9mm$✓

$Use\ t_1 = 10mm$✓

1.5 For thickness of curved section of fork (2)

$b = 1,2d_1 = 1,2(25) = 30mm$

$F = 2bt_2\sigma_t$

$\therefore t_2 = \frac{F}{2b\sigma_t} = \frac{(55\times10^3)}{2(0,03)✓(112\times10^6)} = 8,1845 \times 10^{-3}m = 9mm$✓

1.6 For diameter of pin head and collar (1)

$d_3 = 1,5d_1 = 1,5(25) = 37,5 = 38mm$✓

[20]

QUESTION 2

Data:

$D = 385\ mm;\ \ P = 1\,367\ kPa;\ \ L = 960\ mm;\ \ r = \frac{1}{2}(466) = 0,233mm;$

2.1 For force on piston rod (3)

$F = \frac{\pi}{4}D^2 P = \frac{\pi}{4}(0,385^2)✓(1367 \times 10^3)✓ = 159140,1837N = 159,14kN$✓

2.2 For force on connecting rod. (6)

$$\sin\theta = \frac{r}{L}$$

$$\theta = \sin^{-1}\frac{(0{,}233)✓}{(0{,}96)✓} = 14{,}046^\circ ✓$$

$$\cos\theta = \frac{F}{k}$$

$$k = \frac{F}{\cos\theta} = \frac{(159{,}14\times10^3)✓}{(\cos 14{,}046^\circ)✓} = 164044{,}743N = 164{,}045kN ✓$$

2.3 For force on cross head guide (2)

$$\tan\theta = \frac{R}{F}$$

$$R = F\tan\theta = (159{,}14\times10^3)\cdot(\tan 14{,}046^\circ)✓ = 39813{,}794N = 39{,}814kN✓$$

2.4 For force on crank pin (1)

$$F_{crank\ pin} = F_{connecting\ rod} = 164{,}045kN✓$$

[12]

QUESTION 3

Data:

$$P = 480kW; N = \frac{294r}{min}; PCD = 3D; SF = 3; \tau_{shaft} = \frac{465}{3} = 155MPa;$$

$$\tau_{bolts} = \frac{486}{3} = 162MPa; \tau_{key} = \frac{381}{3} = 127MPa; \sigma_{ckey} = \frac{882}{3} = 294MPa$$

3.1 For shaft diameter (4)

$$P = \frac{2\pi N T_{mean}}{60}$$

$$\therefore T_{mean} = \frac{60P}{2\pi N} = \frac{60(480\times10^3)}{2\pi(294)✓} = 15590{,}688Nm✓$$

$$\therefore T_{max} = T_{mean}\ (no\ overload)$$

$$T_{max} = \frac{\pi}{16}d^3\tau$$

$$\therefore d = \sqrt[3]{\frac{16T_{max}}{\pi\tau}} = \sqrt[3]{\frac{16(15590{,}688)✓}{\pi(155\times10^6)}} = 0{,}08001m$$

$$Use\ d = 80mm✓(std\ size)$$

3.2 For length of key (7)

$w \times t = 22 \times 14mm$

Consider shearing:

$T_{max} = T_{key} = \frac{1}{2} wDL\tau$

$\therefore L = \frac{2T_{max}}{wD\tau} = \frac{2(15590{,}688)^{\checkmark}}{(0{,}022)^{\checkmark}(0{,}08)(127 \times 10^6)} = 0{,}1395015m = 140mm^{\checkmark}$

Consider crushing

$T_{max} = T_{key} = \frac{1}{4} tDL\sigma_c$

$L = \frac{4T_{max}}{tD\sigma_c} = \frac{4(15590{,}688)^{\checkmark}}{(0{,}014)^{\checkmark}(0{,}08)(254 \times 10^6)} = 0{,}1893912m = 190mm^{\checkmark}$

$Use key size w \times t \times L: 22 \times 14 \times 190mm^{\checkmark}$

3.3 For number and diameter of bolts (3)

Select n = 4 bolts (std number for shaft diameter between 45mm and 102mm)

Consider shearing

$T_{max} = \frac{n\pi}{8} d^2 PCD\tau$

$\therefore d = \sqrt[2]{\frac{8T_{max}}{n\pi PCD\tau}} = \sqrt[2]{\frac{8(15590{,}688)}{(4)^{\checkmark}\pi(3 \times 0{,}08)^{\checkmark}(162 \times 10^6)}} = 0{,}0159775m$

$Use\ d = 16mm^{\checkmark}(std\ size)$

[14]

QUESTION 4

Data:

$d_s = 80\ mm; D = 80mm;\ \tau_h = 1{,}15\tau_s$

4.1 For internal diameter of hollow shaft (3)

$$T_{solid} = T_{hollow}$$
$$\frac{\pi}{16} {d_s}^3 \tau_s = \frac{\pi}{16}\left(\frac{D^4 - {d_h}^4}{D}\right)\tau_h$$
$$D d^3 \tau_s = \left(D^4 - {d_h}^4\right)(1{,}15\tau_s)$$
$$\therefore d_h = \sqrt[4]{D^4 - \frac{d^3 D}{1{,}15}}$$
$$d_h = \sqrt[4]{(0{,}08✓)^4 - \frac{(0{,}08)^3(0{,}08)}{1{,}15✓}} = 0{,}048077m$$

4.2 For percentage saving in weight (2)

$$\%_{saving} = \frac{A_s - A_h}{A_s} \times 100$$
$$\%_{saving} = \frac{(d_s)^2 - [(D)^2 - (d_h)^2]}{(d_s)^2} \times 100$$
$$\%_{saving} = \frac{(0{,}048^2)✓}{(0{,}08^2)}$$
$$\therefore \%_{saving} = 36\%✓$$

4.3 For power transmitted by hollow shaft (2)

$$P_h = P_f - P_{loss} = (480 \times 10^3) - [(480 \times 10^3)(5\%)]$$
$$P_h = (480 \times 10^3) - (24 \times 10^3)✓$$
$$\therefore P_h = 456kW✓$$

4.4 For maximum angle of twist in degrees (3)

$$\theta = \frac{584 T_{max} L}{G(D^4 - {d_h}^4)}$$
$$\theta = \frac{584(15590{,}688)(0{,}8)✓}{(85\times10^9)[(0{,}08^4)-(0{,}048^4)✓]} = 2{,}403645°$$
$$\therefore \theta = 2{,}404°✓$$

[10]

QUESTION 5

Data:

Driving pulley: $D = 360mm;\ N = 281\ r/min;\ \mu = 0{,}4$

Driven pulley: $d = 280mm$

Belt: $C = 1{,}2m;\ t = 15mm;\ T_1 = 105kN;\ \sigma_t = 26MPa$

5.1 For contact angle of the driving pulley (4)

$$\sin\beta = \frac{(D+d)}{2C}$$

$$\therefore \beta = \sin^{-1}\left[\frac{(D+d)}{2C}\right] = \sin^{-1}\left[\frac{(0{,}36+0{,}28)^{\checkmark}}{2(1{,}2)}\right] = 15{,}466^{\circ}\checkmark$$

$$\theta = 180^{\circ} + 2\beta = 180^{\circ} + 2(15{,}466^{\circ})^{\checkmark} = 210{,}932^{\circ}\checkmark$$

5.2 For belt width (3)

$$T_1 = wt\sigma_t$$

$$\therefore w = \frac{T_1}{t\sigma_t} = \frac{(105\times10^3)^{\checkmark}}{(0{,}015)(26\times10^6)^{\checkmark}} = 0{,}26923m = 270mm\checkmark$$

5.3 For length of the belt (3)

$$L = \frac{\pi}{2}(D+d+2t) + \frac{(D+d+2t)^2}{4C} + 2C$$

$$L = \frac{\pi}{2}[0{,}36+0{,}28+2(0{,}015^{\checkmark})] + \frac{[0{,}36+0{,}28+2(0{,}015^{\checkmark})]^2}{4(1{,}2)} + 2(1{,}2)$$

$$L = 1{,}052433539 + 0{,}093521 + 2{,}4$$

$$\therefore L = 3{,}546m\checkmark$$

5.4 For tension in the slack side (4)

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{(0{,}4)\left(210{,}932^{\circ\checkmark}\times\frac{\pi}{180^{\circ}}\right)} = 4{,}36\checkmark$$

$$T_1 = 4{,}36T_2$$

$$\therefore T_2 = \frac{T_1}{4{,}36} = \frac{(105\times10^3)}{4{,}36^{\checkmark}} = 24082{,}5688N = 24{,}083kN\checkmark$$

5.5 For belt speed (2)

$$v = \frac{\pi(D+t)N}{60} = \frac{\pi(0{,}36+0{,}015^{\checkmark})(281)}{60} = 5{,}517m/s\checkmark$$

5.6 For power transmitted (3)

$$P = (T_1 - T_2)v = [(105\times10^3) - (24{,}083\times10^3)^{\checkmark}](5{,}517)^{\checkmark} = 446419{,}089W$$

$$\therefore P = 446{,}419kW\checkmark$$

[19]

QUESTION 6

Data:

$D = 65mm; L = 1m; \sigma_c = 19{,}3MPa$

6.1 For forces on each bearing (5)

$F_t = T_1 + T_2 = (105 \times 10^3) + (24{,}083 \times 10^3) = 129{,}083kN$ ✓

For left hand bearing: take moments about right hand bearing

$$\Sigma CWM = \Sigma ACWM$$
$$R_A \times L = F_t \times \frac{2}{3}L$$
$$\therefore R_A = \frac{2}{3}F_t = \left(\frac{2}{3}\right)✓ (129{,}083 \times 10^3) = 86{,}055kN ✓$$

For right hand bearing: take moments about left hand bearing

$$\Sigma CWM = \Sigma ACWM$$
$$F_t \times \frac{2}{3}L = R_B \times L$$
$$\therefore R_B = \frac{1}{3}F_t = \left(\frac{1}{3}\right)✓ (129{,}083 \times 10^3) = 43{,}028kN ✓$$

6.2 For width of bearing (3)

$$F = wd_s\sigma_c$$

$$w = \frac{F}{d_s\sigma_c}$$
$$w = \frac{(129{,}083 \times 10^3)✓}{(0{,}065)✓(19{,}3 \times 10^6)}$$
$$w = 0{,}1028959745m$$
$$\therefore w = 103mm ✓$$

OR

Length of each bearing

$$LH_{Bearing} = \frac{86\,x\,10^3}{0{,}065\,x\,19{,}3\,10^6}$$
$$= 68{,}55\ mm$$

$$RH = \frac{43\,x\,10^3}{0{,}065\,x\,19{,}3\,10^6}$$
$$= 34{,}27\ mm$$

[8]

QUESTION 7

Data:

$\wp = 1420kPa;\ \sigma_c = 60MPa; t = 18mm; n = 2;\ \eta_L = 26{,}3\%; P = 48mm$

$Rivets: SF = 4;\ \sigma_t = \frac{400}{4} = 100MPa;\ \tau = \frac{320}{4} = 80MPa;$

7.1 For diameter of storage tank (3)

$$\sigma_c = \frac{\wp D}{2t\eta_L}$$
$$\therefore D = \frac{2t\eta_L\sigma_c}{\wp} = \frac{2(0{,}018)(0{,}263)\checkmark(60\times10^6)}{(1420\times10^3)\checkmark} = 0{,}400056m = 400mm\checkmark$$

7.2 For diameter of rivets (4)

$$F_t = F_s$$
$$\sigma_t(P-d)t = n\left(\frac{\pi}{4}d^2\right)\tau$$
$$(100\times10^6)(0{,}048\checkmark - d)(0{,}018\checkmark) = 2\checkmark\left(\frac{\pi}{4}\right)d^2(80\times10^6)$$
$$0{,}048 - d = 69{,}813d^2$$
$$69{,}813d^2 + d - 0{,}048 = 0$$
$$d^2 + 0{,}0143d - 6{,}8755\times10^{-4} = 0$$
$$d = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(0{,}0143)\pm\sqrt{(0{,}0143)^2-4(1)(-6{,}8755\times10^{-4})}}{2(1)}$$
$$d = \frac{-(0{,}0143)\ \pm(0{,}054357)}{2} = 0{,}0200285m$$
$$Use\ d = 20mm\checkmark(std\ size)$$

7.3 For tearing strength of plate between rivets (2)

$$F_t = \sigma_t(P-d)t = (100\times10^6)(0{,}048-0{,}02✓)(0{,}018) = 50{,}4kN✓$$

7.4 For strength of solid plate (2)

$$F_{solid} = Pt\sigma_t = (0{,}048)(0{,}018)✓(100\times10^6) = 86{,}4kN✓$$

7.5 For shear strength of rivets (2)

$$F_s = n\frac{\pi}{4}(d^2)\tau = (2)\frac{\pi}{4}(0{,}02)^2✓(80\times10^6) = 50{,}265kN✓$$

7.6 For tearing efficiency of the joint (3)

$$\eta = \frac{F_t}{F_{solid}}\times100 = \left(\frac{50{,}265\ x10^3}{86{,}4x10^6}\right)x\ 100 = 58{,}33\%$$

7.7 For thickness of the cover plate (1)

$$t_1 = \frac{5}{8}(plate\ thickness) = \frac{5}{8}(18) = 11{,}25mm$$
$$\therefore t_1 = 12mm✓(std\ size)$$

[17]

TOTAL: 100

Past Examination Papers

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

APRIL 2013

NATIONAL CERTIFICATE

MECHANICAL DRAWING AND DESIGN N5

(8090075)

2 April (X-Paper)
09:00 – 13:00

REQUIREMENTS:

Candidates may use textbooks and personal notes.

Calculators and mathematical tables may be used.

No exchange of notes, books or communication between candidates will be allowed during the examination.

OPEN-BOOK EXAMINATION

This question paper consists of 5 pages.

TIME: 4 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Questions may be answered in any order, but subsections of questions must be kept together.
4. Number the answers according to the numbering system used in this question paper.
5. Show ALL the intermediate steps.
6. Sketches must be made FREEHAND and in good proportion in the ANSWER BOOK.
7. All the answers must be approximated to THREE decimal places.
8. Write neatly and legibly.

QUESTION 1

1.1 A crossed flat belt drive is used to transmit power. The driving pulley is 320 mm in diameter and rotates at 680 revolutions per minute and the coefficient of friction is 0,4. The driven pulley is 600 mm in diameter and the centres of the pulleys are 900 mm apart. The drive belt can withstand a 7 kN tensile load and is 1 0 mm thick. The allowable tensile stress in the belt material is 4,5 MPa.

Calculate:

1.1.1 The contact angle of the driving pulley (3)

1.1.2 The width of the belt (2)

1.1.3 The total length of the belt (3)

1.1.4 The power transmitted by the belt (7)

1.2 The 600 mm diameter driven pulley in QUESTION 1.1 is mounted on a shaft between two bearings and is positioned 400 mm from the left-hand bearing and 600 mm from the right-hand bearing. Calculate the length of the bearing if the compressive pressure between the 80 mm diameter shaft and the left hand bearing is 2 MPa. (5)

[20]

QUESTION 2

2.1 A hollow shaft with a diameter ratio of 2 : 1 transmits 650 kW at 200 revolutions per minute. The maximum torque exceeds the mean torque by 15 per cent. The angle of twist is 2,5 degrees over the shaft length of 600 mm and the modulus of rigidity is 89 GPa. (9)

Calculate the external diameter and the internal diameter of the shaft.

2.2 Compare the strength of the hollow shaft mentioned in QUESTION 2.1 with the strength of a solid shaft with the same mass and length and made of the same material. (5)

2.3 A splined connection in a vehicle transmission consists of 1 0 splines cut into a 60 mm diameter shaft. The height of each spline is 5 mm, the splines in the hub are 70 mm long and the allowable pressure on the splines is 6 MPa.

Calculate:

2.3.1 The power that can be transmitted at 2 750 revolutions per minute. (4)

2.3.2 The force required to slide the hub axially, under full load, if the coefficient of friction is 0,3. (2)

[20]

QUESTION 3

A boiler shell is to be made of plates 15 mm thick and is to withstand a steam pressure of 1,2 MPa. The efficiencies of the longitudinal and circumferential joints are 70% and 30% respectively. The joint is riveted into place with a double riveted butt joint. The tensile stress is 105 MPa, the shear stress is 75 MPa and the crushing stress is 130 MPa. A rivet in double shear is 1,75 times as strong as a rivet in single shear.

Calculate:

3.1 The maximum permissible diameter of the boiler (4)

3.2 The pitch of the rivets if [illegible]=6[illegible] (6)

3.3 The safe working load for the joints (6)

[16]

QUESTION 4

A rectangular steam chest opening, 350 mm by 310 mm, is to be closed by means of a flat cast-iron cover. The jointing material may be considered to extend to the bolts and to be subjected to the same pressure. The steam pressure is 1,5 MN/m² and the allowable stress for the bolts is 40 MPa. The steam chest walls are 23 mm thick. Use M22 bolts.

Calculate:

4.1 The effective area of the jointing material (5)

4.2 The force exerted by the steam on the cover (2)

4.3 The number of bolts if the core area= 0,8 nominal area (3)

4.4 The pitch of the bolts (4)

4.5 Whether the cover will be steam tight (2)

4.6 The total length and breadth of the cover (4)

[20]

QUESTION 5

5.1 A knuckle joint subjected to a load of 80 kN connects two 40 mm diameter rods. The pin is also 40 mm in diameter and the outside diameter of the eye and the fork is 80 mm. The eye thickness is 30 mm and the fork thickness is 15 mm. A safety factor of 5 is used and all the parts are made of the same material.

Calculate:

5.1.1 The ultimate tensile stress (4)

5.1.2 The ultimate shear stress (2)

5.1.3 The ultimate compressive stress (2)

5.2 Sketch freehand THREE views of the knuckle joint mentioned in QUESTION 5.1 and insert dimensions. (6)

5.3 Explain the purpose of knuckle joints and state THREE applications of this type of joint. (4)

[18]

QUESTION 6

A tie plate is welded to a gusset plate by means of two 14 mm side angle fillets, each 150 mm long. The allowable stress is 80 MPa.

6.1 Determine the safe load for this weld if no return welds are used. (4)

6.2 Make a neat freehand sketch of this weld and insert the appropriate welding symbol. (2)

[6]

TOTAL: 100

Marking Guidelines

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

APRIL 2013

NATIONAL CERTIFICATE

MECHANICAL DRAWING AND DESIGN N5

(8090075)

This marking guideline consists of 9 pages.

QUESTION 1

Data:

Driving pulley: $D_1 = 320mm;\ N_1 = 680r/min$

Driven pulley: $D_2 = 600mm$

$\mu = 0{,}4; C = 900mm;\ T_1 = 7kN; t = 10mm;\ \sigma_t = 4{,}5MPa$

1.1.1 for contact angle: (3)

$$\sin\beta = \frac{D_2 + D_1}{2c}$$

$$\therefore \beta = \sin^{-1}\frac{0{,}32+0{,}6}{2(0{,}9)} = \mathbf{30{,}738^\circ}$$

$$\therefore \theta = 180^\circ + 2\beta = 180^\circ + 2(30{,}738^\circ) = \mathbf{241{,}476^\circ}$$

1.1.2 for width of belt: (2)

$$T_1 = wt\sigma_t$$

$$\therefore w = \frac{T_1}{t\sigma_t} = \frac{7\times10^3}{(0{,}01)(4{,}5\times10^6)} = 0{,}155\,555m$$

$$Say\ \mathbf{w = 160mm}$$

1.1.3 for total length of belt: (3)

$$L = {}^1\!/_2\,\pi(D_1 + D_2 + 2t) + \frac{(D_1+D_2+2t)^2}{4C} + 2C$$

$$L = {}^1\!/_2\,\pi[0{,}32 + 0{,}6 + 2(0{,}01)] + \frac{[0{,}32+0{,}6+2(0{,}01)]^2}{4(0{,}9)} + 2(0{,}9)$$

$$L = 1{,}476\,6 + 0{,}245\,44 + 1{,}8$$

$$\mathbf{L = 3{,}522m}$$

1.1.4 for power transmitted by the belt: (7)

tension ratio: $\theta = \frac{241{,}476^\circ}{57{,}3} = 4{,}214\ radians$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{(0{,}4)(4{,}214)}$$

$$\therefore \boldsymbol{T_1 = 5{,}396\,T_2}$$

for T_2: $T_1 = 5{,}396T_2$

$$\therefore T_2 = \frac{T_1}{5{,}396} = \frac{7\,000}{5{,}396} = \boldsymbol{1\,297{,}257\ N}$$

for velocity: $v = \frac{\pi(D_2+t)N_2}{60} = \frac{\pi(0{,}32+0{,}01)(680)}{60} = \boldsymbol{11{,}75\ m/s}$

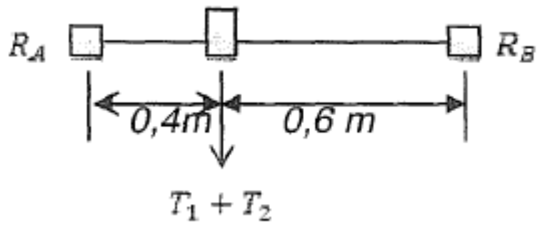
for power:

$$P = (T_1 - T_2)v = (7\,000 - 1\,297{,}257)(11{,}75) = 67\,007{,}23\ W$$

$$\therefore \boldsymbol{P = 67\ kW}$$

1.2 Data: $d_s = 80mm;\ L = 1m;\ \sigma_c = 2\ MPa$ (5)

Solution:



$$F_t = T_1 + T_2 = 7\,000 + 1\,297{,}257 = \boldsymbol{8\,297{,}257\ N}$$

for R_A: take moments about R_B

$$\sum CWM = \sum ACWM$$

$$R_A \times 1 = 8\,297{,}257 \times 0{,}6$$

$$\therefore \boldsymbol{R_A = 4\,978{,}354\ N}$$

for R_B: take moments about R_A

$$\sum CWM = \sum ACWM$$

$$8\,297{,}257 \times 0{,}4 = R_B \times 1$$

$$\therefore R_B = \mathbf{3\,318{,}903\ N}$$

$$\therefore \mathbf{maximum\ force\ R_A = 4\,978{,}354\ N}$$

For bearing length:

$$F = dL\sigma_c$$

$$\therefore L = \frac{F}{d\sigma_c} = \frac{4\,978{,}354}{(0{,}08)(2 \times 10^6)} = 0{,}031\,114\,713\ m = \mathbf{31mm}$$

[20]

QUESTION 2

Data:

$d = 2d; P = 650\ kW; N = 200r/min;\ T_{max} = 1{,}15\,T_{mean};\ \theta = 2{,}5°; L = 600\ mm; G = 89\ GPa$

2.1 for inside and outside diameter of hollow shaft (9)

$$P = \frac{2\pi N T_{mean}}{60}$$

$$\therefore T_{mean} = \frac{60P}{2\pi N} = \frac{60(650 \times 10^3)}{2\pi(200)} = \mathbf{31\,035{,}214\ N}$$

$$T_{max} = \mathbf{1{,}15}T_{mean} = \mathbf{1{,}15}(31\,035{,}214) = \mathbf{35\,690{,}496\ Nm}$$

$$\theta = \frac{584 T_{max} L}{G(D^4 - d^4)} \qquad where\ D^4 - d^4 = (2d^4) - d^4 = 15d^4$$

$$15d^4 = \frac{584 T_{max} L}{\theta G}$$

$$d = \sqrt[4]{\frac{584 T_{max} L}{15\theta G}} = \sqrt[4]{\frac{584(35\,690{,}496)(0{,}6)}{15(2{,}5)(89 \times 10^9)}} = 0{,}043\,997\ m$$

$$Say\ \mathbf{d = 44\ mm}$$

$$\therefore D = 2d = 2(44)$$

2.2 Data: $D = 124\,mm; d = 62\,mm$ (5)

For shaft diameter

$$A_s = A_h$$

$$\tfrac{1}{4}\pi d_s^2 = \tfrac{1}{4}\pi(D^2 - d^2)$$

$$\therefore d_s = \sqrt[2]{(D^2 - d^2)} = \sqrt[2]{(0{,}088)^2 - (0{,}044)^2} = 0{,}076\ 21\,m$$

$$\therefore \mathbf{d_s = 80\,mm}\ (standard\ size)$$

For strength

$$T_{hollow} = T_{solid}$$

$$d_s^3 = \frac{D^4 - d^4}{D}$$

$$(80)^3 = \frac{(88)^4 - (44)^4}{(88)}$$

$$512\,000 = 638\,880$$

$$\therefore Strength\ of\ hollow\ to\ solid\ shaft\ \ \mathbf{1:1{,}247\ 8} \qquad (80{,}141\%)$$

2.3 Data:

$n = 10; d = 60\,mm; h = 5\,mm; L = 70\,mm; \wp = 6\,MPa$

2.3.1 for power transmitted (4)

$$T = \wp A R_{mean} = \rho(nLh)\tfrac{1}{2}(D - h)$$

$$T = (6 \times 10^6)(10)(0{,}07)(0{,}005)\ \tfrac{1}{2}\ (0{,}06 - 0{,}005)$$

$$\therefore \mathbf{T = 577{,}5\ Nm}$$

$$P = \frac{2\pi NT}{60}$$

$$P = \frac{2\pi(2\ 750)(577{,}5)}{60}$$

2.3.2 for force required (2)

$$F_\mu = \mu \times perpendicular\ force$$

$$F_\mu = \mu \wp (nLh)$$

$$F_\mu = (0{,}3)(6 \times 10^6)(10)(0{,}07)(0{,}005)$$

$$\therefore \boldsymbol{F_\mu = 6\ 300\ N}$$

[20

QUESTION 3

Data:

$t = 15\ mm;\ \wp = 1{,}2\ MPa;\ \eta_L = 70\%;\ \eta_c = 30\%;\ \sigma_t = 105\ MPa;\ \tau = 75\ MPa;\ \sigma_c = 130\ MPa$

3.1 for diameter of shell

Consider longitudinal joint: $\sigma_c = \frac{\wp d}{2t\eta_L}$

$$\therefore d = \frac{2t\eta_l\sigma_c}{\wp} = \frac{2(0{,}015)(0{,}7)(105 \times 10^6)}{1{,}2 \times 10^6} = \mathbf{1{,}838\ m}$$

Consider circumferential joint: $\sigma_L = \frac{\wp d}{4t\eta_c}$

$$\therefore d = \frac{4t\eta_c\sigma_t}{\wp} = \frac{4(0{,}015)(0{,}3)(105 \times 10^6)}{(1{,}2 \times 10^6)} = 1{,}57$$

$$\therefore \boldsymbol{Use\ diameter\ of\ 1{,}575\ m}$$

3.2 for pitch of rivets

$$d = 6\sqrt{t} = 6\sqrt{15} = 23{,}237\ 9\ m = \mathbf{24\ mm}\ (standard\ size)$$

$$F_{tearing} = F_{shearing}$$

$$(P - d)t\sigma_t = 1{,}75n\left(\frac{1}{4}\pi d^2\right)\tau$$

$$\therefore P = \frac{1{,}75n\pi d^2\tau}{4t\sigma_t} + d = \frac{1{,}75(2)\pi(0{,}024)^2(75\times10^6)}{4(0{,}015)(105\times10^6)} = 0{,}099\ 398\ m = \mathbf{100\ mm}$$

3.3 for safe working load

Tearing strength:

$$F_t = (P-d)t\sigma_t = (0{,}1-0{,}024)(0{,}015)(105\times10^6) = \mathbf{119{,}7\ kN}$$

Shearing strength:

$$F_s = 1{,}75n\left(\tfrac{1}{4}\pi d^2\right)\tau = 1{,}75(2)\,\tfrac{1}{4}\,\pi(0{,}024)^2(75\times10^6) = \mathbf{118{,}752\ k}$$

Compressive strength:

$$F_c = ndt\sigma_c = 2(0{,}024)(0{,}015)(130\times10^6) = \mathbf{93{,}6\ kN}$$

$$\therefore \textbf{Safe load} = F_c = \mathbf{93{,}6\ kN}$$

[

QUESTION 4

Data:

$X = 350\ mm; Y = 310\ mm;\ \wp = 1{,}5MN/m^2;\ \sigma_t = 40MPa;\ M22$

4.1 for effective area of jointing material: (5)

$$a = X + 2t + 2d + 2\ clearance = 350 + 2(23) + 2(22) + 2(4) = 448\ mm$$

$$b = Z + 2t + 2d + 2\ clearance = 301 + 2(23) + 2(22) + 2(4) = 408\ mm$$

$$A = a\times b = (0{,}448)(0{,}408) = \mathbf{0{,}182\ 784m^2}$$

4.2 for force exerted on cover (2)

$$F = \wp A = (1{,}5\times10^6)(0{,}182\ 784) = \mathbf{274{,}176\ kN}$$

4.3 for number of bolts (3)

$$F = F_s = 0{,}8n\left(\tfrac{1}{4}\pi d^2\right)\sigma_t$$

$$\therefore n = \frac{4F}{0{,}8\pi d^2\sigma_t} = \frac{4(274{,}176\times10^3)}{0{,}8\pi(0{,}022)^2(40\times10^6)} = 22{,}539\ 5 = \mathbf{24\ bolts}$$

4.4 for pitch of bolts: (4)

$$L_x = \frac{X + 2t + 3d}{5} = \frac{350 + 2(23) + 3(22)}{5} = \mathbf{92,4\,mm}$$

$$L_y = \frac{Y + 2t + 3d}{4} = \frac{310 + 2(23) + 3(22)}{4} = \mathbf{105,5\,mm}$$

4.5 for steam tightness (2)

$1,5MN/m^2 \text{ is above } 1,125\ MPa \ \therefore \text{ pitch must fall between } 3d \text{ and } 5d$

$3d = 3(22) = \mathbf{66\,mm}$ and $5d = 5(22) = \mathbf{110\,mm}$

Yes, steam tight

4.6 for total length and breadth of cover (4)

$$Length = X + 2t + 6d = 350 + 2(23) + 6(22) = \mathbf{528\,mm}$$

$$Breadth = Y + 2t + 6d = 310 + 2(23) + 6(22) = \mathbf{488\,mm}$$

[20]

QUESTION 5

Data:

$F = 80\,kN;\ d = d_1 = 40\,mm;\ d_2 = 80\,mm;\ t = 30\,mm;\ t_1 = 15\,mm; SF = 5$

5.1.1 for ultimate tensile stress (4)

Consider rod $F = {}^1/_4 \pi d_1^2 \sigma_t$

$$\therefore \sigma_t = \frac{4F}{\pi d_1^2} = \frac{4(80 \times 10^3)}{\pi(0,04)^2} = 63,662 \times 5 = \mathbf{318,31\,MPa}$$

Consider eye: $F = (d_2 - d_1)t\sigma_t$

$$\therefore \sigma_t = \frac{F}{(d_2 - d_1)t} = \frac{(80 \times 10^3)}{(0,08 - 0,04)(0,03)} = 66,666\ 6 \times 5 = \mathbf{333,333 MPa}$$

Ultimate tensile stress = 318,31 MPa

5.1.2 for ultimate shear stress (2)

Consider pin: $F = 2\left(\frac{1}{4}\pi d^2\right)\tau$

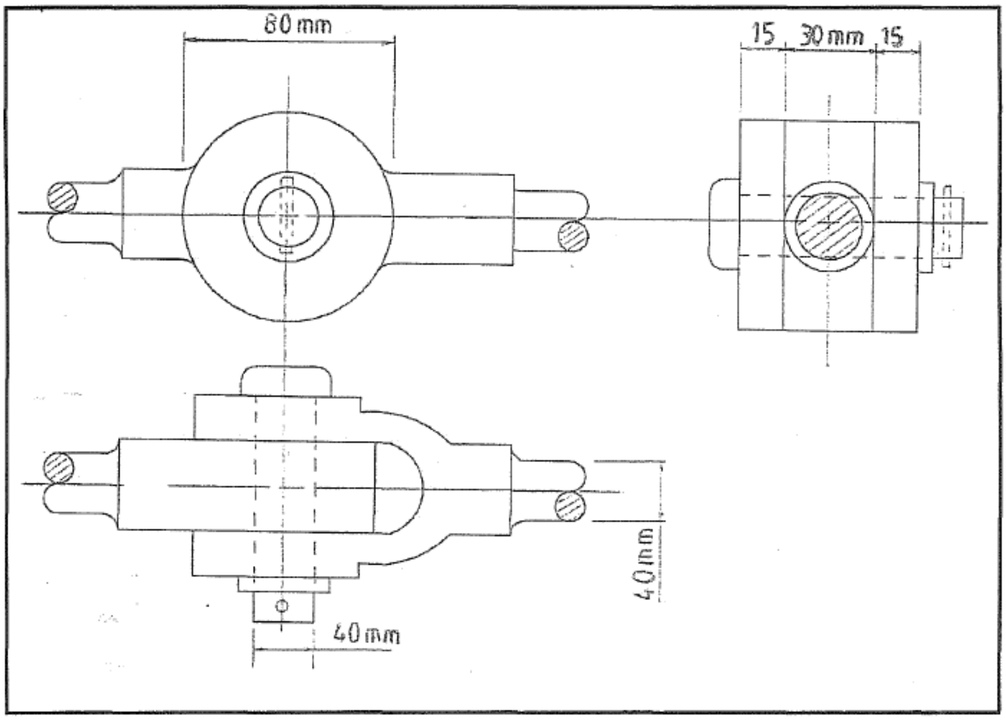
$$\therefore \tau = \frac{4F}{2\pi d_1^2} = \frac{4(80 \times 10^3)}{2\pi(0{,}04)^2} = 31{,}831 \times 5 = \mathbf{159{,}155\ MPa}$$

5.1.3 for ultimate compressive stress (2)

Consider eye: $F = d_1 t \sigma_c$

$$\therefore \sigma_c = \frac{F}{d_1 t} = \frac{(80 \times 10^3)}{(0{,}04)(0{,}03)} = 66{,}667 \times 5 = \mathbf{333{,}333\ MPa}$$

5.2 Sketch (6)



5.3 Purpose: Used to join two parts or rods, where one may have a small angular or axial movement about the other. (4)

Applications:

1. Machinery:valve mechanisms, reciprocating engines
2. Structures: suspension chains, bridges, roof trusses, braced girders
3. Joints of gearing and elevator chains

[18]

QUESTION 6

Data:
$t = 14\ mm; L = 150\ mm;\ \sigma = 80\ MPa$

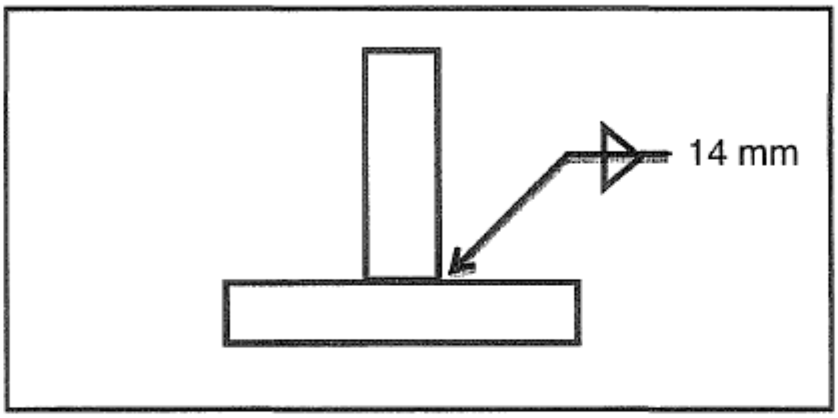
6.1 for safe load: (4)

$$t = 0{,}707 t_1 = 0{,}707(14) = \mathbf{9{,}898\ mm}$$

$$L_s = 2(L - 2t_1) = 2[150 - 2(14)] = \mathbf{244\ mm}$$

$$\therefore F = tL_s\sigma_C = (9{,}898 \times 10^{-3})(0{,}244)(80 \times 10^6) = \mathbf{193{,}209\ kN}$$

6.2 sketch (2)



[6]

TOTAL: 100

N5 Mechanical Drawing & Design is one of many publications introducing the gateways to Multi-Disciplinary Drawing Office Practice. This course is designed to develop the skills for learner draughtspersons to achieve their full potential in a draughting career.

This book, with its modular competence-based approach, is aimed at assisting facilitators and learners alike. With its comprehensive understanding of the engineering environment, it assists them to achieve the outcomes set for course.

The subject matter is presented as worked examples in the problem-solving-result methodology sequence, supported by numerous and clear illustrations. Practical activities are included throughout the book.

The author, Chris Brink, is well known and respected in the manufacturing, engineering and related technology fields. His extensive experience gives an excellent base for further study, as well as a broad understanding of technology and the knowledge to success.

HYBRID LEARNING SOLUTIONS

Other titles in the Gateway series are:

- NCOR Engineering Science
- N1 Engineering Science
- N2 Engineering Science
- N3 Engineering Science
- N4 Engineering Science

- NCOR Mathematics
- N1 Mathematics
- N2 Mathematics
- N3 Mathematics

- N1 Fitting and Machining
- N2 Fitting and Machining

- N3 Mechanotechnology

- N4 Machines & Properties of Metals

- NCOR Engineering Drawing
- N1 Engineering Drawing
- N2 Engineering Drawing
- N3 Engineering Drawing

- N4 General Draughting

- N4 Pictorial Draughting

- N4 Mechanical and Drawing Office Orientation

- N4 Mechanical Draughting

- N5 Building Draughting

- N5 Structural Steel Detailing

- N5 Electrical Draughting

- N5 Mechanical Drawing & Design

- N1 Electrical Trade Theory
- N2 Electrical Trade Theory

- N3 Electrotechnology

- N1 Refrigeration Trade Theory
- N2 Refrigeration Trade Theory
- N3 Refrigeration Trade Theory

- N1 Metalwork Theory

- N2 Welder's Theory

- N1 Rigging Theory
- N2 Rigging Theory

- N1 Plating & Structural Steel Drawing
- N2 Plating & Structural Steel Drawing
- N3 Plating & Structural Steel Drawing
- N4 Plating & Structural Steel Drawing

- N4 Machines & Properties of Metals

- NCOR Industrial Communication

- N1 Industrial Electronics
- N2 Industrial Electronics
- N3 Industrial Electronics

- N1 Motor Trade Theory
- N2 Motor & Diesel Trade Theory
- N3 Motor & Diesel Trade Theory

- N3 Supervision in the Industry
- N4 Supervisory Management
- N5 Supervisory Management

- N3 Industrial Organisation & Planning

- N1 Water & Wastewater Treatment Practice
- N2 Water & Wastewater Treatment Practice
- N3 Water Treatment Practice
- N3 Wastewater Treatment Practice

- N1 Plant Operation Theory
- N2 Plant Operation Theory
- N3 Plant Operation Theory

- N4 Chemical Plant Operation
- N5 Chemical Plant Operation
- N6 Chemical Plant Operation

- N4 Chemistry
- N5 Chemistry

- N6 Chemical Technology

- N1 Plumbing
- N2 Plumbing

- N1 Woodworker's Theory

- N2 Carpentry & Roofwork Theory

- N4 Building & Structural Construction
- N5 Building & Structural Construction
- N6 Building & Structural Construction

- N4 Building & Structural Surveying
- N5 Building & Structural Surveying
- N6 Building & Structural Surveying

- N4 Building Administration
- N5 Building Administration
- N6 Building Administration

Published by
Hybrid Learning Solutions (Pty) Ltd

Orders: urania@hybridlearning.co.za

9 781928 203971